Online Appendices for Pandering to Persuade

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For convenience, we recall from the main text the following equilibrium conditions:

$$\mu_i(\mathbf{b}) = 1 \text{ if } q_i b_i > q_{-i} b_{-i} \tag{1}$$

$$q_i > 0 \implies \mathbb{E}\left[b_i \mid q_i b_i \geq q_{-i} b_{-i}\right] \geq \max\left\{b_0, \mathbb{E}\left[b_{-i} \mid q_i b_i \geq q_{-i} b_{-i}\right]\right\}, \tag{2}$$

$$q_i = 1 \impliedby \mathbb{E}\left[b_i \mid q_i b_i \geq q_{-i} b_{-i}\right] > \max\left\{b_0, \mathbb{E}\left[b_{-i} \mid q_i b_i \geq q_{-i} b_{-i}\right]\right\}. \tag{3}$$

Also recall the definition of strong ordering:

Definition 1. The two projects are **strongly ordered** if

$$\mathbb{E}[b_1|b_1 > b_2] > \mathbb{E}[b_2|b_2 > b_1], \tag{R1}$$

and, for any $i \in \{1, 2\}$,

$$\mathbb{E}[b_i|b_i > \alpha b_{-i}] \text{ is nondecreasing in } \alpha \text{ for } \alpha \in (0, \overline{b}_i/\underline{b}_{-i}). \tag{R2}$$

B Omitted Proofs

This Appendix provides proofs that were omitted from Appendix A. For convenience, we restate the relevant results before providing their proofs.

Lemma 1. *Fix generic distributions (F_1, F_2) and a generic outside option b_0. Then any equilibrium is outcome-equivalent to one in which: (i) the agent plays a pure strategy whose range*

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consists of at most two messages; (ii) the DM's strategy is such that following any message m, if project $i \in \{1, 2\}$ is chosen with positive probability then project $-i$ is chosen with zero probability.

Proof. Here, we will prove part (ii) of the lemma. Part (i) then follows from Lemma 8 in Appendix D. For any $m \in M$, let $\alpha(i|m)$ is the probability that the DM chooses project i following message m. As we are interested in outcome-equivalence, we can ignore the behavior of any zero measure set of agent types.

Suppose there is an on-path message m^* such that $\min\{\alpha(1|m^*), \alpha(2|m^*)\} > 0$. (If m^* does not exist, we are done.) We can assume that there is some other on-path message that induces a different action distribution from the DM, because otherwise $\mathbb{E}[b_1] = \mathbb{E}[b_2]$, which is non-generic. Moreover, we can assume that no on-path message leads to the outside option with probability 1, since the agent will never (except possibly for a zero measure of types) use such a message given the availability of m^*.

Step 1: There exist constants $q_1 > \alpha(1|m^*)$ and $q_2 > \alpha(2|m^*)$ such that for any on-path message m, either(i) $\alpha(m) = \alpha(m^*)$, or (ii) $\alpha(1|m) = 0$ and $\alpha(2|m) = q_2$, or (iii) $\alpha(1|m) = q_1$ and $\alpha(2|m) = 0$.

To prove this, suppose m is on path and $\alpha(m) \neq \alpha(m^*)$. We cannot have the agent strictly prefer m^* to m or vice-versa independent of his type, so suppose $\alpha(1|m) > \alpha(1|m^*)$ and $\alpha(2|m^*) > \alpha(2|m)$, with the opposite case treated symmetrically below. Then m^* will be used by the agent only if

$$b_1\alpha(1|m^*) + b_2\alpha(2|m^*) \geq b_1\alpha(1|m) + b_2\alpha(2|m),$$

or $b_2 \geq b_1 k$, where $k := \frac{\alpha(1|m)-\alpha(1|m^*)}{\alpha(2|m^*)-\alpha(2|m)}$. If $k \geq 1$, $\mathbb{E}[b_2|m^*] > \mathbb{E}[b_1|m^*]$, which cannot be, hence $k < 1$. Analogously, message m will be used by the agent only if $b_1 \geq \frac{b_2}{k}$. Since $k < 1$, $\mathbb{E}[b_2|m] < \mathbb{E}[b_1|m]$, which implies that $\alpha(2|m) = 0 < \alpha(1|m)$.

A symmetric argument applies to the case of $\alpha(1|m) < \alpha(1|m^*)$ and $\alpha(2|m^*) < \alpha(2|m)$, establishing that in this case $\alpha(2|m) > 0 = \alpha(1|m)$.

Finally, note that all on-path messages that lead to (possibly degenerate) randomization between project 1 and the outside option must put the same probability on project 1, call it q_1, and this must be strictly larger than $\alpha(1|m^*)$ — otherwise they would not be used (except possibly by a zero measure of types). Analogously for project 2 and the outside option.

Step 2: Suppose there is an on-path message m_1 such that $\alpha(1|m_1) = q_1$ and an on-path message m_2 such that $\alpha(2|m_2) = q_2$. We cannot have $q_1 = q_2 = 1$, for then only at most a zero measure of types will induce randomization from the DM. So suppose $q_1 = 1 > q_2$. Then m^* will only be used by types such that $b_2 > b_1$, contradicting $\mathbb{E}[b_2|m^*] = \mathbb{E}[b_1|m^*]$. Similarly for

$q_1 < 1 = q_2$. Therefore, $\max\{q_1, q_2\} < 1$, which implies

$$\mathbb{E}[b_1|m_1] = \mathbb{E}[b_2|m_2] = b_0. \tag{6}$$

Since m_1 is used by the agent when $\frac{q_1-\alpha(1|m^*)}{\alpha(2|m^*)}b_1 \geq b_2$, and m_2 is used by the agent when $\frac{\alpha(1|m^*)}{q_2-\alpha(2|m^*)}b_1 \leq b_2$, we can visualize the $b_1 - b_2$ rectangle as being partitioned into three regions by the two line segments $b_2 = xb_1$ and $b_2 = yb_1$ where $y := \frac{q_1-\alpha(1|m^*)}{\alpha(2|m^*)}$ and $x := \frac{\alpha(1|m^*)}{q_2-\alpha(2|m^*)}$. Message m_1 (or other messages that lead to the same distribution of projects) is used in the bottom region, m^* (or messages that lead to the same distribution of projects) is used in the middle region, and m_2 (or messages that lead to the same distribution of projects) in the top region. By the genericity of the prior distributions, there are at most a countable number of (x, y) that can satisfy $\mathbb{E}[b_1|m^*] = \mathbb{E}[b_2|m^*]$. But then, only non-generic b_0 satisfy (6).

Step 3: Suppose that any on-path message m with $\alpha(m) \neq \alpha(m^*)$ has $\alpha(2|m) = 0$. (A symmetric argument applies to the other case where $\alpha(1|m) = 0$.) Then there is some m_1 with $\alpha(1|m_1) = q_1$. We must have $q_1 < 1$ because otherwise the agent will use m_1 whenever $b_1 \geq b_2$, contradicting $\mathbb{E}[b_1|m^*] = \mathbb{E}[b_2|m^*]$. Thus $\mathbb{E}[b_1|m_1] = b_0$. But now analogously to step 2, we can view the type space as partitioned into two regions by a line segment $b_2 = \frac{q_1-\alpha(1|m^*)}{\alpha(2|m^*)}b_1$, with message m_1 (or others that lead to the same distribution over projects) being used in the lower cone and m^* (or other messages that lead to the same distribution over projects) being used in the upper cone. By the genericity of prior distributions, there are most a countable number of values of $y := \frac{q_1-\alpha(1|m^*)}{\alpha(2|m^*)}$ that can satisfy $\mathbb{E}[b_1|m^*] = \mathbb{E}[b_2|m^*]$. But then, only non-generic b_0 can also satisfy $\mathbb{E}[b_1|m_1] = b_0$. *Q.E.D.*

Lemma 2. *If an equilibrium has acceptance vector $\mathbf{q} \in [0,1]^2$, then (2) and (3) are satisfied for all projects i such that $\Pr\{\mathbf{b} : q_i b_i \geq q_{-i}b_{-i}\} > 0$. Conversely, for any $\mathbf{q} \in [0,1]^2$ satisfying (2) and (3) for all i such that $\Pr\{\mathbf{b} : q_i b_i \geq q_{-i}b_{-i}\} > 0$, there is an equilibrium where the DM plays $\mathbf{q}$ and the agent's strategy satisfies (1).*

Proof. The first statement is immediate. For sufficiency, fix any $\mathbf{q}$ satisfying (2) and (3) for all i with $\Pr\{\mathbf{b} : q_i b_i \geq q_{-i}b_{-i}\} > 0$. We consider two cases:

(i) Suppose first there is some i with $q_i > 0$. Then the agent has a best response, μ, that satisfies (1) and also has the property that any project that is recommended on path has positive ex-ante probability of being recommended. Such a μ and $\mathbf{q}$ are mutual best responses and Bayes Rule is satisfied. The only issue is assigning an appropriate out-of-equilibrium belief when any off-path project j is recommended; one can specify the off-path belief that $b_k = b_0$ for all k, which clearly rationalizes q_j.

(ii) Now suppose $q_i = 0$ for all i. Then for all i, $\Pr\{\mathbf{b} : q_i b_i \geq q_{-i}b_{-i}\} = 1$ and $\mathbb{E}[b_i \mid q_i b_i \geq q_{-i}b_{-i}] = \mathbb{E}[b_i]$. It follows from (3) that for all i, $\mathbb{E}[b_i] \leq b_0$, and hence there is an equilibrium where the DM always chooses the outside option with "passive beliefs"of maintaining the prior no matter the recommendation, and the agent always recommends project one. *Q.E.D.*

Lemma 4. *If $(\overline{x}, \overline{y})$ is an incentive compatible mechanism, then for all θ, $(\overline{x}(\theta, b_2), \overline{y}(\theta, b_2)) = (\overline{x}(\theta, b_2'), \overline{y}(\theta, b_2'))$ for any $b_2 \neq b_2'$.*

Proof. Consider two types (θ, b_2) and (θ, b_2'). Then, incentive compatibility means that

$$b_2[\theta\overline{x}(\theta, b_2) + \overline{y}(\theta, b_2)] \geq b_2[\theta\overline{x}(\theta, b_2') + \overline{y}(\theta, b_2')]$$

and

$$b_2'[\theta\overline{x}(\theta, b_2) + \overline{y}(\theta, b_2)] \leq b_2'[\theta\overline{x}(\theta, b_2') + \overline{y}(\theta, b_2')],$$

implying that $\theta[\overline{x}(\theta, b_2) + \overline{y}(\theta, b_2)] = \theta\overline{x}(\theta, b_2') + \overline{y}(\theta, b_2')$. It follows that if $\overline{x}(\theta, b_2) = \overline{x}(\theta, b_2')$, then $\overline{y}(\theta, b_2) = \overline{y}(\theta, b_2')$. It thus suffices to show that for every θ, $\overline{x}(\theta, b_2) = \overline{x}(\theta, b_2')$ for any $b_2, b_2' \in [\underline{b}_2, \overline{b}_2]$. To prove this, consider a correspondence $X : \Theta \rightrightarrows [0, 1]$ defined by

$$X(\theta) = \{x \in [0, 1] \mid \exists b_2 \in [\underline{b}_2, \overline{b}_2] \text{ s.t. } \overline{x}(\theta, b_2) = x\}.$$

Pick any selection $\hat{x}(\cdot)$ from $X(\cdot)$, and for any θ, let $\hat{y}(\theta)$ be corresponding value of $\overline{y}$, i.e. $\hat{y}(\theta) := \overline{y}(\theta, b_2)$ for any b_2 such that $\overline{x}(\theta, b_2) = \hat{x}(\theta)$. For any θ' and θ, incentive compatibility implies

$$\theta\hat{x}(\theta) + \hat{y}(\theta) \geq \theta\hat{x}(\theta') + \hat{y}(\theta') \text{ and } \theta'\hat{x}(\theta') + \hat{y}(\theta') \geq \theta'\hat{x}(\theta) + \hat{y}(\theta).$$

Rearranging the inequalities yields

$$\theta'[\hat{x}(\theta') - \hat{x}(\theta)] \geq \hat{y}(\theta) - \hat{y}(\theta') \geq \theta[\hat{x}(\theta') - \hat{x}(\theta)].$$

As $\theta \to \theta'$, it follows that $\hat{y}(\theta) \to \hat{y}(\theta')$, which in turn implies that $\hat{x}(\theta) \to \hat{x}(\theta')$.

Since the selection $\hat{x}(\cdot)$ was arbitrary, $X(\cdot)$ must be a single-valued correspondence, which proves the result. *Q.E.D.*

Theorem 6. *Assume the two projects are strongly ordered and that $J(\cdot)$ is piecewise monotone. If the best cheap-talk equilibrium is $\mathbf{q}^* < \mathbf{1}$, then an optimal simple mechanism is optimal in the class of all mechanisms without transfers.*

Proof. In light of Lemma 4, we can without loss focus on a direct revelation mechanism $(x, y) : \Theta \to A$; in other words, treat the agent's type as just θ. Let Ω be the set of such mappings. Recall the DM's net benefits from choosing the two projects:

$$A_1(\theta) := \mathbb{E}[b_1|b_1/b_2 = \theta] - b_0, \text{ and } A_2(\theta) := \mathbb{E}[b_2|b_1/b_2 = \theta] - b_0.$$

Note that the assumption that the best cheap-talk equilibrium is $\mathbf{q}^* < \mathbf{1}$ implies, from Theorem 1, that $\mathbb{E}[A_2(\theta)] < 0$.

To begin the analysis, assume $\overline{\theta} < \infty$; the case of $\overline{\theta} = \infty$ is treated later. Define the utility of the agent with type θ when she reports θ' as $U(\theta'|\theta) := \theta x(\theta') + y(\theta')$. Let $u(\theta) := U(\theta|\theta)$. Notice the resemblance with standard mechanism design with transfers: here, the probability of choosing project 2 serves as a transfer. The analogy is not perfect since $y(\theta) \in [0, 1 - x(\theta)]$ need to be satisfied. This difference makes the subsequent analysis more involved than in the standard mechanism design exercise.

By the standard argument, incentive compatibility holds if and only if

$$u(\theta) = u(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} x(\tilde{\theta}) d\tilde{\theta}, \tag{Env}$$

and

$$x(\cdot) \text{ is nondecreasing.} \tag{M}$$

Therefore, the DM's problem is:

$$\max_{(x,y)\in\Omega} \int_{\underline{\theta}}^{\overline{\theta}} [A_1(\theta)x(\theta) + A_2(\theta)y(\theta)] f(\theta) d\theta \tag{P_0}$$

subject to (x, y) satisfies (Env) and (M).

To solve this problem, we first substitute (Env) into the objective function in (P_0). Rewrite (Env) as:

$$y(\theta) = -\theta x(\theta) + u(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} x(s) ds. \tag{7}$$

Substituting (7) into the objective function in (P_0) yields:

$$\begin{aligned}
& \int_{\underline{\theta}}^{\overline{\theta}} \left[A_1(\theta)x(\theta) - A_2(\theta) \left(\theta x(\theta) - u(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} x(s) ds \right) \right] f(\theta) d\theta \\
= & \left(\int_{\underline{\theta}}^{\overline{\theta}} A_2(\theta) f(\theta) d\theta \right) u(\underline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(A_1(\theta) - A_2(\theta)\theta \right) x(\theta) + A_2(\theta) \int_{\underline{\theta}}^{\theta} x(s) ds \right] f(\theta) d\theta \\
= & \left(\int_{\underline{\theta}}^{\overline{\theta}} A_2(\theta) f(\theta) d\theta \right) u(\underline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}} \left[-(1-\theta) b_0 x(\theta) + A_2(\theta) \int_{\underline{\theta}}^{\theta} x(s) ds \right] f(\theta) d\theta \\
= & \left(\int_{\underline{\theta}}^{\overline{\theta}} A_2(\theta) f(\theta) d\theta \right) u(\underline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}} \left[-(1-\theta) b_0 f(\theta) + \int_{\theta}^{\overline{\theta}} A_2(s) f(s) ds \right] x(\theta) d\theta,
\end{aligned} \tag{8}$$

where the second equality follows from the observation that

$$A_1(\theta) - A_2(\theta)\theta = \mathbb{E}[b_1 - \theta b_2 | b_1/b_2 = \theta] - (1-\theta) b_0 = -(1-\theta) b_0, \tag{9}$$

and the third equality follows from an application of Fubini's theorem.[1]

Recall the definition of the "virtual valuation":

$$J(\theta) := -(1-\theta)b_0 f(\theta) + \int_{\theta}^{\overline{\theta}} \left(\mathbb{E}\left[b_2 \middle| \frac{b_1}{b_2} = s\right] - b_0\right) f(s)ds.$$

We can thus rewrite the objective function in (P_0) as

$$\mathbb{E}[A_2(\theta)](\underline{\theta}x(\underline{\theta}) + y(\underline{\theta})) + \int_{\underline{\theta}}^{\overline{\theta}} J(\theta)x(\theta)d\theta.$$

We now recall the constraint that for all θ, $(x(\theta), y(\theta)) \in A$, which given $x(\cdot) \in [0,1]$ is equivalent to requiring that for all θ, $y(\theta) \in [0, 1 - x(\theta)]$. In what follows, we solve a relaxed program by only imposing $y(\cdot) \geq 0$ and $y(\underline{\theta}) \leq 1$; we will show that the solution to this relaxed program is such that for all θ, $y(\theta) \in [0, 1 - x(\theta)]$ and hence solves the original program.

