Supplementary Appendix for "Signaling Character in Electoral Competition"

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This supplementary appendix adds detail and formal results to the discussion in Section 5 of Kartik and McAfee (forthcoming). For consistency with the main article, Theorems here are numbered starting from 3 and equations starting from 4. References to Theorem 1 are to Theorem 1 in the main article. All proofs of results here are collected at the end of the document.

Tied Elections

Our main model results in all elections between strategic candidates being tied. Moreover, if the weight on character, λ, is large enough, then all elections end in ties. Nevertheless, we do not wish to suggest that elections in real life typically end in ties. This is a consequence of the assumption we made that candidates have no uncertainty about the electorate, and in particular, about the median voter's location, m. Consider an extended model where candidates share a common belief about the median voter's location, but the uncertainty is only resolved ex-post after positions have been chosen. Our analysis carries through unchanged, but it would now be the case that after uncertainty has been resolved at the last stage, ties do not generally occur.[1] However, the ex-ante probability of winning when two strategic candidates compete against each other remains one half. Indeed, this is an unavoidable and not unreasonable feature of any (symmetric) model with strategic office-motivated candidates. Note, in particular, that it also applies to the standard Downsian model.

[1] It remains true that neither candidate has an incentive to deviate from his strategy after observing the other candidate's position, but this property would not hold true once the median voter's location is revealed. On a related point, adding private information for the candidates about the location of the median voter would complicate matters (Bernhardt et al., forthcoming).

Broader Interpretations of Character

Under some conditions, our model can be given a richer interpretation.[2] The main hypothesis we posed is that there is an unobservable trait among politicians that is valued by voters and is also negatively correlated with the willingness to pander to the public in order to get elected. Assume instead that while there is an unobservable trait that is valued by voters, it has nothing to do with willingness to pander. For example, the trait may be competence: some candidates are competent, some are not. All candidates are purely office-motivated and fully strategic. Suppose voters conjecture that competent candidates are playing the strategy F, which recall is the (exogenous) distribution chosen by non-strategic types in our model, whereas incompetent candidates are playing the strategy G, which recall is the (endogenous) distribution chosen by strategic types in our model. If the preference weight on competence (λ) is large enough such that the equilibrium of our model has a full support G, then the median voter is indifferent upon observing any platform. But then, candidates are indifferent over platform choices, and it is in fact an equilibrium for the competent ones to play G and the incompetent ones to play F.

The benefit of this perspective is that our policy divergence equilibrium is rationalized under much broader interpretations of unobservable traits, viz. any unobservable but desirable trait, even if it has nothing to do with willingness to pander for office. However, there are two caveats: first, it only applies if the preference weight on the unobservable trait is sufficiently large; second, and perhaps more importantly, unlike in our model with non-strategic types, uniqueness of equilibrium will not hold here. In particular, a "median voter equilibrium" also exists, where all candidates choose the median voter's platform. This is sustained in equilibrium by the out-of-equilibrium beliefs that when any non-median platform is observed, the candidate must not possess the desirable trait (e.g., must be incompetent).

Ex-ante Asymmetry

Suppose that the prior likelihood of having character can differ across candidates, and moreover, the distribution of policies conditional on character can also differ. This may be appealing when thinking of candidates as representing different political parties, for example. We now index b and f as b^i and f^i for each $i \in \{A, B\}$; the setup is otherwise unchanged. Call this the *asymmetric model.*

Theorem 3. *In the asymmetric model,*

1. *There is an ex-post equilibrium where candidate $i \in \{A, B\}$ uses the strategy, $\hat{G}^i$, with density*

$$\hat{g}^i(x) = \max\left\{0, \frac{b^i f^i(x)}{1-b^i}\left[\frac{\lambda}{\hat{\alpha}^i - \mu(x)} - 1\right]\right\} \tag{4}$$

[2] We thank Dino Gerardi for this suggestion.

where $\hat{\alpha}^i \in (\mu(m), \mu(m) + \lambda)$ *is a constant.*

2. *If* $\hat{\alpha}^A = \hat{\alpha}^B$*, then the above is the unique equilibrium.*

3. *If* $\hat{\alpha}^i > \hat{\alpha}^j$*, then in any equilibrium, candidate* i *wins with probability* 1 *when strategic.*

The first part of Theorem 3 is a trivial extension of the existence portion of Theorem 1. As before, the constant $\hat{\alpha}^i$ is unique and determined by the requirement that $\int_x \hat{g}^i(x)\, dx = 1$. As was the case earlier, the median voter's expected utility from electing candidate i is $\hat{\alpha}^i$, for any observed platform that is in the support of strategic i's strategy in this equilibrium. The difference with the base (symmetric) model is that it will no longer generally be true that $\hat{\alpha}^A = \hat{\alpha}^B$. Therefore, one of the candidates may win with probability 1 whenever he is a strategic type. For example, if $f^A = f^B$ but $b^A > b^B$, then when playing the strategies $\hat{G}^A$ and $\hat{G}^B$, candidate A always wins so long as the chosen platform is in the support of G^A.

