

# Pandering to Persuade

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# Motivation

Decision makers often rely upon advice from interested agents

- ▶ business, politics, organizations, daily life

Some common features

- ▶ DM has partial knowledge about attributes of alternatives
- ▶ Agent is better informed
- ▶ Interests of agent and DM well-aligned over some alternatives but not among others

# Examples

- ▶ Buyer decides which product to buy, if any, from a seller
  - ▶ Buyer has read public product reviews
  - ▶ Seller is better informed about products
  
- ▶ Investor must decide whether and which venture capital fund to invest in
  - ▶ Investor knows market trends
  - ▶ Venture capitalist is better informed about potential investments
  
- ▶ Dean decides whether to hire a new Econ faculty, and if so who
  - ▶ Dean can see a candidate's CV, recommendation letters, etc.
  - ▶ Econ department can evaluate research better

# This Paper

What is the impact of differences in observable (or verifiable) information on cheap-talk communication of private (soft) information?

i.e., cheap talk when alternatives “look different” to DM

# Our Baseline Framework

- ▶ Discrete decision space
  - ▶ There are  $n \geq 2$  **alternative projects** and a **status quo/outside option**
- ▶ Simple preference conflict structure
  - ▶ DM and agent have no conflict amongst projects
  - ▶ Agent prefers any project to the outside option
  - ▶ DM prefers outside option (known value) to low-quality projects
- ▶ Soft and hard information
  - ▶ Some aspects of each project are observed by DM (or verifiable and endogenously disclosed by agent)
  - ▶ Others are unverifiable and can only be conveyed through cheap talk

Agent wants to persuade the DM to choose the best project over the outside option

# Main Questions

- ▶ Credibility of communication
  - ▶ Is agent's advice influential? Does agent recommend the best option?
  - ▶ Impact of observable information
  - ▶ Impact of outside option (preference conflict)
  - ▶ **Properties of equilibria**
    - ▶ **Pandering to persuade**
    - ▶ **Pitching to persuade** (credibility through comparisons)
- ▶ Commitment from DM
  - ▶ Pandering arises in an **optimal mechanism** (no transfers)
    - ▶ commitment and cheap talk qualitatively similar
    - ▶ but commitment mitigates magnitude of pandering distortion
    - ▶ simple implementation via delegation to an intermediary
  - ▶ Burning ships: reduce the value of outside option
  - ▶ Ignorance: better to not observe some information

# Literature

- ▶ Seminal one-dimensional cheap talk
  - ▶ Crawford and Sobel (1982): continuous actions, different conflict
- ▶ Multidimensional cheap talk
  - ▶ Chakraborty and Harbaugh (2007): no pandering
- ▶ Pandering
  - ▶ Brandenburger and Polak (1996): not cheap talk
- ▶ Optimal delegation
  - ▶ Nocke and Whinston (2011), Armstrong and Vickers (2009): verifiable information
  - ▶ Holmstrom (1977) and successors: different setup

## Distinct from:

- ▶ Career concerns: Scharfstein and Stein (1990), Ottaviani and Sorensen (2001, 2007), ...
- ▶ Congruence signaling: Morris (2001), Ely and Välimäki (2003), Maskin and Tirole (2004), ...

# Plan

Model

Example

General Analysis

Commitment & Other Responses

Conclusion



# The Model: Basics

- ▶ 2 players: Principal/DM and Agent/advisor
- ▶  $N := \{1, 2\}$  alternative projects and a status quo (project 0)
  - ▶ can generalize to more than two alternative projects
- ▶ Status quo has known value  $b_0 > 0$  to Principal, 0 to Agent
- ▶ Value of project  $i \in N$  to both players is  $b_i$ , drawn from common prior cdf  $F_i$ 
  - ▶  $F_i$  has a density  $f_i$  with support  $[\underline{b}_i, \bar{b}_i]$
  - ▶  $0 \leq \underline{b}_i < b_0 < \bar{b}_i \leq \infty$
  - ▶  $\exists \alpha$  s.t.  $\mathbb{E}[b_i | b_i > \alpha b_{-i}] > b_0$  (relevant only if  $\underline{b}_{-i} > 0$ )
  - ▶  $F_i$ 's are independent but not necessarily identical
    - ▶ projects have observable components, generally asymmetric
    - ▶ can be endogenized with verifiable information revelation