Using (7) and $u(\underline{\theta}) = \underline{\theta}x(\underline{\theta}) + y(\underline{\theta})$, the constraint that $y(\cdot) \geq 0$ can be expressed as

$$\forall \theta \ : \ -\theta x(\theta) + \underline{\theta}x(\underline{\theta}) + y(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} x(s)ds \geq 0. \tag{10}$$

Since (M) implies that the left-hand side of the inequality above is nonincreasing in θ,[2] the constraint will be satisfied for all θ if it is satisfied at $\theta = \overline{\theta}$. Hence, (10) can be replaced with

$$\overline{\theta}x(\overline{\theta}) \leq \underline{\theta}x(\underline{\theta}) + y(\underline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}} x(s)ds. \tag{11}$$

Therefore, the original program (P_0) can be replaced by the relaxed program[3]

$$\max_{y(\underline{\theta})\in[0,1], x(\cdot)\in[0,1]} \mathbb{E}[A_2(\theta)](\underline{\theta}x(\underline{\theta}) + y(\underline{\theta})) + \int_{\underline{\theta}}^{\overline{\theta}} J(\theta)x(\theta)d\theta \tag{P_0'}$$

[1]Note that

$$\begin{aligned}\int_{\underline{\theta}}^{\overline{\theta}} \left[A_2(\theta)\int_{\underline{\theta}}^{\theta} x(s)ds\right] f(\theta)d\theta &= \int_{\underline{\theta}}^{\overline{\theta}}\int_{\underline{\theta}}^{\overline{\theta}} A_2(\theta)x(s)\mathbb{1}_{\{s<\theta\}}ds f(\theta)d\theta = \int_{\underline{\theta}}^{\overline{\theta}} x(s)\left[\int_{\underline{\theta}}^{\overline{\theta}} A_2(\theta)\mathbb{1}_{\{s<\theta\}}f(\theta)d\theta\right]ds \\ &= \int_{\underline{\theta}}^{\overline{\theta}}\left(\int_{s}^{\overline{\theta}} A_2(\theta)f(\theta)d\theta\right)x(s)ds = \int_{\underline{\theta}}^{\overline{\theta}}\left(\int_{\theta}^{\overline{\theta}} A_2(s)f(s)ds\right)x(\theta)d\theta.\end{aligned}$$

[2]For any $\theta < \theta'$, $-\theta x(\theta) + \underline{\theta}x(\underline{\theta}) + y(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} x(s)ds - \left[-\theta' x(\theta') + \underline{\theta}x(\underline{\theta}) + y(\underline{\theta}) + \int_{\underline{\theta}}^{\theta'} x(s)ds\right] = \theta' x(\theta') - \theta x(\theta) - \int_{\theta}^{\theta'} x(s)ds$, which is non-negative when $x(\theta') \geq x(\theta)$.

[3]Recall, this is relaxed because we are ignoring the constraint that for all θ, $y(\theta) \leq 1 - x(\theta)$.

subject to (M) and (11).

Lemma 5. *In any optimal solution to* (P_0')*, the constraint* (11) *binds.*

Proof. Suppose, to contradiction, that there is an optimum at which (11) does not bind. Then (M) and (11) imply that $y(\underline{\theta}) > 0$. Since $\mathbb{E}[A_2(\theta)] < 0$, slightly reducing $y(\underline{\theta})$ would lead to a strict improvement in the value of the objective in (P_0') while still satisfying all the constraints, a contradiction. *Q.E.D.*

Lemma 5 implies that at any optimal solution to (P_0'),

$$y(\underline{\theta}) = \overline{\theta}x(\overline{\theta}) - \int_{\underline{\theta}}^{\overline{\theta}} x(\theta)d\theta - \underline{\theta}x(\underline{\theta}), \tag{12}$$

and hence the program simplifies to:

$$\max_{x(\cdot)\in[0,1]} \mathbb{E}[A_2(\tilde{\theta})]\overline{\theta}x(\overline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}} \Big(J(\theta) - \mathbb{E}[A_2(\tilde{\theta})]\Big)\, x(\theta)d\theta \tag{P}$$

subject to (M) and

$$\overline{\theta}x(\overline{\theta}) - \underline{\theta}x(\underline{\theta}) - \int_{\underline{\theta}}^{\overline{\theta}} x(\theta)d\theta \leq 1. \tag{13}$$

Lemma 6. *For some some integer $n > 1$, there exist sequences*

$$\underline{\theta} = \hat{\theta}_1 < \cdots < \hat{\theta}_n < \hat{\theta}_{n+1} = \overline{\theta} \text{ and } 0 \leq \hat{x}_1 \leq \cdots \leq \hat{x}_n \leq 1$$

such that an optimal solution to (P) *is given by $\hat{x}(\cdot)$ is defined as: $\hat{x}(\underline{\theta}) = \hat{x}_1$, and for all $i = 1, \ldots, n$, $\hat{x}(\theta) = \hat{x}_i$ for $\theta \in (\hat{\theta}_i, \hat{\theta}_{i+1}]$.*

Proof. Existence of an optimal solution to (P) is assured by compactness of the feasible set, so let $x(\cdot)$ be an optimal solution to (P). Since $J(\cdot)$ is piecewise monotone, we can partition $[\underline{\theta}, \overline{\theta}]$ into subintervals $\{[\theta_i, \theta_{i+1}]\}_{i=1,\ldots,m}$ for some $m \in \mathbb{N}$, such that $J(\theta)$ is either nondecreasing or nonincreasing within each subinterval. By (M), $x(\theta_i) \leq x(\theta_{i+1})$. We then construct $\hat{x}(\cdot)$ for each subinterval $(\theta_i, \theta_{i+1}]$ as follows. There are two cases.

Suppose first $J(\theta)$ is nondecreasing on $[\theta_i, \theta_{i+1}]$. Then, we set $\hat{x}(\theta) = x(\theta_i)$ for $\theta \in (\theta_i, \hat{\theta}_i)$ and $\hat{x}(\theta) = x(\theta_{i+1})$ for $\theta \in [\hat{\theta}_i, \theta_{i+1}]$, for $\hat{\theta}_i \in [\theta_i, \theta_{i+1}]$ such that and

$$\int_{\theta_i}^{\theta_{i+1}} \hat{x}(\theta)d\theta = \int_{\theta_i}^{\theta_{i+1}} x(\theta)d\theta.$$

Such a $\hat{\theta}_i$ exists because the LHS of the above equation is continuous in $\hat{\theta}_i$, and is no less than then RHS when $\hat{\theta}_i = \theta_i$ while being no greater than the RHS when $\hat{\theta}_i = \theta_{i+1}$. (If $i = 1$, we also set $\hat{x}(\theta_1) = \lim_{\theta \downarrow \theta_1} \hat{x}(\theta)$.) In this case, changing from $x(\cdot)$ to $\hat{x}(\cdot)$ can only increase the integral term in the objective function of (P) for the subinterval since

$$\begin{aligned} &\int_{\theta_i}^{\theta_{i+1}} \left(J(\theta) - \mathbb{E}[A_2(\tilde{\theta})]\right) \hat{x}(\theta) d\theta - \int_{\theta_i}^{\theta_{i+1}} \left(J(\theta) - \mathbb{E}[A_2(\tilde{\theta})]\right) x(\theta) d\theta \\ = &\int_{\theta_i}^{\theta_{i+1}} J(\theta)(\hat{x}(\theta) - x(\theta)) d\theta \ \geq \ \int_{\theta_i}^{\theta_{i+1}} J(\hat{\theta}_i)(\hat{x}(\theta) - x(\theta)) d\theta \ = \ J(\hat{\theta}_i) \left[\int_{\theta_i}^{\theta_{i+1}} (\hat{x}(\theta) - x(\theta)) d\theta\right] = 0, \end{aligned}$$

where the weak inequality follows from the facts that for $\theta \geq \hat{\theta}_i$, $J(\theta) \geq J(\hat{\theta}_i)$ and $\hat{x}(\theta) \geq x(\theta)$, while for $\theta < \hat{\theta}_i$, $J(\theta) \leq J(\hat{\theta}_i)$ and $\hat{x}(\theta) \leq x(\theta)$, and the final equality follows from $\int_{\theta_i}^{\theta_{i+1}} \hat{x}(s) ds = \int_{\theta_i}^{\theta_{i+1}} x(s) ds$. Furthermore, in case $i = m$, the fact that $\theta_{i+1} = \overline{\theta}$ and $\hat{x}(\theta_{i+1}) = x(\theta_{i+1})$ implies that the first term of the objective function of (P) is unchanged.

Suppose next $J(\theta)$ is nonincreasing on $[\theta_i, \theta_{i+1}]$. Then, we set $\hat{x}(\theta) = \check{x}_i$ for all $\theta \in (\theta_i, \theta_{i+1}]$, for $\check{x}_i \in [x(\theta_i), x(\theta_{i+1})]$ such that and

$$\int_{\theta_i}^{\theta_{i+1}} \hat{x}(\theta) d\theta = \int_{\theta_i}^{\theta_{i+1}} x(\theta) d\theta.$$

Clearly, such a $\check{x}_i$ exists. (If $i = 1$, we also set $\hat{x}(\theta_i) = \check{x}_i$.) Again,changing from $x(\cdot)$ to $\hat{x}(\cdot)$ can only increase the integral term in the objective function of (P) for the subinterval since, denoting $\tilde{\theta}_i := \inf\{\theta \in [\theta_i, \theta_{i+1}] | x(\theta) \geq \hat{x}(\theta) = \check{x}_i\}$,

$$\begin{aligned} &\int_{\theta_i}^{\theta_{i+1}} \left(J(\theta) - \mathbb{E}[A_2(\tilde{\theta})]\right) \hat{x}(\theta) d\theta - \int_{\theta_i}^{\theta_{i+1}} \left(J(\theta) - \mathbb{E}[A_2(\tilde{\theta})]\right) x(\theta) d\theta \\ = &\int_{\theta_i}^{\theta_{i+1}} J(\theta)(\hat{x}(\theta) - x(\theta)) d\theta \ \geq \ \int_{\theta_i}^{\theta_{i+1}} J(\tilde{\theta}_i)(\hat{x}(\theta) - x(\theta)) d\theta \ = \ J(\tilde{\theta}_i) \left[\int_{\theta_i}^{\theta_{i+1}} (\hat{x}(\theta) - x(\theta)) d\theta\right] = 0, \end{aligned}$$

where the weak inequality holds since for $\theta \geq \tilde{\theta}_i$, $J(\theta) \leq J(\tilde{\theta}_i)$ and $\hat{x}(\theta) = \check{x}_i \leq x(\theta)$, while for $\theta < \tilde{\theta}_i$, $J(\theta) \leq J(\tilde{\theta}_i)$ and $\hat{x}(\theta) = \check{x}_i \geq x(\theta)$, and the final equality follows from $\int_{\theta_i}^{\theta_{i+1}} \hat{x}(s) ds = \int_{\theta_i}^{\theta_{i+1}} x(s) ds$. Furthermore, in case $i = m$, the fact that $\hat{x}(\theta_{i+1}) = \check{x} \leq x(\theta_{i+1})$ implies the first term of the objective function of (P) can have only weakly increased since $\mathbb{E}[A_2(\tilde{\theta})] < 0$.

Clearly, the $\hat{x}(\cdot)$ constructed above for all subintervals is of the form stated in the lemma.[4] Moreover, $\hat{x}(\cdot)$ satisfies (M) because $x(\cdot)$ satisfies (M) by hypothesis. Further, the facts that $\hat{x}(\overline{\theta}) \leq x(\overline{\theta})$, $\hat{x}(\underline{\theta}) \geq x(\underline{\theta})$, and $\int_{\underline{\theta}}^{\overline{\theta}} \hat{x}(\theta) d\theta = \int_{\underline{\theta}}^{\overline{\theta}} x(\theta) d\theta$ imply that $\hat{x}(\cdot)$ satisfies (13) since $x(\cdot)$ does by hypothesis. As we have shown that $\hat{x}(\cdot)$ can only increase the value of the objective function, it follows that $\hat{x}$ is an optimal solution to program (P). *Q.E.D.*

Therefore, we can without loss restrict attention in solving program (P) to step functions

[4]That $n > 1$ in the statement of the lemma can always be satisfied is because the sequence of $\hat{x}_i$'s is not required to be strictly increasing.

that take the form described in Lemma 6. Simplifying both the objective function in (P) and the constraint (13) accordingly, and using the fact that the proof of Lemma 6 bounds the number of steps an optimal solution need take, it follows that there is some integer $N > 1$ such that program (P) can be simplified to:

$$\max_{(\hat{\boldsymbol{x}},\hat{\boldsymbol{\theta}})\in A^n, n\in\{2,\ldots,N\}} \sum_{i=1}^{n-1}\left(\int_{\hat{\theta}_i}^{\hat{\theta}_{i+1}}(J(\theta)-\mathbb{E}[A_2(\tilde{\theta})])d\theta\right)\hat{x}_i + \left(\hat{\theta}_n\mathbb{E}[A_2(\tilde{\theta})] + \int_{\hat{\theta}_n}^{\overline{\theta}} J(\theta)d\theta\right)\hat{x}_n, \qquad (P')$$

where for any $n \in \mathbb{N} \setminus \{1\}$,

$$A^n := \left\{(\boldsymbol{x},\boldsymbol{\theta}) \in [0,1]^n \times \Theta^n : x_1 \leq \cdots \leq x_n,\ \underline{\theta} = \theta_1 \leq \cdots \leq \theta_n \leq \overline{\theta},\ \sum_{i=1}^{n-1}\theta_{i+1}(x_{i+1}-x_i) \leq 1\right\}.$$

Lemma 7. *Program (P') has a solution in A^2.*

Proof. Assume, to contradiction, that there is no solution to the program in A^2. Then a solution exists in A^n for some $n > 2$ because the feasible set is compact. We first argue that if there is any solution in A^n for some $n > 3$, then there is a solution in A^{n-1}, which implies by induction that there is a solution in A^3. To prove the inductive step, fix a solution to (P'), $(\hat{\boldsymbol{x}},\hat{\boldsymbol{\theta}}) \in A^n$ for some $n > 3$.

Case 1: If $\hat{x}_i = \hat{x}_{i+1}$ for some $1 \leq i < n$ or $\hat{\theta}_i = \hat{\theta}_{i+1}$ for some $1 \leq i \leq n$, then there is clearly an equivalent solution in A^{n-1}.

Case 2: If Case 1 does not apply, then $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{\theta}}$ are strictly increasing sequences. We can then change $\hat{x}_{n-1}$ and $\hat{x}_{n-2}$ in opposite directions (raising one and lowering the other) while keeping $\sum_{i=1}^{n-1}\hat{\theta}_{i+1}(\hat{x}_{i+1}-\hat{x}_i)$ constant and hence staying within A^n (note that $n > 3$ ensures that x_{n-2} can be lowered). Since the objective function in (P') is linear in each $\hat{x}_i$, it must be one of these two kinds of changes does not affect the value of the objective function. One can then continue making the change until either $\hat{x}_{n-1} = \hat{x}_n$ or $\hat{x}_{n-1} = \hat{x}_{n-2}$ or $\hat{x}_{n-2} = \hat{x}_{n-3}$ binds, at which point Case 1 applies.