That one of the candidates may win with probability 1 when there is an ex-ante asymmetry is not all too surprising. Such a property of the Downsian model is well-known when one candidate has an observable valence advantage (Aragones and Palfrey, 2002; Groseclose, 2001). The effect of asymmetric b^i's (or f^i's) is similar in our model, since it endows one candidate with an ex-ante advantage. As is the approach taken in that literature, extending our setting to one where there is uncertainty over the median voter's location—as we have already outlined earlier—would yield the possibility that even with ex-ante asymmetry, neither strategic candidate wins with probability 1.

Part 2 of Theorem 3 says that if the constellation of parameters do in fact generate the same $\hat{\alpha}$, then the equilibrium identified in Part 1 is unique. Thus, the result subsumes Theorem 1. When $\hat{\alpha}^A \neq \hat{\alpha}^B$, then the equilibrium identified in Part 1 of Theorem 3 may not be unique. Intuitively, say that candidate B is at an ex-ante disadvantage relative to candidate A, and will lose with probability 1 when both are strategic types in the equilibrium of Part 1. Then, candidate B may be indifferent over multiple losing strategies when strategic, and this can lead to a multiplicity of equilibria. However, the multiplicity is inessential in terms of whether candidate A wins when strategic. This is the content of Part 3 of Theorem 3.

Richer and Endogenous Preferences for Character

We now enrich the preferences for character thus far considered. Let utility for a voter with ideal point v facing a candidate i with policy x^i be given by

$$U(x^i, v) \equiv \lambda(x^i, v) \Pr\left(c^i = 1 \middle| x^i\right) + u\left(x^i, v\right)$$

Here, the weight placed on character need not be constant—which was the assumption before—but instead can depend upon both the candidate's platform and a voter's ideal point. One natural motivation for such a specification is to capture the idea that politicians take actions of two kinds: one observable ("in plain view") and one unobservable ("out of sight"). The $u(\cdot,\cdot)$ component of a voter's utility represents the utility over observable actions that have been committed to by the politician during the electoral process. The $\lambda(\cdot,\cdot)$ component represents the voter's utility over the unobservable actions that the politician will take in office. If the politician has character, then his position on the unobservable dimension will be the same as what he committed to on the observable action, since those with character say what they will actually do. If he does not have character, however, he might do something very different on the unobservable dimension that what he promised (and necessarily lives up to) on the observable dimension. That $\lambda(\cdot,\cdot)$ can vary over policies and over voter ideal points allows for the possibility that different voters care differently about whether a candidate keeps his word behind the scenes, and moreover, this can depend on the position a candidate promised in different ways to voters. For example, a voter with ideal point $v = 1$ may prefer a candidate with platform $x^i = 0$ to *not* have character and thus likely do something different on the unobservable dimension than what was promised. On the other, the same voter may prefer a candidate with $x^i = 1$ to in fact have character, thus guaranteeing that he will take the same policy position on the unobservable dimension. This preference ordering over character can be reversed for a voter with ideal point $v = 0$.

The function $u(x, v)$ is exactly the same as earlier. Following the above discussion, we assume that $\lambda(x, v)$ is twice continuously differentiable, $\lambda(v, v) > 0$ for all v, and $\lambda_{12}(x, v) \geq 0$ for all x, v.[3] In words, preferences for character are smooth; every voter values character positively at least when a candidate's platform is her most-preferred policy; and as the platform of a candidate shifts to the right, the marginal increase in the preference for character is weakly higher for voters with ideal points further to the right. The assumptions on candidates are as before. Call this the *rich preferences model.* The following result extends our main insight to this setting, under a further assumption on the value for character.

Theorem 4. *If* $\max_{x,v} |\lambda_2(x, v)|$ *is sufficiently small,*[4] *the rich preferences model has an ex-post equilibrium where both candidates use the same strategy,* $\overline{G}$*, with density*

$$\overline{g}\,(x) = \max\left\{0, \frac{bf\,(x)}{1-b}\left[\frac{\lambda(x,m)}{\overline{\alpha} - u\,(x,m)} - 1\right]\right\}$$

where $\overline{\alpha} > u(m, m)$ *is the unique constant such that* $\int_x \overline{g}(x)dx = 1$.