# The Model: Leading Examples

- ▶ Can view  $F_i(b_i) \equiv F(b_i|v_i)$ ; with  $v_i$ 's commonly known
- ▶ Two leading families:

Given parameters  $v_1 \geq v_2 \geq 0$ ,

1. **Scale-invariant unif. distributions:**  $b_i \sim U[v_i, v_i + \bar{u}]$ , for some  $\bar{u} > 0$
2. **Exponential distributions:**  $b_i \sim \text{Exp}(v_i)$

All assumptions satisfied for  $b_0$  not too large.

# The Model: Timing

1. The agent privately observes  $(b_1, b_2)$
2. Agent sends a cheap-talk message to DM (large message space)
3. DM chooses a project or status quo

We are interested in the perfect Bayesian equilibria of this game.

## Lemma

*Generically, there is no equilibrium in which a positive measure of types induce the DM to randomize between projects 1 and 2.*

# The Model: Equilibrium

- ▶ The cheap-talk game can thus be simplified to
  1. Agent “recommends a project”, i.e. sends message  $m \in N$
  2. Principal's strategy is vector of acceptance probabilities,  $\mathbf{q} \in [0, 1]^n$
- ▶  $\mathbf{q}$  characterizes an equilibrium
  - ▶ Agent recommends project  $i$  if

$$q_i b_i > q_{-i} b_{-i}$$

- ▶  $q_i > 0$  only if

$$\mathbb{E}[b_i | q_i b_i > q_{-i} b_{-i}] \geq \max\{b_0, \mathbb{E}[b_{-i} | q_i b_i > q_{-i} b_{-i}]\}$$

with  $q_i = 1$  when inequality is strict

# An Example

- ▶  $b_1 \sim U[\frac{1}{3}, \frac{4}{3}]$ ;  $b_2 \sim U[0, 1]$
- ▶ So project 1 “looks better”
- ▶  $\mathbb{E}[b_1 | b_1 > b_2] = 0.91 > 0.78 = \mathbb{E}[b_2 | b_1 < b_2]$
- ▶ Thus, a truthful equilibrium,  $\mathbf{q} = (1, 1)$ 
  - ▶ Agent recommends project  $i$  if  $b_i > b_{-i}$
  - ▶ DM chooses the recommended project

exists if and only if  $b_0 \leq 0.78$

# An Example

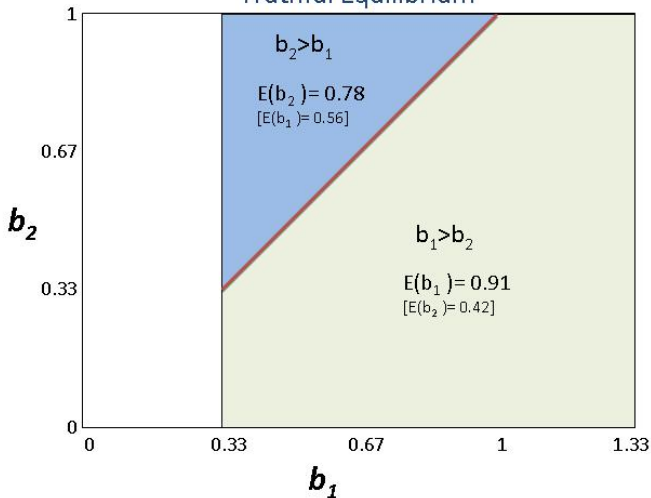
What if  $b_0 > 0.78$ ? There is no truthful eqm, but