Therefore, we conclude that there is a solution to (P'), $(\hat{\boldsymbol{x}},\boldsymbol{\theta}) \in A^3$. Since by hypothesis there is no solution in A^2, it must be that $\underline{\theta} = \hat{\theta}_1 < \hat{\theta}_2 < \hat{\theta}_3 < \overline{\theta}$, for otherwise the solution can be redefined to be in A^2 just as Case 1 above. Furthermore, we must have $0 = \hat{x}_1 < \hat{x}_2 < \hat{x}_3 = 1$, because an argument akin to the one used in Case 2 above shows that otherwise there would be an equivalent solution in A^2. This implies that $\int_{\hat{\theta}_2}^{\hat{\theta}_3}(J(\theta)-\mathbb{E}[A_2(\tilde{\theta})])d\theta < 0$, for otherwise it would remain optimal to raise $\hat{x}_2$ until it coincides with $\hat{x}_3$ (this is feasible because raising $\hat{x}_2$ only relaxes the last requirement in the definition of A^3), which cannot be. But then reducing $\hat{x}_2$ would strictly improve the value of the objective function, so it must be that this is not feasible in A^3, which implies that $1 = \hat{\theta}_3 - (\hat{\theta}_3 - \hat{\theta}_2)\hat{x}_2$, and consequently $\hat{\theta}_3 > 1$.

The derivative of the objective function in (P') with respect to $\hat{\theta}_3$ is

$$\begin{aligned}\left(\mathbb{E}[A_2(\tilde{\theta})] - J(\hat{\theta}_3)\right)(1-\hat{x}_2) &= \left[\int_{\underline{\theta}}^{\overline{\theta}} A_2(s)f(s)ds + (1-\hat{\theta}_3)b_0 f(\hat{\theta}_3) - \int_{\hat{\theta}_3}^{\overline{\theta}} A_2(s)f(s)ds\right](1-\hat{x}_2) \\ &= \left[\int_{\underline{\theta}}^{\hat{\theta}_3} A_2(s)f(s)ds + (1-\hat{\theta}_3)b_0 f(\hat{\theta}_3)\right](1-\hat{x}_2) < 0,\end{aligned}$$

where the first equality is from definitions, and the inequality holds because $\hat{x}_2 \in (0,1)$ and

$$\int_{\underline{\theta}}^{\hat{\theta}_3} A_2(s)f(s)ds = F(\hat{\theta}_3)\mathbb{E}[b_2 - b_0|b_1 < \hat{\theta}_3 b_2] \leq F(\hat{\theta}_3)\mathbb{E}[b_2 - b_0|b_1 < b_2] < 0,$$

which in turn is because of strong ordering, $\hat{\theta}_3 > 1$, and $\mathbb{E}[b_2|b_1 < b_2] < b_0$ since $\mathbf{q}^* < \mathbf{1}$.

It follows that lowering $\hat{\theta}_3$ slightly strictly improves the value of the objective function, and this is feasible because lowering $\hat{\theta}_3$ slightly only relaxes the last requirement in the definition of A^3. But this contradicts $(\hat{\boldsymbol{x}}, \boldsymbol{\theta}) \in A^3$ being optimal. *Q.E.D.*

By Lemma 7, we can simplify program (P') to:

$$\max_{(\hat{x}_1,\hat{x}_2,\hat{\theta})\in A'} \left(\int_{\underline{\theta}}^{\hat{\theta}} (J(\theta) - \mathbb{E}[A_2(\tilde{\theta})])d\theta\right)\hat{x}_1 + \left(\hat{\theta}\mathbb{E}[A_2(\tilde{\theta})] + \int_{\hat{\theta}}^{\overline{\theta}} J(\theta)d\theta\right)\hat{x}_2, \qquad (P'')$$

where

$$A' := \{(\hat{x}_1, \hat{x}_2, \hat{\theta})|0 \leq \hat{x}_1 \leq \hat{x}_2 \leq 1; \hat{\theta} \in [\underline{\theta}, \overline{\theta}]; \hat{\theta}(\hat{x}_2 - \hat{x}_1) \leq 1\}.$$

We will argue that any solution to (P'') also solves the original program (P_0) and can be implemented by a simple mechanism. There are three possibilities to consider.

First, suppose that a solution to (P'') has $\hat{x}_1 > 0$. Then $\hat{x}_2 > 0$ and moreover the coefficient of $\hat{x}_1$ in (P'') must be nonnegative, or else one could strictly improve the objective by lowering $\hat{x}_1$ while keeping $\hat{x}_2$ and $\hat{\theta}$ unchanged. Since the coefficient of $\hat{x}_1$ in (P'') is nonnegative, an optimum is also obtained by keeping the same $\hat{x}_2$ and $\hat{\theta}$ but setting $\hat{x}_1 = \hat{x}_2$. Then, by (12), $y(\underline{\theta}) = 0$. This solves the original program (P_0) because the outcome can implemented by a feasible mechanism (x, y) where for all θ, $x(\theta) = \hat{x}_2$ and $y(\theta) = 0$; note that this obviously satisfies the feasibility constraint that $y(\theta) \leq 1 - x(\theta)$ for all θ. In turn, this optimal mechanism can be implemented by a simple mechanism with $\mathbf{q} = (\hat{x}_2, 0)$.[5]

Next, suppose that that $\hat{x}_2 = \hat{x}_1 = 0$ at a solution to (P''). Then by (12), $y(\underline{\theta}) = 0$. This outcome can be implemented by a feasible mechanism (x, y) where $x(\theta) = 0$ and $y(\theta) = 0$ for all

[5]Indeed, this actually shows $\hat{x}_1 > 0$ cannot be a solution to (P'') because we know that $\mathbf{q} = (\hat{x}_2, 0)$ for any $\hat{x}_2 \in (0, 1]$ is never optimal in the class of simple mechanisms, as it is is strictly dominated by full delegation.

θ. Note that obviously satisfies the feasibility constraint $y(\theta) \leq 1 - x(\theta)$ for all θ. Since the DM always picks the outside option at this optimal mechanism, it can be implemented by a simple mechanism with $\mathbf{q} = \mathbf{0}$.

Finally, suppose $\hat{x}_2 > 0 = \hat{x}_1$ at a solution to (P''). It must be that the coefficient of $\hat{x}_2$ in (P'') is nonnegative, or else one could strictly improve the objective by lowering $\hat{x}_2$ while keeping $\hat{x}_1$ and $\hat{\theta}$ unchanged. Therefore, it is also optimal to set $\hat{x}_2 = \min\{1, 1/\hat{\theta}\}$. This solution can be implemented by a feasible mechanism (x, y), where $x(\theta) = 0$ for $\theta < \hat{\theta}$ and $x(\theta) = \min\{1, 1/\hat{\theta}\}$ for $\theta > \hat{\theta}$, and $y(\theta) = \min\{\hat{\theta}, 1\}$ for $\theta < \hat{\theta}$ and $y(\theta) = 0$ for $\theta > \hat{\theta}$; since this satisfies both (12) and the feasibility constraint that $y(\theta) \in [0, 1 - x(\theta)]$ for all θ, it also solves the original program (P_0). In turn, this optimal mechanism can be implemented by a simple mechanism with $\mathbf{q} = (\min\{1, \frac{1}{\hat{\theta}}\}, \min\{\hat{\theta}, 1\})$. Notice in this simple mechanism the agent will recommend project two if $b_1/b_2 < \hat{\theta}$ and project one if $b_1/b_2 > \hat{\theta}$. In the former case, project two is implemented with probability $\min\{\hat{\theta}, 1\}$ and the outside option with complementary probability; in the latter case, project one is implemented with probability $\min\{1, 1/\hat{\theta}\}$ while the outside option is implemented with complementary probability.[6]

Summarizing, we have shown that the optimal mechanism is implemented by a simple mechanism. This completes the proof under the assumption that $\overline{\theta} < \infty$.

The case of $\overline{\theta} = \infty$.

Hereafter assume $\overline{\theta} = \infty$. This introduces two difficulties with the method used above for $\overline{\theta} < \infty$: first, the application of Fubini's theorem to derive (8) is not necessarily valid since it is possible that $\mathbb{E}[\theta] = \infty$; second, objects such as $x(\overline{\theta})$ and hence constraints such as (11) are not well defined. We thus take a different approach. Note that the value of (P_0) is bounded because $\mathbb{E}[b_i] < \infty$ for $i = 1, 2$.

To begin, consider a subproblem $[P_t]$ in which the agent draws θ from $[\underline{\theta}, t]$ for $t < \infty$, according to density $g_t(\theta) := f(\theta)/F(t)$. Analogous to ($P_0$), the DM's objective is to now maximize

$$\Phi_t(x, y) := \frac{1}{F(t)} \int_{\underline{\theta}}^{t} [x(\theta)A_1(\theta) + y(\theta)A_2(\theta)] f(\theta) d\theta.$$

The associated virtual value is given by

$$J_t(\theta) := -(1 - \theta) g_t(\theta) + \int_{\theta}^{t} A_2(s) g_t(s) ds = \frac{J(\theta)}{F(t)} - \frac{\int_t^{\infty} A_2(s) f(s) ds}{F(t)}.$$

It follows from our preceding analysis that there is a simple mechanism that is optimal in this subproblem. In particular, by Theorem 4, there exists an optimal simple mechanism indexed by $(\hat{\theta}, \overline{x}) \in [0, 1]^2$ such that $(x(\theta), y(\theta)) = (0, \hat{\theta}\overline{x})$ for $\theta < \hat{\theta}$, and $(x(\theta), y(\theta)) = (\overline{x}, 0)$ for $\theta \in (\hat{\theta}, t]$.

[6] Indeed, Theorem 4 implies that $\hat{\theta} < 1$ (since by assumption $\mathbf{q}^* < \mathbf{1}$).

There exists a sequence $\{t_n\}_{n=1}^{\infty}$ with $t_n \to \infty$ as $n \to \infty$ such that for each n the simple mechanism (x_{t_n}, y_{t_n}) indexed by $(\hat{\theta}_{t_n}, \overline{x}_{t_n}) \in [0,1]^2$ is optimal for subproblem $[P_{t_n}]$. As $n \to \infty$, (x_{t_n}, y_{t_n}) converges (in subsequence) to a simple mechanism (x^{**}, y^{**}) indexed by $(\hat{\theta}^{**}, \overline{x}^{**}) = \lim_{n\to\infty}(\hat{\theta}_{t_n}, \overline{x}_{t_n})$.[7]

Fix any feasible mechanism $(x, y) : [\underline{\theta}, \infty) \to A$ for the original problem. Its restriction to $[\underline{\theta}, t_n]$, $(x^{t_n}, y^{t_n}) := (x, y)|_{\theta < t_n}$ is clearly feasible for the subproblem $[P_{t_n}]$. Since (x_{t_n}, y_{t_n}) is optimal for subproblem $[P_{t_n}]$, we have that for each n,

$$\Phi_{t_n}(x_{t_n}, y_{t_n}) \geq \Phi_{t_n}(x^{t_n}, y^{t_n}). \tag{14}$$

As $n \to \infty$, the right side of (14) converges to

$$\lim_{n\to\infty} \Phi_{t_n}(x^{t_n}, y^{t_n}) = \lim_{n\to\infty} \Phi_{t_n}(x, y) = \int_{\underline{\theta}}^{\infty} [x(\theta)A_1(\theta) + y(\theta)A_2(\theta)]f(\theta)d\theta,$$

which is the value of the original objective function (P_0) under (x, y).

Meanwhile, as $n \to \infty$, the left side of (14) converges to

$$\begin{aligned}
\lim_{n\to\infty} \Phi_{t_n}(x_{t_n}, y_{t_n}) &= \lim_{n\to\infty} \frac{\overline{x}_{t_n}}{F(t_n)} \left(\int_{\underline{\theta}}^{\hat{\theta}_{t_n}} A_1(\theta)f(\theta)d\theta + \hat{\theta}_{t_n} \int_{\hat{\theta}_{t_n}}^{t_n} A_2(\theta)f(\theta)d\theta \right) \\
&= \overline{x}^{**} \left(\int_{\underline{\theta}}^{\hat{\theta}^{**}} A_1(\theta)f(\theta)d\theta + \hat{\theta}^{**} \int_{\hat{\theta}^{**}}^{\infty} A_2(\theta)f(\theta)d\theta \right) \\
&= \int_{\underline{\theta}}^{\infty} [x^{**}(\theta)A_1(\theta) + y^{**}(\theta)A_2(\theta)]f(\theta)d\theta,
\end{aligned}$$

which is the value of the original objective function (P_0) under (x^{**}, y^{**}).

Since the inequality in (14) is preserved in the limit as $n \to \infty$, the simple mechanism (x^{**}, y^{**}) gives a weakly higher value than (x, y) in the original problem (P_0). Since (x, y) is an arbitrary feasible mechanism, (x^{**}, y^{**}) is optimal in the original problem, thus establishing the optimality of a simple mechanism. *Q.E.D.*

C Leading Examples

This Appendix provides detailed computations for the leading examples with $n = 2$. We prove that they satisfy strong ordering and verify the expressions provided in Examples 1 and 2 from the main text.

[7]This follows from the fact that $\{(\hat{\theta}_{t_n}, \overline{x}_{t_n})\}_{n=1}^{\infty}$ has a convergent subsequence because each element lies in $[0,1]^2$.

C.1 Scale-invariant uniform distributions

Assume that b_2 is uniformly distributed on $[0,1]$, while b_1 is uniformly distributed on $[v, 1+v]$ with $v \in (0,1)$. Assumption (A1) requires $b_0 < 1$; this also guarantees (A3).

Strong ordering: We compute

$$\mathbb{E}[b_2|b_2 > \alpha b_1] = \frac{\int_v^{1+v} \int_{\alpha b_1}^1 b_2 db_2 db_1}{\int_v^{1+v} \int_{\alpha b_1}^1 db_2 db_1} = \frac{3v^2\alpha^2 + 3v\alpha^2 + \alpha^2 - 3}{3\alpha + 6v\alpha - 6} \text{ for } \alpha \in \left[0, \frac{1}{1+v}\right],$$

and

$$\mathbb{E}[b_2|b_2 > \alpha b_1] = \frac{\int_v^{1/\alpha} \int_{\alpha b_1}^1 b_2 db_2 db_1}{\int_v^{1/\alpha} \int_{\alpha b_1}^1 1 db_2 db_1} = \frac{2}{3} + \frac{\alpha}{3} v \text{ for } \alpha \in \left(\frac{1}{1+v}, \frac{1}{v}\right]. \tag{15}$$

Both expressions are increasing in α in the relevant range. Note that $\mathbb{E}[b_2|b_2 > \alpha b_1]$ is not defined for $\alpha > 1/v$.

Similarly, it can be computed that

$$\mathbb{E}[b_1|b_1 > \alpha b_2] = \begin{cases} v + \frac{1}{2} & \text{if } \alpha \leq v \\ \frac{-2v^3+3v^2\alpha+6v\alpha-\alpha^3+3\alpha}{-3v^2+6v\alpha-3\alpha^2+6\alpha} & \text{if } \alpha \in (v, 1+v) \\ \frac{6v^2+6v+2}{6v+3} & \text{if } \alpha \geq 1+v, \end{cases} \tag{16}$$

which is nondecreasing in α.

Therefore, (R2) is satisfied. It is also routine to verify that (R1) is satisfied using formulas (15) and (16) with $\alpha = 1$. We conclude that strong ordering holds.