The restriction on $|\lambda_2(\cdot,\cdot)|$ requires that the value on a candidate's character for any particular policy platform not change too sharply with voter ideal points. The logic of the

[3] In fact, the assumption that $\lambda(v, v) > 0$ for all v is stronger than necessary. All that is needed for the ensuing result is that $\lambda(x, v) > 0$ for some x. So long as the median voter values character positively at some policy platform, it's not necessary that $\lambda(\cdot, v)$ attain its maximum at v, nor that every voter value character positively for some platform.

[4] More precisely, write $\lambda(x, v) = \tilde{\lambda}(x) + \theta\hat{\lambda}(x, v)$. We require that $\tilde{\lambda}(x) > 0$ for all x. The conclusion of the Theorem is true for all θ smaller than some threshold $\overline{\theta} > 0$.

equilibrium parallels Theorem 1. By construction of the equilibrium strategy, $\overline{G}$, voter m is indifferent over all platforms in the support of $\overline{G}$. The assumption on $|\lambda_2(\cdot,\cdot)|$ ensures that voter m remains a well-defined median voter in the space of campaign platforms, taking into account both direct policy utility and inferred character utility. This guarantees that indifference of m is sufficient for elections to end in ties among all platforms in the support of $\overline{G}$, which implies that it is optimal for strategic candidates to play $\overline{G}$. Plainly, the basic case we considered in the main part of the paper, that $\lambda(x, v)$ is a strictly positive constant, is covered by the Theorem.

No Commitment

Suppose that campaign statements are pure cheap talk. If elected, a politician need not necessarily implement his campaign platform, and can instead choose any policy in the policy space. We suppose that the policy a candidate will actually implement if elected is private information, and drawn from the density $f(x)$. A candidate also has character with probability b; as usual, this is also private information. Candidates with character announce precisely what they will do if elected, whereas candidates without character will say anything to get elected. Voters have preferences *only* over final policy, $u(x, v)$, and not directly over character at all. All the notation follows our standard use.

Suppose that a politician without character plays the same distribution over platforms independent of his ideal point. It follows that a voter with ideal point v has expected utility from a platform of x^i $(i = A, B)$:

$$U\left(x^i, v\right) = \varphi^i\left(x^i\right) u\left(x^i, v\right) + \left(1 - \varphi^i\left(x^i\right)\right) \mathbb{E}_x\left[u\left(x, v\right)\right]$$

where $\varphi^i(x^i)$ is the Bayes update about character of candidate i given his platform x^i, and the expectation term is taken with respect to the prior density f, without conditioning on the announced platform (since by hypothesis strategic politicians do not vary their strategy based on their ideal points). Rewriting gives

$$U\left(x^i, v\right) = \mathbb{E}_x\left[u\left(x, v\right)\right] + \varphi^i\left(x^i\right)\left[u\left(x^i, v\right) - \mathbb{E}_x\left[u\left(x, v\right)\right]\right]$$

Analogous to the construction of equilibrium strategies in the main article, define the strategic candidate's density by

$$\tilde{g}\left(x\right) = \max\left\{0, \frac{bf\left(x\right)}{1-b}\left[\frac{u\left(x, m\right) - \mathbb{E}_x\left[u\left(x, m\right)\right]}{\tilde{\alpha} - \mathbb{E}_x\left[u\left(x, m\right)\right]} - 1\right]\right\} \tag{5}$$

There is a unique constant $\tilde{\alpha} \in \left(\mathbb{E}_x\left[u\left(x, m\right)\right], u\left(m, m\right)\right)$ such that $\int_x \tilde{g}(x) = 1$, by the usual argument. This formula in (5) has an intuitive interpretation: a strategic candidate can offer utility of at least $\mathbb{E}_x\left[u\left(x, m\right)\right]$ and at most $u\left(m, m\right)$.

Theorem 5. *If* $\max_{x,v} \left| u_2(x,v) - \frac{d}{dv}\mathbb{E}_x[u(x,v)] \right|$ *is sufficiently small,*[5] *the model without policy commitment has an ex-post equilibrium where both candidates use the strategy given by* (5).

The requirement that $\max_{x,v} \left| u_2(x,v) - \frac{d}{dv}\mathbb{E}_x[u(x,v)] \right|$ be sufficiently small plays a similar role to the restriction on λ_2 in Theorem 4: it is necessary to ensure that the voter m remains a well-defined median voter in the sense that her indifference—which is by construction of the density in (5)—is sufficient for elections to end in ties. It has the interpretation that for any x, the benefit (or loss) of having that policy with certainty over a random policy draw must not change by much for a small change in voter ideal points.