- ▶ If  $b_0 \leq \frac{5}{6} = \mathbb{E}[b_1]$ , there is an **uninformative eqm**:  $\mathbf{q} = (1, 0)$
- ▶ If  $b_0 > \frac{5}{6}$ , there is a **zero eqm**:  $\mathbf{q} = (0, 0)$
- ▶ Both are rather inefficient outcomes given common interest over projects ... anything better?
- ▶ If  $b_0 \in (0.78, 0.85)$ , there is a **(partially) informative eqm** in which  $q_1 = 1$  and  $q_2 \in (0, 1)$ 
  - ▶ **Intuition:** Suppose  $q_2$  falls below 1 (“DM gets tough on project 2”)
    - ⇒ Agent becomes more selective against 2 (“pandering toward 1”)
    - ⇒ Posterior of project 2 improves, becoming acceptable to DM
  - ▶ **Eqm constraints:**
    - (1)  $\mathbb{E}[b_2|2 \text{ proposed}] = b_0$
    - (2)  $\mathbb{E}[b_1|1 \text{ proposed}] \geq b_0$

Parametric Example:  $b_2 \sim U[0,1]$     $b_1 \sim U[1/3,4/3]$

$b_0 \leq 0.78$

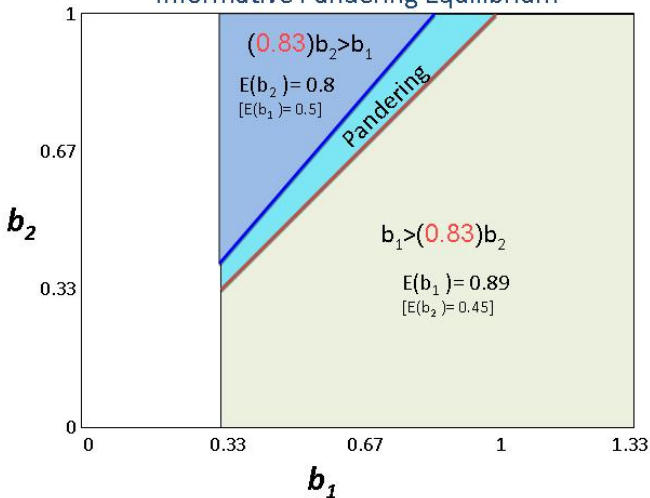
Truthful Equilibrium



Parametric Example:  $b_2 \sim U[0,1]$     $b_1 \sim U[1/3,4/3]$

$b_0 = 0.8$

### Informative Pandering Equilibrium

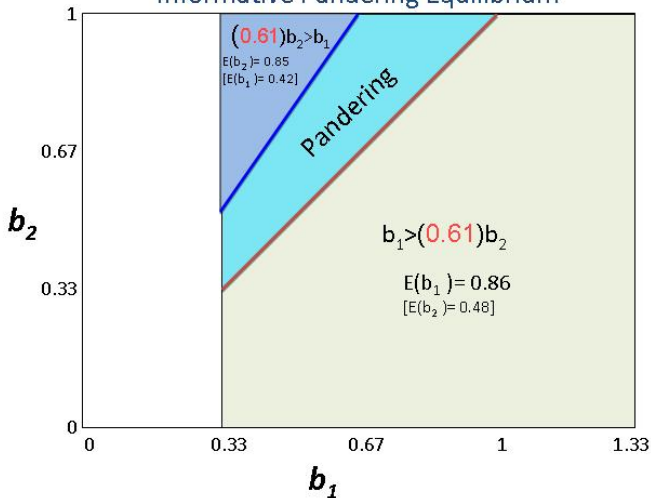




Parametric Example:  $b_2 \sim U[0,1]$     $b_1 \sim U[1/3,4/3]$

$b_0 = 0.85$

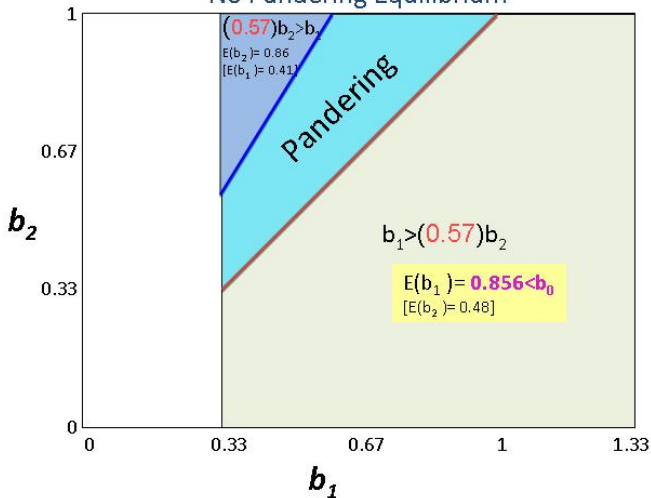
### Informative Pandering Equilibrium



Parametric Example:  $b_2 \sim U[0,1]$   $b_1 \sim U[1/3,4/3]$

$$b_0 = 0.86$$

No Pandering Equilibrium



Model

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**General Analysis**

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# Pandering in General Model

## Definition

An equilibrium  $\mathbf{q}$

1. is **influential** if  $\mathbf{q} \gg \mathbf{0}$ 
  - ▶ wlog, we assume  $q_i > 0 \Rightarrow q_i \bar{b}_i > q_{-i} \underline{b}_{-i}$
2. is **truthful** if  $q_1 = q_2 = 1$
3. has **pandering** if it is influential and  $q_1 \neq q_2$ 
  - ▶ pandering toward the project with higher  $q_i$
4. is **better than** equilibrium if it is interim Pareto superior
  - ▶ agent knows his type but DM does not

# Pandering in General Model

- ▶ If  $b_0$  is large enough, truthful equilibria won't exist