The Largest Equilibrium: From Theorem 1 and formula (15),

$$b_0^* = \mathbb{E}[b_2|b_2 > b_1] = \frac{2}{3} + \frac{v}{3}.$$

For $b_0 > b_0^*$, a pandering equilibrium $\mathbf{q}^* = (1, q_2^*)$ with $q_2^* \in (0,1)$ requires $\mathbb{E}[b_2|b_2 > b_1/q_2^*] = b_0$. Substituting from (15) yields the solution

$$q_2^*(b_0) = \frac{v}{3b_0 - 2} \tag{17}$$

so long as the right hand side above is larger than v, which is guaranteed since $b_0 < 1$. That $q_1^* = 1$ implies $\mathbb{E}[b_1|b_1 > q_2^*(b_0)b_2] \geq b_0$, into which we substitute (17) to obtain

$$\mathbb{E}\left[b_1|b_1 > \frac{v}{3b_0 - 2} b_2\right] \geq b_0.$$

By substituting from (16), it can be verified that the left-hand side of the above expression is continuous and weakly decreasing in b_0, while the right-hand side is, obviously, strictly increasing. Moreover, by the definition of b_0^*, $\mathbb{E}\left[b_1|b_1 > \frac{v}{3b_0^*-2}b_2\right] = \mathbb{E}[b_1|b_1 > b_2] > b_0^*$. Therefore, there is a unique b_0^{**} such that

$$\mathbb{E}\left[b_1|b_1 > \frac{v}{3b_0^{**}-2}b_2\right] = b_0^{**},$$

and $b_0^{**} > b_0^*$. It can be verified that $b_0^{**} < 1$ if and only if $v < \frac{1}{2}$. It follows that a pandering equilibrium $\mathbf{q}^* = (1, q_2^*)$ with $q_2^* \in (0,1)$ exists if and only if $b_0 \in (b_0^*, \min\{1, b_0^{**}\})$. If $b_0^{**} < 1$ (i.e., $v < 1/2$), then for $b_0 \in (b^{**}, 1)$ the only equilibrium is $\mathbf{q} = (0,0)$.

Remark 1. Finally, what happens if $b_0 > 1$, so that Assumptions (A1) and (A3) fail? If $v < 1/2$, then $\mathbb{E}[b_1] = v + \frac{1}{2} < b_0$, hence $\mathbf{q} = (0,0)$ is the only equilibrium. If $v > \frac{1}{2}$, then for $b_0 \in \left(1, \frac{1}{2} + v\right)$, $\mathbf{q} = (1,0)$ is the only equilibrium, whereas for $b_0 > \frac{1}{2} + v$, $\mathbf{q} = (0,0)$ is the only equilibrium. Thus, a violation of (A1) and (A3) allow for the non-influential equilibrium $(1,0)$ to be the largest equilibrium for certain values of b_0.

C.2 Exponential distributions

Assume that b_1 and b_2 are exponentially distributed with respective means v_1 and v_2, where $v_1 > v_2 > 0$. Assumption (A1) is obviously satisfied for any $b_0 \in \mathbb{R}_{++}$; we will show below that (A3) requires $b_0 < 2v_2$.

Strong Ordering: Denoting the project densities respectively by $f_1(\cdot)$ and $f_2(\cdot)$, we have that

$$\begin{aligned}
\mathbb{E}[b_1|b_1 > b_2] &= \frac{\int_0^\infty (\mathbb{E}[b_1 > b_2|b_2] \Pr(b_1 > b_2|b_2) f_2(b_2) db_2}{\int_0^\infty \Pr(b_1 > b_2|b_2) f_2(b_2) db_2} \\
&= \frac{v_1+v_2}{v_1} \int_0^\infty (v_1 + b_2) e^{-\frac{1}{v_1}b_2} \left(\frac{1}{v_2} e^{-\frac{1}{v_2}b_2}\right) db_2 \\
&= \int_0^\infty (v_1 + b_2) \left(\frac{v_1+v_2}{v_1 v_2}\right) e^{-\left(\frac{v_1+v_2}{v_1 v_2}\right)b_2} db_2 \\
&= v_1 + \frac{v_1 v_2}{v_1 + v_2}.
\end{aligned}$$

Similarly,

$$\mathbb{E}[b_2|b_2 > b_1] = v_2 + \frac{v_1 v_2}{v_1 + v_2}.$$

Plainly, (R1) is satisfied. Moreover, since αb_i is exponentially distributed with mean αv_i, the above calculations imply

$$\mathbb{E}[b_i|b_i > \alpha b_j] = v_i + \frac{\alpha v_i v_j}{v_i + \alpha v_j}. \tag{18}$$

Since the right-hand side above is strictly increasing in α for any $\alpha \in \mathbb{R}_+$, (R2) is satisfied and hence strong ordering holds. Note that since $\lim_{\alpha\to\infty} \mathbb{E}[b_i|b_i > \alpha b_j] = 2v_i$, Assumption (A3) requires $b_0 < 2v_2$.

The Largest Equilibrium: From Theorem 1 and equation (18),

$$b_0^* = \mathbb{E}[b_2|b_2 > b_1] = v_2 + \frac{v_1 v_2}{v_1 + v_2}.$$

For $b_0 > b_0^*$, a pandering equilibrium $\mathbf{q} = (1, q_2^*)$ with $q_2^* \in (0,1)$ requires $\mathbb{E}[b_2|b_2 > b_1/q_2^*] = b_0$. Substituting from (18) yields the solution

$$q_2^*(b_0) = \frac{v_1}{v_2}\left(\frac{2v_2 - b_0}{b_0 - v_2}\right). \tag{19}$$

That $q_1^* = 1$ implies $\mathbb{E}[b_1|b_1 > q_2^*(b_0)b_2] \geq b_0$, into which we substitute (18) to obtain

$$3v_1 - b_0\frac{v_1}{v_2} \geq b_0.$$

Since the left-hand side of the this inequality is decreasing in b_0 and the right-hand side is increasing in b_0, the inequality is satisfied if and only if

$$b_0 \leq b_0^{**} = \frac{3v_1 v_2}{v_2 + v_1}$$

Note that $b_0^{**} < 2v_2$ if and only if $v_1 < 2v_2$. It follows that a pandering equilibrium $\mathbf{q}^* = (1, q_2^*)$ with $q_2^* \in (0,1)$ exists if and only if $b_0 \in (b_0^*, \min\{b_0^{**}, 2v_2\})$. If $b_0^{**} < 2v_2$ (i.e., if $v_1 < 2v_2$), then for $b_0 \in (b_0^{**}, 2v_2)$, the only equilibrium is $\mathbf{q} = (0,0)$.

DM's Expected Payoff: If $b_0 < b_0^*$, the DM's ex-ante expected payoff is

$$\begin{aligned}
\pi^t &:= \mathbb{E}\left[\max\left\{b_1, b_2\right\}\right] \\
&= \left(\frac{v_1}{v_1 + v_2}\right)\left(v_1 + \frac{v_1 v_2}{v_1 + v_2}\right) + \left(\frac{v_2}{v_1 + v_2}\right)\left(v_2 + \frac{v_1 v_2}{v_1 + v_2}\right) \\
&= v_1 + v_2 - \frac{v_1 v_2}{v_1 + v_2}.
\end{aligned}$$

For $b_0 \in (b_0^*, \min\{b_0^{**}, 2v_2))$, the DM's expected payoff is

$$\begin{aligned}
\pi^p &:= \Pr(b_1 > q_2^*(b_0)b_2)\mathbb{E}[b_1|b_1 > q_2^*(b_0)b_2] + \Pr(q_2^*(b_0)b_2 > b_1)b_0 \\
&= \left(\frac{v_1}{v_1 + q_2 v_2}\right)\left(v_1 + \frac{v_1 q_2 v_2}{v_1 + q_2 v_2}\right) + \left(\frac{q_2 v_2}{v_1 + q_2 v_2}\right) b_0 \\
&= \left(\frac{v_1}{v_1 + q_2 v_2}\right)\left(3v_1 - b_0\frac{v_1}{v_2}\right) + \left(\frac{q_2 v_2}{v_1 + q_2 v_2}\right) b_0 \\
&= \frac{1}{(v_2)^2}\left(2b_0(v_2)^2 - (v_2 + v_1)(b_0)^2 + 4b_0 v_1 v_2 - 3v_1(v_2)^2\right) \\
&= \pi^t - \frac{(v_1 + v_2)}{v_2^2}\left(b_0 - b_0^*\right)^2.
\end{aligned}$$

Finally, if $b_0 > (b_0^{**}, 2v_2)$, the DM's expected payoff is just b_0.

Remark 2. What happens if $2v_2 < b_0$, so that Assumption (A3) fails? Then $q_2 = 0$ in any equilibrium. If $v_1 < 2v_2$, then $E[b_1] = v_1 < b_0$, hence $\mathbf{q} = (0, 0)$ is the only equilibrium. If $v_1 > 2v_2$, then for $b_0 \in (2v_2, v_1)$, $\mathbf{q} = (1, 0)$ is the unique equilibrium whereas for $b_0 > v_1$, $q = (0, 0)$ is the unique equilibrium. Thus, a violation of (A3) allows for the non-influential equilibrium $(1, 0)$ to be the largest equilibrium, for certain values of b_0.

D Many Projects

This Appendix shows that most of the main results generalize to $n > 2$, with an appropriate strengthening of the strong ordering condition. The two caveats are:

- The conclusion of Lemma 1 is now an assumption, i.e. we assume that the agent uses a pure strategy and that the DM responds to any message with a mixture whose support consists of only one project and the outside option.
- The largest equilibrium that we characterize is not necessarily interim Pareto dominant. While it is guaranteed to be interim superior to any other for the agent, it need not be for the DM.

For the remainder of this Appendix, assume $n > 2$.

D.1 Preliminaries

We study a class of perfect Bayesian equilibria. The agent's strategy is represented by a function $\mu : \mathcal{B} \to \Delta(M)$ and the DM's strategy by $\alpha : M \to \Delta(N \cup \{0\})$, where $\Delta(\cdot)$ is the set of probability distributions. We restrict attention to equilibria where the DM does not randomize on the equilibrium path between two or more alternative projects. In other words, in equilibrium, any randomization by the DM must be between the outside option and one project, although which project it is could depend upon the message received. Given that the only conflict between the two players is about the outside option, we view this as a natural class of equilibria to study.

Lemma 8. *If (α, μ) is an equilibrium in which the DM does not randomize on the equilibrium path between two or more alternative projects (i.e., for any on-path $m \in M$, $|Support[\alpha(m)] \cap N| < 2$), then the equilibrium is outcome-equivalent to one where no more than n messages are used in equilibrium and the agent plays a pure strategy.*

Before providing a proof, here is the intuition: there are n alternative projects and any message will (by assumption) lead to a distribution of decisions over the outside option and at most one project. Whenever two or more messages result in a particular project being implemented with positive probability, the agent will only use the message(s) that maximize(s) the acceptance probability of that project. Finally, equilibria in which two or more messages yield the same acceptance probability are outcome-equivalent to an equilibrium in which only one of these messages is ever used.

Proof of Lemma 8. Consider any equilibrium (α, μ) with more than n on-path messages. Letting $\alpha(i|m)$ be the probability that $\alpha(\cdot)$ puts on any project i following message m. By outcome-equivalence, we can ignore the behavior of any zero measure set of types. Let M^* be the set of on-path messages. First suppose that the DM chooses the outside option for sure for all $m \in M^*$. Then for any $m \in M^*$ and any $i \in N$, we must have $\mathbb{E}[b_i|m] \leq b_0$, so that $\mathbb{E}[b_i] \leq b_0$, and it follows that there is an outcome-equivalent uninformative or pooling equilibrium with only one on-path message.

Next, consider the case where some $m^* \in M^*$ leads to an alternative project with positive probability, i.e. $Support[\alpha(m^*)] \cap N \neq \emptyset$. This requires that $Support[\alpha(m)] \cap N \neq \emptyset$ for all $m \in M^*$, since for almost all types, the agent would never use a message that fails this property given the availability of m^*. For each project $i \in N$, define $p_i = 0$ if $\alpha(i|m) = 0$ for all $m \in M^*$, and otherwise define $p_i = \alpha(i|m)$ for all $m \in M^*$ such that $\alpha(i|m) > 0$. Note that for any $i \in N$, p_i is well-defined because if there are two distinct messages $m \in M^*$ and $m' \in M^*$ such that $\alpha(i|m) > 0$ and $\alpha(i|m') > 0$, then we must have $\alpha(i|m) = \alpha(i|m') > 0$ because otherwise one of these messages would not be used by any type (except possibly a set of zero measure, which can be ignored). Therefore, the agent is effectively faced with a choice of which p_i he would like to induce. Since some project has $p_i > 0$, it follows that a full measure of types have a uniquely optimal choice from the set $\{p_1, \ldots, p_n\}$, and we can ignore any zero measure set of types who do not. Let B_i be the (possibly empty) set of types for whom p_i is uniquely optimal from the set $\{p_1, \ldots, p_n\}$; the collection $\{B_i\}_{i \in N}$ is a partition of $\mathcal{B}$.

Now for each $i \in N$, let $M_i^* := \{m \in M^* : \alpha(i|m) > 0\}$. Note that for an arbitrary $i \in N$, M_i^* could be empty; however, since $|M^*| > n$, there is some project $i^* \in N$ such that $|M_i^*| > 1$ and $p_i > 0$. Moreover, optimality for the agent implies that any type $\mathbf{b} \in B_i$ will not use any message except those in M_i^*, although it may be mixing over messages within M_i^*. This implies that the support of the DM's beliefs about the agent's type when receiving a message $m \in M_i^*$ must be a non-empty subset of B_i; denote this belief $\beta(m)$. By the optimality of α for the DM, we have:

$$\text{for any } m \in M_{i^*}^*, \qquad \mathbb{E}[b_{i^*}|\beta(m)] \geq \max\{b_0, \max_{j \in N} \mathbb{E}[b_j|\beta(m)]\}, \tag{20}$$

$$\text{for any } m \in M_{i^*}^*, \qquad \mathbb{E}[b_{i^*}|\beta(m)] = b_0 \text{ if } p_{i^*} < 1. \tag{21}$$

Now pick some $\overline{m} \in M_i^*$ and consider a strategy $\tilde{\mu}$ defined as follows: for any $\mathbf{b} \notin B_{i^*}$, $\tilde{\mu}(\mathbf{b}) = \mu(\mathbf{b})$; for any $\mathbf{b} \in B_{i^*}$, $\tilde{\mu}(\mathbf{b}) = \overline{m}$. So $\tilde{\mu}$ is identical to μ except that all types that were using any message in $M^*_{i^*}$ (necessarily types in B_{i^*}) play a pure strategy of sending message $\overline{m}$. Since $|M^*_{i^*}| > 1$, we have reduced the number of used messages by at least 1 in moving from μ to $\tilde{\mu}$. We now argue that $(\alpha, \tilde{\mu})$, augmented with the obvious beliefs, constitutes an equilibrium. Optimality for the agent is immediate because every $m \in M^*_{i^*}$ has $\alpha(i^*|m) = p_{i^*} > 0$. For the DM, notice that the beliefs over agent types have not changed for any $m \in M^*_i$ with $i \neq i^*$, so $\alpha(m)$ remains optimal for any such m. For message $\overline{m}$, the new beliefs are just the prior restricted to B_{i^*}; equations (20) and (21) imply that $\alpha(\overline{m})$ remains optimal (recall that for any $m \in M^*_{i^*}$, $\beta(m)$ has support within B_{i^*}).

Since the choice of project i^* was arbitrary above and only required that $|M^*| > n$, we can repeat the above argument to reduce the number of used messages so long as there are more than n messages being used. Notice further that after repetition of the argument, the resulting agent's strategy is a pure strategy because the original $\{B_i\}_{i \in N}$ was a partition of $\mathcal{B}$. *Q.E.D.*

In light of Lemma 8, we focus hereafter on equilibria where no more than n messages are used, which, without loss of generality, can be taken to be the set N. In other words, *the cheap-talk game is effectively reduced to one in which the agent recommends a project $i \in N$* (or ranks $i \in N$ above all $j \in N \setminus \{i\}$). In turn, the DM's equilibrium strategy can now be viewed as a vector of acceptance probabilities, $\mathbf{q} := (q_1, \ldots, q_n) \in [0,1]^n$, where q_i is the probability with which the DM implements project i if the agent recommends that project. Thus, if an agent recommends project i, a DM who adopts strategy $\mathbf{q}$ accepts the recommendation with probability q_i but rejects it in favor of the outside option with probability $1 - q_i$.