Proofs of Theorems 3–5

Proof of Theorem 3. The first part is almost identical to the existence portion of Theorem 1, hence omitted. To prove the second and third parts, two intermediate claims are needed. For $i \in \{A, B\}$, let $\hat{\alpha}^i$ denote the constant defined by $\int_X \hat{g}^i(x)\,dx = 1$.

Claim 1: $\hat{G}^i$ is the unique solution to the following program:

$$\max_{G^i,\varphi^i} \left[\min_{x \in Supp(G^i)} \lambda\varphi^i(x) + \mu(x) \right] \text{ s.t. } \varphi^i \text{ being a posterior given } G^i \tag{6}$$

Proof: Let $\tilde{G}^i$ be a solution to program (6). We argue that $\tilde{G}^i = \hat{G}^i$. The support of $\tilde{G}^i$ must be contained in the support of $\hat{G}^i$; otherwise by the construction of $\hat{G}^i$, it is immediate that $\tilde{G}^i$ cannot be a solution to (6). Clearly then $\tilde{G}^i$ has no atoms; hence it has a density. But then if there is an interval on which $\tilde{g}^i(x) > \hat{g}^i(x)$, there must be an interval on which $\tilde{g}^i(x) < \hat{g}^i(x)$, for the densities to integrate to 1. This contradicts $\tilde{G}^i$ being a solution to (6). ‖

Claim 2: $\hat{G}^i$ is the unique solution to the following program:

$$\min_{G^i,\varphi^i} \left[\max_{x} \lambda\varphi^i(x) + \mu(x) \right] \text{ s.t. } \varphi^i \text{ being a posterior given } G^i \tag{7}$$

Proof: Let $\breve{G}^i$ be a solution to program (7). We argue that $\breve{G}^i = \hat{G}^i$. Suppose not. Let $\breve{\alpha}^i \equiv \max_x \lambda\breve{\varphi}^i(x) + \mu(x)$, where $\breve{\varphi}^i$ is a posterior that is a solution to program (7). We have $\breve{\alpha}^i < \hat{\alpha}^i$. If $\breve{G}^i$ has mass outside the support of $\hat{G}^i$, then there must be an interval inside the support of $\hat{G}^i$ on which $\breve{G}^i$ has a density $\breve{g}^i(x) < \hat{g}^i(x)$, which contradicts $\breve{\alpha}^i < \hat{\alpha}^i$. So the support of $\breve{G}^i$ is within that of $\hat{G}^i$. But then, since the distributions are not

[5] More precisely, the statement is analogous to fn. 4.

the same by hypothesis, there must be an interval on which $\breve{G}^i$ has a density $\breve{g}^i(x) < \hat{g}^i(x)$, which contradicts $\breve{\alpha}^i < \hat{\alpha}^i$. $\|$

Proof of Part (2) of the Theorem: Suppose that $\hat{\alpha}^A = \hat{\alpha}^B$ and let (G^A, G^B) be an equilibrium. By Claim 2, some platform for A provides a utility of at least $\hat{\alpha}^A$; by Claim 1, some platform in the support of G^A provides a utility weakly less than $\hat{\alpha}^A$. The same applies to player B. Since $\hat{\alpha}^A = \hat{\alpha}^B$, we conclude that both players win with positive probability when strategic. By the same logic as Lemma A.3, it follows that the equilibrium must be ex-post, and all platforms in the support of each G^i must provide the same utility. So now suppose towards contradiction that without loss of generality, $G^A \neq \hat{G}^A$. Claim 1 implies that some platform in the support of G^A provides utility strictly less than $\hat{\alpha}^A$, which by ex-postness extends to all platforms in the support. But Claim 2 implies than B has a platform that provides utility at least that of $\hat{\alpha}^B = \hat{\alpha}^A$, implying that B must win with probability 1 if strategic, a contradiction.