- ▶ Any influential eqm in such cases will feature pandering towards some project
- ▶ Would like to identify systematically which project the agent panders toward
  - ▶ across equilibria for a given  $b_0$
  - ▶ across different values of  $b_0$

and do comparative statics, welfare, etc.

- ▶ This requires a stochastic ranking of projects

# Strong Order

▶ Weaker

## Definition

The two projects are **strongly ordered** if

$$\mathbb{E}[b_1 | b_1 > b_2] > \mathbb{E}[b_2 | b_2 > b_1], \quad (\text{R1})$$

and, for any  $i \in \{1, 2\}$ ,

$$\mathbb{E}[b_i | b_i > \alpha b_{-i}] \text{ is nondecreasing in } \alpha \in \mathbb{R}_+ \quad (\text{R2})$$

so long as the expectation is well-defined.

- ▶ We say that project 1 “**looks (conditionally) better**” than project 2
- ▶ (R1) is mild given non-identical projects
- ▶ (R2) is the important part: **conditioning** vs. **selection** effects
- ▶ Leading examples satisfy strong order when  $v_1 > v_2$

# General Pandering

## Theorem

Assume projects are strongly ordered. There exist thresholds

$$b_0^{**} \geq b_0^* := \mathbb{E}[b_2 | b_2 \geq b_1]$$

such that:

1. If  $b_0 \leq b_0^*$ , the best eqm is the *truthful equilibrium*:  $\mathbf{q}^* = (1, 1)$

# General Pandering

## Theorem

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such that:

1. If  $b_0 \leq b_0^*$ , the best eqm is the **truthful equilibrium**:  $\mathbf{q}^* = (1, 1)$
2. If  $b_0 \in (b_0^*, b_0^{**})$ , the best eqm is a **pandering eqm** with  $\mathbf{q}^* = (1, q_2^*)$  for some  $q_2^* \in (0, 1)$ :
  - ▶ the agent proposes project 2 if and only if  $b_2 > b_1/q_2^*$
  - ▶  $\mathbf{q}^*$  is the **largest** eqm, i.e.  $\mathbf{q}^* > \mathbf{q}$  for any other eqm  $\mathbf{q}$
  - ▶ An increase in DM's outside option,  $b_0$ 
    - ▶ increases pandering, i.e. decreases  $q_2^*$
    - ▶ decreases expected payoffs of both Agent and DM
3. If  $b_0 > b_0^{**}$ , the only eqm is **non-influential**,  $\mathbf{q}^* = (0, 0)$ .