We are now in a position to characterize equilibria. The agent's problem is to choose a strategy $\mu : \mathcal{B} \to \Delta(N)$ that maps each profile of project values $\mathbf{b}$ to probabilities $(\mu_1(\mathbf{b}), ..., \mu_n(\mathbf{b}))$ of recommending alternative projects in N. Given any $\mathbf{q}$, a strategy μ is optimal for the agent if and only if

$$\mu_i(\mathbf{b}) = 1 \text{ if } q_i b_i > \max_{j \in N \setminus \{i\}} q_j b_j. \tag{22}$$

Accordingly, in characterizing an equilibrium, we can just focus on the DM's acceptance vector, $\mathbf{q}$, with the understanding that the agent best responds according to (22). For any equilibrium $\mathbf{q}$, the optimality of the DM's strategy combined with (22) implies a pair of conditions for each project i:

$$q_i > 0 \implies \mathbb{E}\left[b_i \mid q_i b_i = \max_{j \in N} q_j b_j\right] \geq \max\left\{b_0, \max_{k \in N \setminus \{i\}} \mathbb{E}\left[b_k \mid q_i b_i = \max_{j \in N} q_j b_j\right]\right\}, \tag{23}$$

$$q_i = 1 \impliedby \mathbb{E}\left[b_i \mid q_i b_i = \max_{j \in N} q_j b_j\right] > \max\left\{b_0, \max_{k \in N \setminus \{i\}} \mathbb{E}\left[b_k \mid q_i b_i = \max_{j \in N} q_j b_j\right]\right\}. \tag{24}$$

Condition (23) says that the DM accepts project i (when it is recommended) only if she finds it weakly better than the outside option as well as the other (unrecommended) projects, given her posterior which takes the agent's strategy (22) into consideration. Similarly, (24) says that if she finds the recommended project to be strictly better than all other options, she must accept that project for sure. These conditions are clearly necessary in any equilibrium;[8] the following result shows that they are also sufficient.

Lemma 9. *If an equilibrium has acceptance vector* $\mathbf{q} \in [0,1]^n$*, then* (23) *and* (24) *are satisfied for all projects* i *such that* $\Pr\{\mathbf{b} : q_i b_i = \max_{j\in N} q_j b_j\} > 0$*. Conversely, for any* $\mathbf{q} \in [0,1]^n$ *satisfying* (23) *and* (24) *for all* i *such that* $\Pr\{\mathbf{b} : q_i b_i = \max_{j\in N} q_j b_j\} > 0$*, there is an equilibrium where the DM plays* $\mathbf{q}$ *and the agent's strategy satisfies* (22)*.*

The proof is omitted since it is the same logic as Lemma 2 for the 2-project case.

For expositional convenience, we will also focus on equilibria with the property that if a project i has ex-ante probability zero of being implemented on the equilibrium path, then the DM's acceptance vector $\mathbf{q}$ has $q_i = 0$. This is without loss of generality because there is always an outcome-equivalent equilibrium with this property: if $q_i > 0$ but the agent does not recommend i with positive probability, it must be that $q_i \overline{b}_i \le q_j \underline{b}_j$ for some $j \neq i$, so setting $q_i = 0$ does not change the agent's incentives and remains optimal for the DM with the same beliefs.

D.2 Terminology

An equilibrium with $\mathbf{q} = \mathbf{0} := (0, \ldots, 0)$ is a **zero equilibrium**. If $q_i = 1$, we say that the DM **rubber-stamps** project i, since she chooses it with probability one when the agent recommends it. The agent is **truthful** if he always always recommends the best project. An equilibrium is **truthful equilibrium** if $\mathbf{q} = \mathbf{1} := (1, \ldots, 1)$.[9] An equilibrium is **influential** if $|\{i \in N : q_i > 0\}| \ge 2$, i.e. there are at least two projects that are implemented on the equilibrium path. We say that the agent **panders toward** i **over** j if $q_i > q_j > 0$. An equilibrium is a **pandering equilibrium** if there are some i and j such that the agent panders toward i over j in the equilibrium. Finally, say that an equilibrium $\mathbf{q}$ is **larger** than another equilibrium $\mathbf{q}'$ if $\mathbf{q} > \mathbf{q}'$, and $\mathbf{q}$ **is better than** $\mathbf{q}'$ if $\mathbf{q}$ Pareto dominates $\mathbf{q}'$ at the interim stage where the agent has learned his type but the DM has not.

D.3 Strong ordering

Definition 2. For $n > 2$, projects are **strongly ordered** if

1. For any $i < j$, and any $k \in \mathbb{R}_+$,

$$\mathbb{E}[b_i | b_i > b_j, b_i > k] > \mathbb{E}[b_j | b_j > b_i, b_j > k]. \tag{R1-many}$$

[8] Strictly speaking, for those projects that are recommended with positive probability on the equilibrium path, i.e. when $\Pr\{\mathbf{b} : q_i b_i = \max_{j\in N} q_j b_j\} > 0$.

[9] There can be a zero equilibrium where the agent always recommends the best project; this exists if and only if for all $i \in N$, $\mathbb{E}[b_i | b_i = \max_{j\in N} b_j] \le b_0$. We choose not to call this a truthful equilibrium.

whenever both expectations are well-defined.

2. For any i and j, and any $k \in \mathbb{R}_+$,

$$\mathbb{E}[b_i | b_i > \alpha b_j, b_i > k] \text{ is nondecreasing in } \alpha \in \mathbb{R}_+ \tag{R2-many}$$

so long as the expectation is well-defined.

The only difference between (R1-many) and (R1), or (R2-many) and (R2), is the extra conditioning on the relevant random variable being above the non-negative constant k. Obviously, when $k = 0$, (R1-many) and (R2-many) are respectively identical to (R1) and (R2), because of our maintained assumption (A1). Since Definition 2 requires (R1-many) and (R2-many) to hold for all $k \in \mathbb{R}_+$, this notion of strong ordering is more demanding than that of Definition 1, even if there are only two projects. Intuitively, the roles of (R1-many) and (R2-many) are analogous to that of (R1) and (R2), but modified to account for the fact that when $n > 2$, a recommendation for a project i is a comparative statement not only against project j, but also the other $n - 2$ projects. In other words, the DM's posterior about i when the agent recommends project i rather than project j must also account for the fact that i is sufficiently better than all the other non-j projects as well, for each realization of their values.[10]

We assert that strong ordering for $n > 2$ is satisfied for the leading parametric families of distributions (scale-invariant uniform and exponential); a proof is available on request.

D.4 Results

We first generalize Theorem 1:

Theorem 7. *Assume strong ordering, as stated in Definition 2.*

1. *For any equilibrium $\mathbf{q}$, for any $i < j$, if $q_i > 0$, then $q_i \geq q_j$, and if $q_i > 0$ and $q_j < 1$, then $q_i > q_j$.*

2. *There is a largest equilibrium, $\mathbf{q}^*$, such that:*

   (a) *A truthful equilibrium, $\mathbf{q}^* = \mathbf{1}$, exists if and only if $b_0 \leq b_0^* := \mathbb{E}[b_n | b_n = \max_{j \in N} b_j]$.*

   (b) *If $b_0 \in (b_0^*, b_0^{**})$ for some $b_0^{**} \geq b_0^*$, then $\mathbf{q}^* \gg \mathbf{0}$ and $q_1^* = 1$; consequently, the largest equilibrium is a pandering equilibrium. Moreover, for any $\tilde{b}_0 > b_0$ in this interval, $\mathbf{q}^* > \tilde{\mathbf{q}}^*$, where these are the largest equilibria respectively for b_0 and $\tilde{b}_0$.*

[10] In this light, some readers may find it helpful to consider the following alternative to part one of the definition: For any $i < j$ and any $(\alpha_k)_{k \neq i,j} \in \mathbb{R}_{++}^{n-2}$, $\mathbb{E}[b_i | b_i > b_j, b_i > \max_{k \neq i,j} \alpha_k b_k] > \mathbb{E}[b_j | b_j > b_i, b_j > \max_{k \neq i,j} \alpha_k b_k]$ whenever these expectations are well-defined. A similar modification can also be used for the second part of the definition. While these requirements are slightly weaker and would suffice, we chose the earlier formulation for greater clarity.

(c) *If $b_0 > b_0^{**}$, then only the zero equilibrium exists, $\mathbf{q}^* = \mathbf{0}$.*

Proof. The proof is in several steps.

Step 1: *Fix any equilibrium $\mathbf{q}$ and any $i < j$. If $q_i > 0$, then $q_i \geq q_j$, and if in addition $q_j < 1$, then $q_i > q_j$.*

Proof: Fix any equilibrium $\mathbf{q}$ and any projects $i < j$. Suppose to the contrary that $q_j > q_i > 0$. Then

$$\begin{aligned}
\mathbb{E}\left[b_i \,\middle|\, q_i b_i \geq \max\left\{q_j b_j, \max_{k\neq i,j} q_k b_k\right\}\right] &= \mathbb{E}\left[b_i \,\middle|\, b_i \geq \max\left\{\left(\frac{q_j}{q_i}\right) b_j, \max_{k\neq i,j}\left(\frac{q_k}{q_i}\right) b_k\right\}\right] \\
&> \mathbb{E}\left[b_j \,\middle|\, \left(\frac{q_j}{q_i}\right) b_j \geq \max\left\{b_i, \max_{k\neq i,j}\left(\frac{q_k}{q_i}\right) b_k\right\}\right] \\
&= \mathbb{E}\left[b_j \,\middle|\, q_j b_j \geq q_k b_k, \forall k \neq j\right] \geq b_0,
\end{aligned}$$

where the strict inequality is because of strong ordering and the weak inequality is because $q_j > 0$. But this implies that $\mathbb{E}[b_i \,| q_i b_i = \max_{k\in N} q_k b_k] > b_0$, which is contradiction with $q_i \in (0, 1)$. This proves that $q_i \geq q_j$. For the second statement, notice that $0 < q_i = q_j < 1$ implies that the final inequality above must hold with equality. Since the strict inequality above still applies, the DM's optimality requires $q_i = 1$, a contradiction. ‖

For the remaining results, we consider a mapping $\psi : [0, 1]^n \to [0, 1]^n$ such that for each $\mathbf{q} = (q_1, ..., q_n) \in [0, 1]^n$,

$$\psi_i(q_1, ..., q_n) := \max\left\{q_i' \in [0, 1] \,\middle|\, \mathbb{E}[b_i \,|\, q_i' b_i \geq q_j b_j, \forall j \neq i] \geq b_0\right\} \tag{25}$$

with the convention that $\max \emptyset := 0$. The mapping ψ_i calculates the highest probability with which the DM is willing to accept project i when it is recommended according to (1) subject to the constraint that the posterior belief does not fall below b_0.

Step 2: *The mapping ψ has a largest fixed point $\mathbf{q}^*$.*

Proof: It suffices to prove that the mapping is monotonic, since Tarski's fixed point theorem then implies that the set of fixed points is nonempty and contains a largest element. Fix any $\mathbf{q}' \geq \mathbf{q}$. We will prove that $\psi(\mathbf{q}') \geq \psi(\mathbf{q})$. If $\psi_i(\mathbf{q}) = 0$ for some i then clearly $\psi_i(\mathbf{q}') \geq \psi_i(\mathbf{q})$. So suppose $\psi_i(\mathbf{q}) > 0$ for some i. Then $\mathbb{E}[b_i \,|\, \psi_i(\mathbf{q}) b_i \geq q_j b_j, \forall j \neq i] \geq b_0$. Since $\mathbf{q}' \geq \mathbf{q}$, for any such i, $(R2')$ implies that $\mathbb{E}[b_i \,|\, \psi_i(\mathbf{q}) b_i \geq q_j' b_j, \forall j \neq i] \geq \mathbb{E}[b_i \,|\, \psi_i(\mathbf{q}) b_i \geq q_j b_j, \forall j \neq i]$. Putting the two facts together, we have $\mathbb{E}[b_i \,|\, \psi_i(\mathbf{q}) b_i \geq q_j' b_j, \forall j \neq i] \geq b_0$, from which it follows that $\psi_i(\mathbf{q}') \geq \psi_i(\mathbf{q})$. ‖

Step 3: *The largest fixed point $\mathbf{q}^*$ of ψ is an equilibrium.*

Proof: By Lemma 9, it suffices to prove that $\mathbf{q}^*$ satisfies (2) and (3). To begin, suppose $q_i^* > 0$. Then, since $q_i^* = \psi_i(\mathbf{q}^*) > 0$, we have

$$\mathbb{E}[b_i \mid q_i^* b_i = \max_{k \in N} q_k^* b_k] \geq b_0. \tag{26}$$

Now consider any project $j \neq i$ with $q_j^* > 0$. If $q_j^* = 1$, then $q_j^* \geq q_i^*$, so $q_i^* b_i \geq q_j^* b_j$ implies $b_i \geq b_i$. It thus follows that

$$\mathbb{E}[b_j \mid q_i^* b_i = \max_{k \in N} q_k^* b_k] \leq \mathbb{E}[b_i \mid q_i^* b_i = \max_{k \in N} q_k^* b_k]. \tag{27}$$

If $q_j^* \in [0, 1)$, then we have

$$\mathbb{E}[b_j \mid q_i^* b_i = \max_{k \in N} q_k^* b_k] \leq \mathbb{E}[b_j \mid q_j^* b_j = \max_{k \in N} q_k^* b_k] \leq b_0, \tag{28}$$

where the second inequality follows from $q_j^* = \psi_j(\mathbf{q}^*)$ and from the construction of ψ for the case $q_j^* < 1$, and the first inequality is explained as follows: Define $x := \max_{k \neq i,j} q_k^* b_k$, and let G be its cumulative distribution function. Then, the middle term of (28) can be written as

$$\mathbb{E}[b_j \mid q_j^* b_j = \max_{k \in N} q_k^* b_k] = \frac{\int_0^\infty b_j G(q_j^* b_j) F_i(\frac{q_j^*}{q_i^*} b_j) f_j(b_j) db_j}{\int_0^\infty G(q_j^* b_j) F_i(\frac{q_j^*}{q_i^*} b_j) f_j(b_j) db_j} = \int_0^\infty b \hat{f}_j(b) db,$$

where

$$\hat{f}_j(z) := \frac{G(q_j^* z) F_i(\frac{q_j^*}{q_i^*} z) f_j(z)}{\int_0^\infty G(q_j^* \tilde{z}) F_i(\frac{q_j^*}{q_i^*} \tilde{z}) f_j(\tilde{z}) d\tilde{z}}.$$

Likewise, the left-most term of (28) can be written as

$$\mathbb{E}[b_j \mid q_i^* b_i = \max_{k \in N} q_k^* b_k] = \frac{\int_0^\infty b_j \left(\int_{\frac{q_j^*}{q_i^*} b_j}^\infty G(q_i^* b_i) f_i(b_i) db_i \right) f_j(b_j) db_j}{\int_0^\infty \left(\int_{\frac{q_j^*}{q_i^*} b_j}^\infty G(q_i^* b_i) f_i(b_i) db_i \right) f_j(b_j) db_j} = \int_0^\infty b \tilde{f}_j(b) db,$$

where

$$\tilde{f}_j(z) := \frac{\left(\int_{\frac{q_j^*}{q_i^*} z}^\infty G(q_i^* b_i) f_i(b_i) db_i \right) f_j(z)}{\int_0^\infty \left(\int_{\frac{q_j^*}{q_i^*} \tilde{z}}^\infty G(q_i^* b_i) f_i(b_i) db_i \right) f_j(\tilde{z}) d\tilde{z}}.$$

Note that $\tilde{\eta}(z) := \int_{\frac{q_j^*}{q_i^*} z}^\infty G(q_i^* b_i) f_i(b_i) db_i$ is non-increasing in z, while $\hat{\eta}(z) := G(q_j^* z) F_i(\frac{q_j^*}{q_i^*} z)$

is nondecreasing in z. Hence, for any $z' > z$,

$$\frac{\tilde{f}_j(z')}{\tilde{f}_j(z)} = \frac{\tilde{\eta}(z')f_j(z')}{\tilde{\eta}(z)f_j(z)} \leq \frac{f_j(z')}{f_j(z)} \leq \frac{\hat{\eta}(z')f_j(z')}{\hat{\eta}(z)f_j(z)} = \frac{\hat{f}_j(z')}{\hat{f}_j(z)}, \tag{29}$$

whenever the left-most and right-most terms are well defined.