Proof of Part (3) of the Theorem: If $\hat{\alpha}^i > \hat{\alpha}^j$ then for any equilibrium (G^i, G^j), some platform in the support of G^j gives weakly less utility than $\hat{\alpha}^j$ (Claim 1) and some platform for i gives gives weakly more utility than $\hat{\alpha}^i$ (Claim 2). So i wins with positive probability when strategic. If j wins with positive probability when strategic, then by the same logic as Lemma A.3, it follows that the equilibrium must be ex-post, and all platforms in the support of both distributions G^i and G^j must provide the same utility. But then, i has a profitable deviation to some platform outside the support of G^i, a contradiction. $\square$

Proof of Theorem 4. The same logic as in the proof of Theorem 1 proves that there must exist a unique $\overline{\alpha} \in (u(m,m), u(m,m) + \max_x \lambda(x,m))$ such that $\int_x \overline{g}(x) = 1$. We need only prove that both strategic candidates playing this density constitutes an equilibrium. It is easily verified that the Bayes update about character for a candidate who chooses platform $x \in Supp[\overline{G}]$ is $\overline{\varphi}(x) = \frac{\overline{\alpha} - u(x,m)}{\lambda(x,m)}$. Thus, the expected utility to a voter, v, from electing a candidate with platform $x \in Supp[\overline{G}]$ is

$$\overline{U}(x,v) = u(x,v) + \lambda(x,v)\left[\frac{\overline{\alpha} - u(x,m)}{\lambda(x,m)}\right]$$

Clearly, the median voter is indifferent over all platforms in the support of $\overline{G}$, i.e. $\overline{U}_1(x,m) = 0$ for all $x \in Supp[\overline{G}]$. It follows that if we prove that $\overline{U}_{12}(x,v) > 0$ for all v and all $x \in Supp[\overline{G}]$, strategic candidates are indifferent over all platforms in the support of $\overline{G}$. Taking the cross-partial derivative yields

$$\overline{U}_{12}(x,v) = u_{12}(x,v) + \lambda_{12}(x,v)\frac{\overline{\alpha} - u(x,m)}{\lambda(x,m)} - \lambda_2(x,v)\frac{\lambda(x,m)u_1(x,m) + (\overline{\alpha} - u(x,m))\,\lambda_1(x,m)}{(\lambda(x,m))^2}$$

Since $u_{12} > 0$, $\lambda_{12} \geq 0$, $\overline{\alpha} > u(x,m)$, and $\lambda(x,m) > 0$ for $x \in Supp[\overline{G}]$, the sum of the first two terms in the right hand side above is strictly positive. Therefore, when $\max_{x,v} |\lambda_2(x,v)|$ is sufficiently small (see fn. 4), the right hand side above is strictly positive, proving that $\overline{U}_{12}(x,v) > 0$ for all v and all $x \in Supp[\overline{G}]$ as desired.

Now, consider platforms outside the support of $\overline{G}$. We claim that strategic candidates have a strict incentive to not choose such platforms. To see this, note that by construction of the density $\overline{g}$, voter m strictly prefers any platform in the support of $\overline{G}$ to any platform outside the support (because $\overline{\alpha} > u(x,m) + \lambda(x,m)$ for any $x \notin Supp[\overline{G}]$). Since $\overline{\varphi}(x) = 1$ for all $x \notin Supp[\overline{G}]$, and $u_{12} + \lambda_{12} > 0$, this in turn implies that all voters $v < m$ also strictly prefer any platform in the support of $\overline{G}$ to any platform outside the support of $\overline{G}$. Therefore, any platform outside the support of $\overline{G}$ loses to a platform in the support of $\overline{G}$, and we conclude that that is optimal for strategic candidates to play $\overline{G}$. □

Proof of Theorem 5. By construction, voter m is indifferent over all platforms in the support of $\tilde{g}$, and strictly prefers these to all platforms outside the support. It remains to show that voter m really is a median voter under this construction — similar to Theorem 4. Computing expected utility in this putative equilibrium to voter v gives

$$W(x,v) = \frac{\tilde{\alpha} - \mathbb{E}_x[u(x,m)]}{u(x,m) - \mathbb{E}_x[u(x,m)]}(u(x,v) - \mathbb{E}_x[u(x,v)]) + \mathbb{E}_x[u(x,v)]$$

Taking the cross-partial and writing $\varphi(x) = \frac{\tilde{\alpha} - \mathbb{E}_x[u(x,m)]}{u(x,m) - \mathbb{E}_x[u(x,m)]}$ gives

$$W_{12}(x,v) = \varphi(x)\, u_{12}(x,v) - \left(u_2(x,v) - \frac{d}{dv}\mathbb{E}_x[u(x,v)]\right)\frac{(\tilde{\alpha} - \mathbb{E}_x[u(x,m)])\, u_1(x,v)}{(u(x,m) - \mathbb{E}_x[u(x,m)])^2}$$

Since $u_{12} > 0$, this will be positive if $\left|u_2(x,v) - \frac{d}{dv}\mathbb{E}_x[u(x,v)]\right|$ is sufficiently small for all $x, v \in [0,1]$. □

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