# General Pandering: Discussion

- ▶ When  $b_0 \in (b_0^*, b_0^{**})$ , the agent would be worse off with a commitment to truthfully rank alternatives
  - ▶ Ability to distort rankings is not self-defeating
  - ▶ Agent wants DM to know that he is pandering!
- ▶ *Both* projects benefit from pandering compared to truthful ranking
  - ▶ Better-looking project recommended more often
  - ▶ Worse-looking project becomes credible and acceptable
- ▶ DM's payoff is non-monotonic in outside option
  - ▶ can benefit from **burning ships**, i.e. reducing the outside option even at a cost

# Ordering: Meaning of “Looks Better” (1)

- ▶ Under Strong Ordering, a project looks better than another if its expectation is higher **when recommended under truthful strategy**
- ▶ The ordering can be intuitive
  - ▶ Leading examples: exponential, scale-invariant uniform
  - ▶ Here, order coincides with ranking under ex-ante expectation

## Ordering: Meaning of “Looks Better” (2)

But the ordering can also be less intuitive.

A job-market example:

$$\text{Stanford} \quad b_S \sim U[2, 3]$$

$$\text{Ohio} \quad b_O \sim U[1, 3]$$

Who “looks better”: Stanford or Ohio State Ph.D. student?

$$\mathbb{E}[b_O | b_O > b_S] = 2.66$$

$$\mathbb{E}[b_S | b_S > b_O] = 2.55$$

Can verify Ordering here, but **OHIO is “conditionally better looking”!**

⇒ **Pandering towards OHIO, even though  $\mathbb{E}[b_O] < \mathbb{E}[b_S]$**

Does NOT mean Ohio is recommended more often; rather, at the margin

## Ordering: Meaning of “Looks Better” (3)

- ▶ Standard stochastic dominance relations and our “conditionally better looking” relation cannot be generally compared
  - ▶  $F_1$  can be dominated in LR (hence FOSD) by  $F_2$  and yet satisfy strong order
  - ▶  $F_1$  can be dominated in SOSD by  $F_2$  and yet satisfy strong order
- ▶ “Looking better” in a comparative ranking vs. in isolation
- ▶ A sufficient condition for (R1):
  - ▶ with common support:  $\frac{f_2}{F_2} \frac{F_1}{f_1}$  is decreasing
  - ▶ if different supports, a generalization
- ▶ A sufficient condition for (R2):
  - ▶ for  $i = 1, 2$ :  $F_i(b_i/\alpha)$  is logsupermodular

# Pitching

A recommendation becomes more likely to be accepted (“sellable”) when it is **pitched in comparison** to projects that are themselves stronger

- ▶ even if these projects are already accepted with prob 1. when proposed

# Pitching

## Theorem

Assume  $\mathbf{F} = (F_1, F_2)$  and  $\tilde{\mathbf{F}} = (\tilde{F}_1, \tilde{F}_2)$  both satisfy strong order, but each  $F_i$  weakly dominates  $\tilde{F}_i$  in likelihood ratio, one of them strictly.

Let  $\mathbf{q}^*$  and  $\tilde{\mathbf{q}}^*$  denote the best equilibria respectively. Then,

1.  $\mathbf{q}^* \geq \tilde{\mathbf{q}}^*$
2.  $\mathbf{q}^* > \tilde{\mathbf{q}}^*$  if  $\mathbf{q}^* > (0, 0)$  and  $\tilde{\mathbf{q}}^* < (1, 1)$ .

Implications:

- (a) Agent's recommendation for Ohio is more acceptable when the alternative is Stanford than Brown.
- (b) Ohio may prefer to compete with Stanford than Brown.
- (c) Agent never benefits from "hiding" a project (by analogous result).