The inequality (29) means that $\hat{f}$ likelihood-ratio dominates $\tilde{f}$, which proves the first inequality of (28). When combined, (26), (27), and (28) imply that $\mathbf{q}^*$ satisfies (2). The construction of ψ implies that $\mathbf{q}^*$ satisfies (3). ‖

STEP 4: *The largest fixed point $\mathbf{q}^*$ of ψ is the largest equilibrium.*

Proof: Suppose to the contrary that there is an equilibrium $\hat{\mathbf{q}} \not\leq \mathbf{q}^*$. Define a mapping $\hat{\psi} : \prod_{i \in N}[\hat{q}_i, 1] \to \prod_{i \in N}[\hat{q}_i, 1]$ such that for each $\mathbf{q} = (q_1, ..., q_n) \in \prod_{i \in N}[\hat{q}_i, 1]$,

$$\hat{\psi}_i(q_1, ..., q_n) := \max \left\{ q_i' \in [\hat{q}_i, 1] \,\middle|\, \mathbb{E}[b_i \,|\, q_i' b_i \geq q_j b_j, \forall j \neq i] \geq b_0 \right\},$$

again with the convention that $\max \emptyset := 0$. Since $\hat{\mathbf{q}}$ is an equilibrium, it must satisfy (2), so $\hat{\psi}_i(\hat{\mathbf{q}}) \geq \hat{q}_i$. Hence the mapping is well defined on the restricted domain. Further, since $\psi_i(\hat{\mathbf{q}}) \geq \hat{q}_i$ for each i, it must be that $\hat{\psi}(\mathbf{q}) = \psi(\mathbf{q})$ for any $\mathbf{q} \in \prod_{i \in N}[\hat{q}_i, 1]$. Hence $\hat{\psi}$ is monotonic, and Tarski's fixed point theorem implies existence of a fixed point, say $\hat{\mathbf{q}}^+$. By construction, $\hat{\mathbf{q}}^+ \geq \hat{\mathbf{q}}$. Evidently, $\hat{\mathbf{q}}^+$ is a fixed point of ψ as well (in the unrestricted domain). Since $\mathbf{q}^*$ is the largest fixed point, we must have $\mathbf{q}^* \geq \hat{\mathbf{q}}^+ \geq \hat{\mathbf{q}}$, a contradiction. The result follows since $\mathbf{q}^*$ is an equilibrium by Step 3. ‖

STEP 5: *If $\mathbf{q}^* \neq \mathbf{0}$, then $\mathbf{q}^* \gg \mathbf{0}$ and $q_1^* = 1$.*

Proof: Suppose $\mathbf{q}^* \neq \mathbf{0}$. Then, there must exist $k \in N$ such that $q_k^* > 0$. Fix any $i \neq k$. By (A3), there exists $\alpha > 0$ such that

$$b_0 \leq \mathbb{E}\left[b_i \middle| b_i > \alpha b_k \right] = \mathbb{E}\left[b_i \middle| \left(\frac{q_k^*}{\alpha}\right) b_i > q_k^* b_k \right] \leq \mathbb{E}\left[b_i \middle| \left(\frac{q_k^*}{\alpha}\right) b_i > q_j^* b_j, \forall j \right],$$

which implies that, for $q_i = \frac{\overline{q}^*}{\alpha} > 0$, $\mathbb{E}\left[b_i \middle| q_i b_i > q_j^* b_j, \forall b_j \right] \geq b_0$. It follows that $q_i^* = \psi_i(\mathbf{q}^*) \geq q_i > 0$. We have thus proven $\mathbf{q}^* \gg \mathbf{0}$. Step 1 then implies that $q_i^* \geq q_j^*$ for any $i < j$. Suppose $q_1^* < 1$. Then, it must be that $\mathbb{E}[b_i \,|\, q_i^* b_i = \max_{k \in N} q_k^* b_k] = b_0$ for all $i \in N$. Now consider $\overline{\mathbf{q}}^* = \left(\frac{1}{q_i^*}\right) \mathbf{q}^*$. Clearly, $\overline{\mathbf{q}}^*$ is also an equilibrium and $\overline{\mathbf{q}}^* \gg \mathbf{q}^*$, which contradicts Step 4. ‖

STEP 6: *Let $\mathbf{q}^*(b_0)$ denote the largest equilibrium under outside option b_0. Then, $\mathbf{q}^*(b_0) \geq \mathbf{q}^*(b_0')$ for $b_0 < b_0'$. If $b_0 < b_0'$ and $\mathbf{q}^*(b_0') \in (\mathbf{0}, \mathbf{1})$, then $\mathbf{q}^*(b_0) > \mathbf{q}^*(b_0')$.*

Proof: Write $\psi(\mathbf{q}; b_0)$ in (25) to explicitly recognize its dependence on b_0. It is easy to see that $\psi_i(\mathbf{q}; b_0)$ is nonincreasing in b_0. It follows that the largest fixed point $\mathbf{q}^*(\mathbf{b}_0)$ is nonincreasing in b_0, proving the first statement. To prove the second, let $b_0 < b_0'$ and $\mathbf{q}^*(b_0') \in (\mathbf{0}, \mathbf{1})$. The statement holds trivially if $\mathbf{q}^*(b_0) = \mathbf{1}$. Hence, assume $\mathbf{q}^*(b_0) < \mathbf{1}$. By Step 1 and Step 5, we must have $q_n^*(b_0) \in (0,1)$, and this implies that $\mathbb{E}[b_n | q_n^*(b_0) b_n = \max_{k \in N} q_k^*(b_0) b_k] = b_0 < b_0'$. Clearly, $\mathbf{q}^*(b_0) \neq \mathbf{q}^*(b_0')$. By the first statement, it follows that $\mathbf{q}^*(b_0) > \mathbf{q}^*(b_0')$. $\|$

STEP 7: *The truthful equilibrium exists if and only if* $b_0 \leq b_0^* := \mathbb{E}[b_n | b_n = \max_{j \in N} b_j]$.

Proof: If $b_0 \leq b_0^*$, then $(R1')$ implies that $b_0 \leq \mathbb{E}[b_i | b_i = \max_{j \in N} b_j]$ for all $i \in N$, so there is a truthful equilibrium. If $b_0 > b_0^*$, $\mathbf{q} = \mathbf{1}$ clearly violates (2), so there cannot be a truthful equilibrium. $\|$

STEP 8: *There exists* $b_0^{**} \geq b_0^*$ *such that the largest equilibrium is* $\mathbf{q}^*(b_0) \in (\mathbf{0}, \mathbf{1})$*—it is a pandering equilibrium—if* $b_0 \in (b_0^*, b_0^{**})$ *and it is zero equilibrium if* $b_0 > b_0^{**}$. *For any* $b_0, b_0' \in (b_0^*, b_0^{**})$ *such that* $b_0 < b_0'$, $\mathbf{q}^*(b_0) > \mathbf{q}^*(b_0')$.

Proof: The first statement follows directly from Steps 1, 5, 6, and 7. The second statement follows directly from Step 7 by noting that $\mathbf{q}^*(b_0') \in (\mathbf{0}, \mathbf{1})$. $\|$ *Q.E.D.*

Remark 3. Unlike with Theorem 1, the largest equilibrium may not be the best equilibrium when there are many projects.[11] Yet, it is compelling to focus on. First, it clearly maximizes the agent's (interim) expected payoff. Second, there is a sense in which any non-zero equilibrium $\mathbf{q}$ where $q_i = 0$ for some i must be supported with "unreasonable" off-path beliefs. Informally, a forward-induction logic goes as follows: by recommending a project i when $q_i = 0$ (which is off the equilibrium path in a non-zero equilibrium), the agent must be signaling that i is sufficiently better than all the projects that he could get implemented with positive probability. So the DM should focus her beliefs on those types that would have the most to gain from such a deviation. Naturally, the agent has more to gain the higher is b_i. But then, with enough weight of beliefs on high b_i's, the DM should accept i with probability one, contradicting $q_i = 0$. A formal discussion of this intuition is available upon request or in working paper versions of this article. Given that when $\mathbf{q}^* \neq \mathbf{0}$ it will generically be the only equilibrium where all projects are implemented with positive probability on the equilibrium path,[12] and $\mathbf{q}^*$ is obviously better for both players than the zero equilibrium, we find it reasonable to focus on $\mathbf{q}^*$.

Focusing on the largest equilibrium, Theorem 2 can also be generalized to the multi-project environment:

Theorem 8. *Fix* b_0 *and an environment* $\mathbf{F} = (F_1, \ldots, F_i, \ldots, F_n)$ *that satisfies strong ordering as in Definition 2. Let* $\tilde{\mathbf{F}} = (F_1, \ldots, \tilde{F}_i, \ldots, F_n)$ *be a new environment such that either*

[11] To see why, suppose $n = 4$ and the largest equilibrium is $\mathbf{q}^* = (1, q_2^*, q_3^*, q_4^*) \gg 0$ while another equilibrium is $\mathbf{q} = (1, q_2, q_3, 0)$ with $q_2 > 0$ and $q_3 > 0$. Even if $q_2^* > q_2$ and $q_3^* > q_3$, so that $\mathbf{q}^*$ has less pandering than $\mathbf{q}$ toward project one, it could be that $\mathbf{q}^*$ has more pandering toward project two over three than $\mathbf{q}$, i.e. $1 > q_3/q_2 > q_3^*/q_2^*$. If projects two and three are ex-ante significantly more likely to be better than projects one and four, it is possible that the DM could prefer $\mathbf{q}$ over $\mathbf{q}^*$.

[12] A proof of this statement is available on request.

(a) $\tilde{\mathbf{F}}$ *satisfies strong ordering and* F_i *likelihood-ratio dominates* $\tilde{F}_i$*; or*

(b) $\tilde{F}_i$ *is a degenerate distribution at zero.*

In either case, let $\mathbf{q}^*$ *and* $\tilde{\mathbf{q}}^*$ *denote the largest equilibria respectively under* $\mathbf{F}$ *and* $\tilde{\mathbf{F}}$*. Then* $\mathbf{q}^* \geq \tilde{\mathbf{q}}^*$*; moreover,* $\mathbf{q}^* > \tilde{\mathbf{q}}^*$ *if* $\tilde{\mathbf{q}}^* \neq \mathbf{1}$ *and* $\mathbf{q}^* > \mathbf{0}$ *and either (b) holds or the likelihood-ratio dominance in (a) is strict.*

Proof. The proof is very similar to that of Theorem 2, so we do not reproduce the entire argument. The key difference is that instead of inequality (A2) in that proof,[13] we must now show that for any $j \in N$,

$$\mathbb{E}[b_j|\tilde{q}_j^* b_j = \max_{k\in N} \tilde{q}_k^* b_k] \geq \mathbb{E}[\tilde{b}_j|\tilde{q}_j^* \tilde{b}_j = \max_{k\in N} \tilde{q}_k^* \tilde{b}_k]. \tag{30}$$

(As before, case (b) is straightforward, so we focus on case (a) of the Theorem so that $\tilde{F}_i$ is not degenerate at zero, and moreover, we can assume $\tilde{\mathbf{q}}^* \gg \mathbf{0}$. Also, we are supposing the conditional expectations are well-defined; an analogous argument to that used in the proof of Theorem 2 can be used to address this issue.) For $j = i$, (30) follows from likelihood-ratio dominance of F_i over $\tilde{F}_i$. For $j \neq i$, (30) is proven as follows. Define $x := \max_{k\neq i,j} \tilde{q}_k^* b_k$, and let G be its cumulative distribution function. We can write

$$\mathbb{E}[b_j \mid \tilde{q}_j^* b_j = \max_{k\in N} \tilde{q}_k^* b_k] = \frac{\int_0^\infty b_j G(\tilde{q}_j^* b_j) F_i(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} b_j) f_j(b_j) db_j}{\int_0^\infty G(\tilde{q}_j^* b_j) F_i(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} b_j) f_j(b_j) db_j} = \int_0^\infty b k_j(b) db,$$

where $k_j(z) := \frac{G(\tilde{q}_j^* z) F_i(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} z) f_j(z)}{\int_0^\infty G(\tilde{q}_j^* \tilde{z}) F_i(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} \tilde{z}) f_j(\tilde{z}) d\tilde{z}}$. Likewise,

$$\mathbb{E}[\tilde{b}_j \mid \tilde{q}_j^* \tilde{b}_j = \max_{k\in N} \tilde{q}_k^* \tilde{b}_k] = \int_0^\infty b \tilde{k}_j(b) db,$$

where $\tilde{k}_j(z) := \frac{G(\tilde{q}_j^* z) \tilde{F}_i(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} z) f_j(z)}{\int_0^\infty G(\tilde{q}_j^* \tilde{z}) \tilde{F}_i(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} \tilde{z}) f_j(\tilde{z}) d\tilde{z}}$. To prove inequality (30), it suffices to show that k_j likelihood-ratio dominates $\tilde{k}_j$. Consider any $b' > b$. Algebra shows that

$$\frac{k_j(b')}{k_j(b)} \geq \frac{\tilde{k}_j(b')}{\tilde{k}_j(b)} \Leftrightarrow \frac{F_i\left(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} b'\right)}{F_i\left(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} b\right)} \geq \frac{\tilde{F}_i\left(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} b'\right)}{\tilde{F}_i\left(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} b\right)},$$

which is the same inequality as we had in the proof of Theorem 2, so again the right-hand side of the equivalence is implied by the hypothesis that F_i likelihood-ratio dominates $\tilde{F}_i$ (see the earlier proof for additional details).

[13] This stated that $\mathbb{E}[b_m|\tilde{q}_m^* b_m = \max_{k\in\{1,2\}} \tilde{q}_k^* b_k] \geq \mathbb{E}[\tilde{b}_m|\tilde{q}_m^* \tilde{b}_m = \max_{k\in\{1,2\}} \tilde{q}_k^* \tilde{b}_k]$.

To finish the proof of the first part of the Theorem, let ψ and $\tilde{\psi}$ denote the mappings (25) for environments $\mathbf{F}$ and $\tilde{\mathbf{F}}$, respectively. Then, (30) means that $\psi(\tilde{\mathbf{q}}^*) \geq \tilde{\psi}(\tilde{\mathbf{q}}^*)$. This implies that there exists a fixed point of ψ weakly greater than $\tilde{\mathbf{q}}^*$. It follows that $\mathbf{q}^* \geq \tilde{\mathbf{q}}^*$.

Just as in the proof of Theorem 2, the second part of the current Theorem follows from the fact that inequality (30) has to hold strictly when the likelihood-ratio domination of F_i over $\tilde{F}_i$ is strict; hence $\psi(\tilde{\mathbf{q}}^*) > \tilde{\psi}\,(\tilde{\mathbf{q}}^*)$, whereby $\mathbf{q}^* > \tilde{\mathbf{q}}^*$. *Q.E.D.*

E Extensions

E.1 Ignorance can be bliss

Suppose there are two projects, A and B, whose values b_A and b_B are ex-ante identically distributed. Suppose the DM, either through her own investigation or verifiable communication with the agent, can *costlessly* obtain a signal $s \in S$ prior to the agent's communication of soft information. Assume for convenience that S is finite. We consider two regimes: (1) No information: The DM does not observe s; and (2) Information: the DM and the agent observe the realized value of s. We say that the signal is **value-neutral** if $\mathbb{E}[\max\{b_A, b_B\}|s]$ is the same for all $s \in S$, and it is **non-trivial** if $\mathbb{E}[b_A|b_A > b_B, s] \neq \mathbb{E}[b_A|b_A > b_B, s']$ for some $s, s' \in S$. Value-neutrality captures the notion of the signal being valuable only insofar as it informs the DM about which of the projects is better, but not about how the best project compares against the outside option.[14]

Theorem 9. *Consider the best equilibrium under each information regime. If the signal is value-neutral, then the DM prefers (at least weakly) not observing the signal to observing the signal. If the signal is also non-trivial, then there exists a non-empty interval $[\hat{b}_0, \bar{b}_0]$ such that the preference for ignorance is strict for $b_0 \in (\hat{b}_0, \bar{b}_0)$.*

Theorem 9 shows that observable information can be harmful, and the DM would benefit from ignorance in the sense of not observing such information. While the result assumes that the projects are ex-ante identical, it is robust to relaxing this assumption because the DM's payoffs from no information and information vary continuously (upon selecting the best equilibrium) when the assumption is slightly relaxed.

It is clear that the nature of information is crucial for the conclusion of Theorem 9. Just as a value-neutral signal can only make the DM worse off, other kinds of information can only benefit the DM. In particular, It can be shown that the DM will always benefit from learning information with the dual characteristics, i.e. observing a signal that is **ranking-neutral** in the

[14]While value-neutrality is generally a strong assumption, it holds for example with the widely-used binary signal structure: $S = \{s_A, s_B\}$ such that for any real valued function $h(b_A, b_B)$, $\mathbb{E}[h(b_A, b_B)|s_A] = \mathbb{E}[h(b_B, b_A)|s_B]$. Given symmetric binary signals, $\mathbb{E}[\max\{b_A, b_B\}|s_A] = \mathbb{E}[\max\{b_B, b_A\}|s_B] = \mathbb{E}[\max\{b_A, b_B\}|s_B]$.

sense that $\mathbb{E}[b_A|b_A > b_B, s]$ and $\mathbb{E}[b_B|b_A < b_B, s]$ are constant across $s \in S$ and also value-non-neutral in that $\mathbb{E}[\max\{b_A, b_B\}|s]$ varies with s.

Proof of Theorem 9. Assume the signal is value-neutral. Suppose first the DM has learned some signal $s \in S$, and a cheap-talk equilibrium $\mathbf{q}(s) = (q_A(s), q_B(s))$ ensues, where $q_i(s)$ denotes the probability of project $i = A, B$ being accepted by the DM when the agent recommends i given signal s. There are two possibilities. First, if $\mathbf{q}(s) = \mathbf{0}$, then the DM's payoff will be b_0. Suppose next $\mathbf{q}(s) > \mathbf{0}$. Then following the argument of Theorem 5, the DM's payoff is no higher than it is under delegation. The latter payoff is $\mathbb{E}[\max\{b_A, b_B\}|s]$, which by the value-neutrality assumption is independent of the signal realization and hence is equal to $\mathbb{E}[\max\{b_A, b_B\}]$. Thus, regardless of $\mathbf{q}(s)$, the DM's expected payoff from having learned s is no greater than $\max\{b_0, \mathbb{E}[\max\{b_A, b_B\}]\}$. But the latter is exactly the DM's expected payoff under "no information." More precisely, since b_A and b_B are identically distributed, $\mathbb{E}[b_A|b_A \geq b_B] = \mathbb{E}[b_B|b_A \leq b_B] = \mathbb{E}[\max\{b_A, b_B\}]$. Hence, if $\mathbb{E}[\max\{b_A, b_B\}] \geq b_0$, then a truthful equilibrium arises under no information, and if $\mathbb{E}[\max\{b_A, b_B\}] < b_0$, only the zero equilibrium arises under no information. Since the preceding argument applies to any signal realization, the first statement of the theorem follows.

Suppose next that the signal is also non-trivial. This implies that

$$\hat{b}_0 := \min_{s \in S} \left(\min\{\mathbb{E}[b_A|b_A \geq b_B, s], \mathbb{E}[b_B|b_A < b_B, s]\}\right) < \mathbb{E}[\max\{b_A, b_B\}] =: \bar{b}_0. \tag{31}$$

Consider any $b_0 \in (\hat{b}_0, \bar{b}_0)$. (31) implies that there is a truthful equilibrium under "no information," which gives the DM a payoff of $\mathbb{E}[\max\{b_A, b_B\}]$. (31) also implies that there is no truthful equilibrium following any observed signal; hence, following any observed signal, the DM's payoff is strictly less than $\mathbb{E}[\max\{b_A, b_B\}]$. Integrating over all possible signals, the second statement of the theorem follows. *Q.E.D.*

E.2 Preference conflicts over projects

An important extension of our baseline model is to allow the DM and the agent to have non-congruent preferences over the set of alternative projects. For instance, a seller may obtain a larger profit margin on a particular product, or a Dean may have a gender bias or prefer candidates who do research in a particular area. A simple way to introduce such conflicts is to assume that the agent derives a benefit $a_i b_i$ from project i, where $a_i > 0$ is common knowledge, while the DM continues to obtain b_i from project i.[15] The parameter $a := a_1/a_2 > 0$ is a sufficient statistic for the conflict of interest between the two projects: if $a > 1$, the agent's preferences are biased (relative to the DM's) toward the conditionally better-looking project, whereas when $a < 1$ the agent is biased toward the conditionally worse-looking project.

[15] This multiplicative form of bias is especially convenient to study, but it is also straightforward to incorporate an additive or other forms of bias.

We assume that the two projects are strongly ordered, and focus on "ranking equilibria" where the agent recommends project one if and only if $\alpha b_1 > b_2$ for some constant $\alpha > 0$, just as in the baseline model. A ranking equilibrium is still characterized by the DM's acceptance probabilities $\mathbf{q} = (q_1, q_2)$, but now the agent recommends project one if $aq_1b_1 > q_2b_2$ and project two otherwise. We will continue to say that the equilibrium is truthful if $q_1 = q_2 = 1$ and that the agent panders toward project i if $q_i > q_{-i} > 0$, i.e. he biases his recommendation toward project i from the perspective of his preferences, not from the DM's.[16] Consequently, the DM may benefit from pandering, as we discuss below. To avoid uninteresting cases, assume that the truthful equilibrium does not exist, i.e. either $\mathbb{E}[b_1|ab_1 > b_2] < b_0$ or $\mathbb{E}[b_2|ab_1 < b_2] < b_0$, and a non-zero equilibrium does exist.

Under strong ordering, one can show that there exists a critical threshold of conflict, $\overline{a} \in (1, \infty]$, such that *if* $a \in (0, \overline{a})$*, the largest equilibrium has pandering toward the conditionally better-looking project* ($\mathbf{q}^* = (1, q_2^*)$ *with* $q_2^* \in (0, 1)$) *while for* $a > \overline{a}$*, the largest equilibrium has pandering toward the conditionally worse-looking project* ($\mathbf{q}^* = (q_1^*, 1)$ *with* $q_1^* \in (0, 1)$).[17] Notice that if $a < 1$, the agent has a preference bias for project two but nevertheless panders toward project one in order to persuade the DM. If $a \in (1, \overline{a})$, then pandering reinforces the agent's bias to over-recommend project one from the DM's perspective.[18] Finally, when $a > \overline{a}$, the agent's preferences are so biased toward the good-looking project that a recommendation of project one is less credible than that of project two; hence the persuasion motive leads him to pander toward project two. It is not hard to check that even though the agent is pandering toward project two relative to his true preferences, he still over-recommends project one from the DM's perspective, i.e. $aq_1 \geq q_2$ in any equilibrium.

An important difference from the baseline model is that if $a < 1$ or $a > \overline{a}$, the DM benefits from some pandering in communication, because it counteracts the agent's preference bias. This affects the DM's gains from full delegation. If $a < 1$, a sufficiently small degree of pandering toward project one helps mitigate the agent's preference bias toward project two. Delegation then dominates communication only when the largest equilibrium has sufficiently severe pandering.[19]

[16] From the DM's point of view, the agent always recommends the better project only if $aq_1 = q_2$. Hence, the agent's recommendations are distorted whenever $aq_1 \neq q_2$.

[17] Formally, $\overline{a}$ is the value of a that solves $\mathbb{E}[b_2|b_2 > ab_1] = \mathbb{E}[b_1|b_2 < ab_1]$ if a solution exists (we assume it is unique, to avoid uninteresting complications), and $\overline{a} = \infty$ otherwise.

[18] Interestingly, in this case, the acceptance probability of project two in the largest equilibrium is increasing in the preference conflict a; the reason is that the agent's preference bias toward the better-looking project makes his recommendation of a worse-looking project more credible than the same recommendation made by an unbiased agent. Nevertheless, the DM's welfare is constant in the agent's preference bias so long as $a \in (0, \overline{a})$. The reason is that $q_2^* \in (0, 1)$ implies $\mathbb{E}[b_2|q_2^*b_2 > ab_1] = b_0$; hence, in equilibrium, a change in a triggers an offsetting change in q_2^* that keeps q_2^*/a constant, and thus does not affect the agent's recommendation strategy. Since the DM is indifferent across all $q_2 \in [0, 1]$ if we hold fixed the agent's strategy, the DM's welfare does not change.

[19] The same logic also implies that the comparative statics of the DM's expected utility in the outside option can be different from the baseline model; in particular, higher outside options even in the pandering region can raise the DM's expected utility. To see this, note that when $a < 1$, the largest equilibrium has pandering only if $b_0 > \mathbb{E}[b_2|b_2 > ab_1]$. As b_0 rises from this threshold, q_2^* falls so as to maintain $\mathbb{E}[b_2|q_2^*b_2 > ab_1] = b_0$. From the

If $a > \overline{a}$, in which case the agent is strongly biased in preference toward project one, the agent's pandering toward project two (recall, this is relative to the agent's preferred alternative) is always beneficial to the DM, so delegation is never optimal. Only when $a \in [1, \overline{a})$ is delegation optimal for any level of pandering.

The above observations highlight that pandering due to conflicts of interests over projects and pandering due to observable differences between projects have fundamentally different implications. In particular, if projects are identical ($F_1 = F_2$) but $a \neq 1$, then pandering is always beneficial to the DM: the agent knows that a proposal of a pet project is less credible, so he restrains himself from recommending such a project, i.e. he panders toward the project his preference is biased against. Delegation is then suboptimal. By contrast, in our baseline model where $F_1 \neq F_2$ and $a = 1$, we have seen that pandering is always detrimental to the DM, and delegation is strictly preferred whenever pandering occurs in the largest equilibrium.

E.3 Private information about outside option

The value of the outside option may be known only privately to the DM when the agent recommends a project. For example, a seller may not be privy to a buyer's reservation value of her product, or a CEO may know more than a division manager about the cost of capital. We can readily accommodate such situations by assuming that the value of the outside option, b_0, is observed privately by the DM prior to the agent's communication about $\mathbf{b}$. Suppose that b_0 is drawn from a distribution $G(\cdot)$ with strictly positive density on $[0, \infty)$. In this setting, a ranking equilibrium is described not by a vector of acceptance probabilities, but rather by a threshold vector (b_0^1, b_0^2) such that the DM follows the agent's recommendation of project $i \in \{1, 2\}$ if and only if $b_0 \leq b_0^i$, choosing her outside option otherwise. Assuming strong ordering, one can show that $b_0^1 > b_0^2$ and, hence, *there is pandering in any ranking equilibrium.*[20] Since the agent is uncertain about the outside option when communicating, he prefers to recommend the conditionally better-looking project, project one, when b_1 is only slightly below b_2, because this increases the probability of acceptance.[21]

What about the DM's decision to delegate, assuming this is made after she learns the value of b_0? One may think that for values of $b_0 \in (b_0^1, b_0^2)$, the DM does not want to delegate and instead just accepts project one when it is recommended. This logic is incomplete, however, because the DM's decision not to delegate would reveal that her outside option is high and thereby exacerbate the agent's pandering. Strikingly, it can be shown that the DM delegates

DM's point of view, a lower q_2^* is welfare improving until $q_2^* = a$, which obtains when $b_0 = \mathbb{E}[b_2 | b_2 > b_1]$.

[20] See the previous subsection for the notion of a ranking equilibrium.

[21] The DM has an incentive to try to convince the agent that her outside option is low, because the agent will pander less if he believes the outside option to have a lower value. Such communication from the DM is not credible, however, if the DM can only make cheap-talk statements about the outside option. By contrast, if the DM can engage in verifiable disclosure about the outside option value, there will full revelation due to an unraveling argument.

project choice in equilibrium if and only if $b_0 \le \mathbb{E}[\max\{b_1, b_2\}]$ — just as in the baseline model where b_0 was common knowledge.

E.4 Flexibility in resource allocation

In many settings, the DM may have flexibility in allocating resources: she could implement both projects if she wants, or vary the size of her investment in a project. A first-best outcome would require that the resources invested should be responsive to the quality of the projects. In reality, however, business units often receive fixed budgets, university departments are given a fixed number of hiring slots, and so on. We will show that such an inflexible allocation rule may be explained as an optimal response to pandering by the agent. As with the previous extensions, we focus on ranking equilibria as described earlier.

Variable project size. Assume that the DM must decide how much to invest in one of two projects, where $q_i b_i$ are the returns to investing q_i in project i; these are common to both the DM and the agent. For simplicity, assume further that resource costs are quadratic in q_i and are incurred only by the DM. In equilibrium, projects with higher expected values then receive more resources from the DM. As a result, one can show that in any ranking equilibrium, the agent always panders toward the conditionally better-looking project no matter the value of the outside option.[22] Again, the resulting distortion can be mitigated by delegation, provided the DM can put a cap on the maximum investment the agent can make (knowing that the agent will always invest the maximum allowed). This is equivalent to giving the agent an inflexible budget but allowing freedom in how to spend that budget.

Non-exclusive projects. There are many situations in which the DM may choose to implement multiple projects. For example, a corporate board may approve several capital investment projects if the expected profits of each exceed their cost of capital, or a Dean may want to hire both economists if they are both sufficiently good. To fix ideas, assume that the DM may choose to implement neither, either, or both projects. If both projects are chosen, both the DM and the agent obtain a payoff of $b_1 + b_2$; if only project $i \in \{1, 2\}$ is chosen, the DM gets $b_i + b_0$ while the agent gets b_i; and if neither is chosen, the DM gets $2b_0$ while the agent gets 0.

In this setting, one may wonder if the intuition of pandering toward better-looking projects in order to persuade would still apply. In particular, is it possible that the agent, in equilibrium, panders toward the worse-looking project in order to increase the chances that *both* projects are selected? Such a possibility is particularly relevant if b_0 is such that

$$\mathbb{E}[b_1|b_1 > b_2] > \mathbb{E}[b_2|b_2 > b_1] > \mathbb{E}[b_1|b_2 > b_1] > b_0 > \mathbb{E}[b_2|b_1 > b_2], \tag{32}$$

[22] This extension permits a comparison with Blanes i Vidal and Moller (2007), who show that a principal may select a project that she privately knows is inferior but is perceived to be of higher quality by an agent who must exert costly effort to implement the project. Intuitively, the DM in our model is the agent in theirs whose implementation effort is increasing in his posterior on project quality.

because in this case project one appears to "shoe in" (it would be implemented even if the agent truthful ranks project two ahead of project one), while project two would be implemented if truthfully ranked ahead but not if truthfully ranked behind.

One can show that any influential ranking equilibrium still has pandering toward the conditionally better-looking project, project one. In such an equilibrium, when the agent ranks project one ahead of project two, the DM accepts it and also accepts project two with some probability. If the agent ranks project two ahead of project one, the DM accepts it but rejects project one. Hence, even if the DM can implement both projects, the communication is still biased toward the conditionally better-looking project. Existence of an influential ranking equilibrium requires $\mathbb{E}[b_2] \geq b_0$. If $\mathbb{E}[b_2] < b_0$, the only equilibrium is the one where the DM always chooses project one.

Interestingly, if $\mathbb{E}[b_2] < b_0$, the DM would benefit from committing herself to implement at most one project. Indeed, the inequalities in (32) imply that the agent will then truthfully reveal the better project, and the DM will follow this recommendation. By contrast, if the DM does not make such a commitment, the desire to get both projects adopted destroys the credibility of the agent's communication.

F Revelation of Verifiable Information

In this Appendix, we show how revelation of hard information by the agent can lead to asymmetries in soft information about projects. This formalizes the assertion at the end of Section II of the paper that asymmetric distributions for the project values can be viewed as resulting from either asymmetries that are directly observable to the DM or private but verifiable information of the agent that is fully revealed.

Formally, suppose that all projects are ex-ante identical. Each project i independently draws from a distribution $G(\cdot)$ a verifiable component, $v_i \in V$, where V is a compact subset of $\mathbb{R}$. Thereafter, each project draws it's value b_i independently from a family of distributions $F(b_i|v_i)$ with density $f(b_i|v_i)$. The agent privately observes the vector $(\mathbf{v}, \mathbf{b})$ and then communicates with the DM in two stages. First, he sends a vector of messages $\mathbf{r} := (r_1, \ldots, r_n)$ about $\mathbf{v}$ subject to the constraint that for each i,

$$r_i \in \{X : X \subseteq V, X \text{ is closed}, v_i \in X\}.$$

This formulation captures that each v_i is hard information: the agent can claim that v_i lies in any subset of V so long as the claim is true. Thereafter, the agent sends a cheap-talk message just as in our baseline model. Finally, the DM implements a project or the outside option.

The key assumption we make is that the distributions $F(b|v)$ satisfy the monotone likelihood-ratio property (MLRP): if $v > v'$, then for all $b > b'$, $\frac{f(b|v)}{f(b'|v)} > \frac{f(b|v')}{f(b'|v')}$. Moreover, assume that for any vector of hard information, $\mathbf{v}$, the project distributions $(F(\cdot|v_1), \ldots, F(\cdot|v_n))$ satisfy strong ordering.

Theorem 10. *In this extended model with privately observed hard information and any $n \geq 2$, there is an equilibrium where the agent fully reveals his hard information by sending $r_i = v_i$, and the subsequent cheap-talk subgame outcome is identical to the largest equilibrium, $\mathbf{q}^*$, of our baseline model where each $F_i = F(\cdot|v_i)$.*

Proof. Consider a skeptical posture by the DM, where for any hard information report $r_i \subseteq V$, the DM believes that $v_i = \min r_i$. Then for any profile $\mathbf{r}$, the DM plays the $\mathbf{q}^*$ associated with our baseline model where each $F_i = F(\cdot|\min r_i)$. Since $F(b|v)$ has the MLRP, Theorem 2 for for $n = 2$ and Theorem 8 for $n > 2$ imply that if the agent deviates from $\mathbf{r} = \mathbf{v}$ to any other hard information report, he only induces a weakly smaller acceptance profile from the DM in the ensuing cheap-talk game. Thus the agent can do no better than playing $r_i = v_i$ and then playing according to $\mathbf{q}^*$ of the game where $F_i = F(\cdot|v_i)$. Plainly, the DM is playing optimally as well. *Q.E.D.*

G On Strong Ordering

G.1 Condition (R2)

In this subsection, we show that the restrictive portion of strong ordering, viz. that (R2) must be satisfied for each $i \in \{1, 2\}$, holds if projects are drawn from a number of familiar families of distributions whose support is contained in the non-negative reals: Pareto distributions, Power Functions distributions, Weibull distributions, and at least for a subset of its parameters, Gamma distributions. Note that it is not necessary that the distribution for both projects need be in the same family.

Recall that Lemma 3 provides a sufficient condition for (R2) that depends only on F_{-i}. Accordingly, to ease notation, in this subsection only we will drop the project subscript and just a distribution $F(b)$ with density $f(b)$ and support $[\underline{b}, \overline{b}]$, where $\underline{b} \geq 0$ and $\overline{b} \leq \infty$. The sufficient condition in Lemma 3 is that project $-i$ be drawn a distribution F whose reverse hazard rate $r(b) := f(b)/F(b)$ is decreasing fast enough so that $br(b)$ is non-increasing. We will use a few equivalent formulations:

$$\text{For any } \overline{b} > b' > b > \underline{b}: \; br(b) \geq b'r(b'), \tag{R2$'$}$$

or when the density f is differentiable,

$$\text{For any } \overline{b} > b > \underline{b}: -\frac{b}{r(b)}r'(b) \geq 1, \tag{R2$''$}$$

where a prime on a function denotes its derivative. Yet another useful version is generated by noticing that $br(b)$ being non-increasing is equivalent to $\frac{F(b)}{bf(b)}$ being non-decreasing, which for a differentiable density is equivalent to

$$b - \frac{F(b)}{f(b)}\left(1 + b\frac{f'(b)}{f(b)}\right) \geq 0. \tag{R2'''}$$

Pareto distribution. The Pareto distribution has support $[\underline{b}, \infty)$ and cdf $F(b) = 1 - (\underline{b}/b)^k$ for some parameters $\underline{b}, k > 0$. So $f(b) = k\underline{b}^k b^{-k-1}$ and (R2′) requires that for any $b' > b \geq \underline{b}$,

$$\left(\frac{b'}{b}\right)^k \geq \frac{1-(\underline{b}/b)^k}{1-(\underline{b}/b')^k}.$$

This inequality holds because it is true when $b' = b$ and the LHS is strictly increasing in b' while the RHS is strictly decreasing in b'.

Power function distribution. The Power function distribution has support $[x, y]$ and cdf $F(b) = \frac{(b-x)^k}{(y-x)^k}$ for some $\infty > y > x \geq 0$ and $k > 0$.[23] Note that the case of $k = 1$ subsumes uniform distributions. Since $f(b) = \frac{k}{(y-x)^k}(b-x)^{k-1}$, (R2′) requires that for any $y \geq b' > b \geq x$,

$$bk(b-x)^{-1} \geq b'k(b'-x)^{-1}.$$

This condition simplifies to $\frac{b'}{b}x \geq x$, which is true because $b' > b$ and $x \geq 0$.

Weibull Distribution. The Weibull distribution has support $[0, \infty)$ and cdf $F(b) = 1 - e^{-(b\lambda)^k}$ for some $\lambda, k > 0$. The density is $f(b) = (k\lambda)(b\lambda)^{k-1}e^{-(b\lambda)^k}$. Note that $k = 1$ subsumes the exponential distribution.

We compute

$$r(b) = \frac{(k\lambda)(b\lambda)^{k-1}e^{-(b\lambda)^k}}{1-e^{-(b\lambda)^k}}$$

and

$$r'(b) = \frac{e^{-(b\lambda)^k}(k\lambda)(b\lambda)^{k-2}\lambda\left[(k-1)(1-e^{-(b\lambda)^k}) - (b\lambda)^k k\right]}{(1-e^{-(b\lambda)^k})^2}.$$

Hence,

$$-\frac{b}{r(b)}r'(b) =: \mathcal{E}(b) = \frac{(b\lambda)^k k}{1-e^{-(b\lambda)^k}} + 1 - k.$$

[23] In general, one does not need $x \geq 0$, but we require it because projects must have non-negative values.

To verify (R2″), we must show that $\mathcal{E}(\cdot) \geq 1$. By L'Hôpital's rule,

$$\mathcal{E}(0) = \left.\frac{kb^{k-1}k\lambda^k}{e^{-(b\lambda)^k}k\lambda(b\lambda)^{k-1}}\right|_{b=0} + 1 - k = \left.\frac{k}{e^{-(b\lambda)^k}}\right|_{b=0} + 1 - k = 1.$$

So it suffices to show that $\mathcal{E}'(\cdot) \geq 0$. Differentiating and rearranging yields

$$\mathcal{E}'(b) \propto k^2\lambda^k b^{k-1}\left[1 - e^{-(b\lambda)^k}\left(1 + (b\lambda)^k\right)\right].$$

Writing $x = b\lambda$, it therefore suffices to show that for any $x \geq 0$,

$$1 \geq e^{-x^k}\left(1 + x^k\right) =: g(x).$$

This is true because $g(0) = 1$ and

$$g'(x) \;=\; e^{-x^k}kx^{k-1} + \left(1 + x^k\right)e^{-x^k}\left(-kx^{k-1}\right) = -e^{-x^k}x^{k-1}kx^k \leq 0.$$

Remark 4. Unlike the Pareto and Power function distributions, whose densities are non-increasing, the Weibull distribution family includes densities that are strictly increasing in some region of the domain; this is the case whenever $k > 1$. Nevertheless, it is known that any Weibull distribution is log-concave and hence has a decreasing reverse hazard rate (e.g. Bagnoli and Bergstrom, 2005); what we have shown above is that the reverse hazard rate decreases fast enough that $br(b)$ is non-increasing.

Gamma Distribution. The Gamma distribution has support $[0, \infty)$ and density function $f(b) = \frac{x^{\alpha-1}e^{-x\beta}}{\Gamma(\alpha)(1/\beta)^\alpha}$ for some $\alpha, \beta > 0$. The cdf is $F(b) = \frac{\int_0^b u^{\alpha-1}e^{-u\beta}du}{\Gamma(\alpha)(1/\beta)^\alpha}$. The density of the Gamma distribution is non-increasing if and only if $\alpha \leq 1$. We will show that (R2‴) is satisfied when $\alpha \leq 1$. Note that $\alpha = 1$ subsumes the exponential distribution.

Since $f'(b) = \frac{b^{\alpha-2}e^{-b\beta}(\alpha-1-b\beta)}{\Gamma(\alpha)(1/\beta)^\alpha}$, it follows that $b\frac{f'(b)}{f(b)} = \alpha - 1 - b\beta$, and hence (R2‴) is verified by showing that for any $b \geq 0$,

$$b - \frac{F(b)}{f(b)}(\alpha - b\beta) \geq 0. \tag{33}$$

The above inequality clearly holds for all $b \geq \alpha/\beta$. So restrict attention to $b < \alpha/\beta$. Observe that

$$\frac{F(b)}{f(b)} = \frac{\int_0^b u^{\alpha-1}e^{-u\beta}du}{b^{\alpha-1}e^{-b\beta}} \leq \frac{\int_0^b u^{\alpha-1}du}{b^{\alpha-1}e^{-b\beta}} = \frac{b}{\alpha e^{-b\beta}},$$

where the inequality is because for any $u \geq 0$, $e^{-u\beta} \leq 1$. Hence, (33) is true if $b - \frac{b}{\alpha e^{-b\beta}}(\alpha - b\beta) \geq 0$, or equivalently if $e^{-b\beta} \geq 1 - \frac{b\beta}{\alpha}$. This inequality holds for any $\alpha \leq 1$ because $e^{-x} \geq 1 - x$ for any

$x \geq 0$.

Remark 5. While we have proven above that condition (R2‴) is satisfied for Gamma distributions with $\alpha \leq 1$, numerical analyses suggest that it holds even when $\alpha > 1$.

G.2 Condition (R1)

As discussed in the main text, the first part of strong ordering — condition (R1) — can be viewed as essentially a labeling convention that project one is the conditionally better-looking project. Nevertheless, given a fixed labeling of projects with distributions F_1 and F_2, it is of interest to know whether (R1) is satisfied. While one may often directly compute the relevant conditional expectations (as in Appendix C for the leading examples), the following result provides a demanding but general sufficient condition for (R1). Recall that $r_i(b) := f_i(b)/F_i(b)$ is the reverse hazard rate for project $i \in \{1, 2\}$.

Lemma 10. (R1) *is satisfied if*

$$\bar{b}_1 \geq \bar{b}_2 \text{ and } r_1/r_2 \text{ is non-decreasing on } (\max\{\underline{b}_1, \underline{b}_2\}, \bar{b}_1), \tag{R1$'$}$$

and either $\bar{b}_1 > \bar{b}_2$ or r_1/r_2 is not constant on the specified interval.

Proof. We first reproduce some notation introduced in the proof of Lemma 3:

$$\Upsilon_i(y) := \mathbb{E}[b_i | b_i > y b_j] = \int_0^\infty b_i f_i(b_i | b_i > y b_j) db_i = \int_0^\infty b \hat{f}_i(b; y) db,$$

where

$$\hat{f}_i(b; y) := \begin{cases} \frac{F_{-i}(\frac{b}{y}) f_i(b)}{\int_0^\infty F_{-i}(\frac{\tilde{b}}{y}) f_i(\tilde{b}) d\tilde{b}} & \text{if } b \in [\max\{\underline{b}_i, y\underline{b}_{-i}\}, \bar{b}_i] \\ 0 & \text{otherwise.} \end{cases}$$

Condition (R1) states that $\Upsilon_1(1) > \Upsilon_2(1)$. Since the support of each $\hat{f}_i(\cdot; 1)$ is $[\max\{\underline{b}_1, \underline{b}_2\}, \bar{b}_i]$, a well-known consequence of domination in likelihood ratio implies that a sufficient condition for $\Upsilon_1(1) > \Upsilon_2(1)$ is that $\bar{b}_1 \geq \bar{b}_2$ and

$$\frac{\hat{f}_1(b'; 1)}{\hat{f}_1(b; 1)} \geq \frac{\hat{f}_2(b'; 1)}{\hat{f}_2(b; 1)} \text{ for all } b, b' \text{ such that } \max\{\underline{b}_1, \underline{b}_2\} < b < b' < \bar{b}_2,$$

with either $\bar{b}_1 > \bar{b}_2$ or the ratio inequality above holding strictly for a positive measure of (b, b'). The proof is completed by observing that the above ratio inequality is equivalent to

$$\frac{r_1(b')}{r_1(b)} \geq \frac{r_2(b')}{r_2(b)} \text{ for all } b, b' \text{ such that } \max\{\underline{b}_1, \underline{b}_2\} < b < b' < \bar{b}_2,$$

because $\frac{\hat{f}_i(b';1)}{\hat{f}_i(b;1)} = \frac{F_{-i}(b')f_i(b')}{F_{-i}(b)f_i(b)}$. *Q.E.D.*

We apply the sufficiency condition (R1′) in two ways. First, we use it to show that (R1) is satisfied if the support of F_1 contains that of F_2 and the two densities are equal within the support of F_2 up to scaling:

Theorem 11. *(R1) is satisfied if (i) $\underline{b}_1 \leq \underline{b}_2$ and $\bar{b}_1 \geq \bar{b}_2$ with at least one strict inequality, and (ii) there is some $x \in (0,1)$ such that $f_1(b) = xf_2(b)$ for all $b \in [\underline{b}_2, \bar{b}_2]$.*

Proof. Pick any b, b' such that $\underline{b}_2 < b < b' < \bar{b}_2$. Then,

$$\begin{aligned}\frac{r_1(b')}{r_1(b)} = \frac{f_1(b')}{F_1(b')}\frac{F_1(b)}{f_1(b)} &= \frac{xf_2(b')}{F_1(\underline{b}_2) + xF_2(b')}\left(\frac{F_1(\underline{b}_2) + xF_2(b)}{xf_2(b)}\right) \\ &= \frac{f_2(b')}{f_2(b)}\left(\frac{F_1(\underline{b}_2)/x + F_2(b)}{F_1(\underline{b}_2)/x + F_2(b')}\right) \\ &\geq \frac{f_2(b')}{f_2(b)}\frac{F_2(b)}{F_2(b')} = \frac{r_2(b')}{r_2(b)},\end{aligned}$$

where the inequality is because $b' > b$. Hence, Lemma 10 applies. *Q.E.D.*

Second, we can use (R1′) to understand further why the conditionally better-looking ranking does not necessarily imply that F_1 likelihood-ratio dominates F_2.

Theorem 12. *Assume f_1 and f_2 are differentiable. If F_1 dominates F_2 in reverse hazard rate,*[24] *and furthermore r_1/r_2 is non-decreasing on $(\underline{b}_1, \bar{b}_2)$, then F_1 dominates F_2 in likelihood ratio.*

Proof. Assume the hypotheses. The reverse hazard rate dominance implies both $\underline{b}_1 \geq \underline{b}_2$ and $\bar{b}_1 \geq \bar{b}_2$. So it suffices to prove that f_1/f_2 is non-decreasing on $(\underline{b}_2, \bar{b}_2)$. Using primes for derivatives and omitting arguments, we have that within the domain $(\underline{b}_2, \bar{b}_2)$,

$$(\ln(r_1/r_2))' = \frac{r_1'}{r_1} - \frac{r_2'}{r_2} = \frac{F_1}{f_1}\frac{F_1f_1' - (f_1)^2}{(F_1)^2} - \frac{F_2}{f_2}\frac{F_2f_2' - (f_2)^2}{(F_2)^2} = (r_2 - r_1) + \left(\frac{f_1'}{f_1} - \frac{f_2'}{f_2}\right)$$

and hence

$$\frac{f_1'}{f_1} - \frac{f_2'}{f_2} = (\ln(r_1/r_2))' + (r_1 - r_2) \geq 0,$$

where the inequality is by the hypotheses. Since $(f_1/f_2)' \propto f_2f_1' - f_1f_2'$, the desired conclusion follows. *Q.E.D.*

Theorem 12 says that if (R1′) holds and yet F_1 does not dominate F_2 in likelihood ratio, then it must be that F_1 does not dominate F_2 in reverse hazard rate. This suggests that a failure

[24] Recall that F_1 dominates F_2 in reverse hazard rate if $\underline{b}_1 \geq \underline{b}_2$, $\bar{b}_1 \geq \bar{b}_2$, and $r_1(b) \geq r_2(b)$ for all $b \in (\underline{b}_1, \bar{b}_2)$.

of reverse hazard dominance of F_1 over F_2 is a likely "culprit" when project one is conditionally better looking but does not dominate project two in likelihood ratio.[25]

References

Bagnoli, Mark and Ted Bergstrom, "Log-concave Probability and its Applications," *Economic Theory*, 08 2005, *26* (2), 445–469.

Blanes i Vidal, Jordi and Marc Moller, "When Should Leaders Share Information with their Subordinates?," *Journal of Economics and Management Strategy*, 2007, *16*, 251–283.

[25]We say "suggests" because (R1′) is a sufficient but certainly not necessary condition for (R1).