# More than Two Projects

## Definition

For  $n > 2$ , projects are **strongly ordered** if

1. For any  $i < j$ , and any  $k \in \mathbb{R}_+$ ,

$$\mathbb{E}[b_i | b_i > b_j, b_i > k] > \mathbb{E}[b_j | b_j > b_i, b_j > k]. \quad (\text{R1}')$$

whenever both expectations are well-defined.

2. For any  $i$  and  $j$ , and any  $k \in \mathbb{R}_+$ ,

$$\mathbb{E}[b_i | b_i > \alpha b_j, b_i > k] \text{ is nondecreasing in } \alpha \in \mathbb{R}_+ \quad (\text{R2}')$$

so long as the expectation is well-defined.

- ▶ Satisfied by leading families when  $v_1 > v_2 > \dots > v_n$
- ▶ Previous results generalize under this strengthened ordering
  - ▶ Focus on largest equilibrium
  - ▶ Caveat that it may not be the best equilibrium for DM

Model

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# Simple Delegation

What happens if the decision is **delegated** to agent?

- ▶ Eliminates pandering
- ▶ But sometimes a project is chosen when principal prefers status quo

## Theorem

*Compared to any eqm  $0 < \mathbf{q} < \mathbf{1}$ , the principal is strictly better off with unconstrained delegation to the agent.*

## Proof.

- ▶ Given pandering strategy, principal is indifferent between  $\mathbf{q}$  and  $\mathbf{1}$ .
- ▶ Delegation implements the latter, but also eliminates pandering. □
  
- ▶ Delegation requires **credible commitment to not override** whenever  $\mathbf{q} = \mathbf{1}$  is not an eqm
- ▶ Delegation may be beneficial even if  $\mathbf{q} = \mathbf{0}$  is the only eqm

# Optimal Commitment (1)

- ▶ Suppose the principle has rich commitment power. General mechanism design problem, **without transfers**.
- ▶ First: a simple but restricted class of mechanisms, where the the agent recommends a project, and the DM commits to an acceptance vector
  - ▶ includes delegation and cheap talk as special cases
- ▶ Is  $\mathbf{q}^D := \mathbf{1}$  the optimal commitment for the principal, at least when  $\mathbf{q}^D$  is preferred to  $\mathbf{q} = \mathbf{0}$ ?

## Optimal Commitment (2)

### Theorem

If the best cheap-talk eqm,  $\mathbf{q}^*$ , is such that  $\mathbf{0} < \mathbf{q}^* < \mathbf{q}^D$ , then the optimal simple mechanism is  $\mathbf{q}^M$  such that  $\mathbf{q}^* < \mathbf{q}^M < \mathbf{q}^D$ .

So no rubberstamping in optimal simple mechanism if no rubberstamping in communication eqm.

- ▶  $\mathbf{q}^D$  not an eqm if and only if

$$\mathbb{E}[b_2 | b_2 \geq b_1] < b_0$$

- ▶ Reducing  $q_2$  slightly from 1  $\Rightarrow$  2nd order loss from pandering distortion, but 1st order gain from choosing  $b_0$  sometimes when project 2 is recommended
- ▶ Since  $\mathbb{E}[b_2 | q_2^* b_2 \geq b_1] = b_0$ , raising  $q_2^*$  slightly has no direct effect on principal's utility
- ▶ But it reduces pandering

Can implement this decision rule by delegating decision-making to a third party whose value from the status quo is  $b'_0 \in (0, b_0)$ .

## Optimal Commitment (3)

- ▶ Now unrestricted class of mechanisms (but no transfers)
  - ▶ e.g. sometimes randomize between the two projects?
- ▶ Can focus on incentive-compatible direct mechanisms
  - ▶ mappings from  $(b_1, b_2)$  to  $\Delta := \{(x, y) \in [0, 1]^2 : x + y \leq 1\}$
- ▶ Key lemma: IC implies  $(x(\mathbf{b}), y(\mathbf{b})) = (x(\mathbf{b}'), y(\mathbf{b}'))$  if  $\frac{b_1}{b_2} = \frac{b'_1}{b'_2}$ 
  - ▶ not trivial, because even though for the agent the ratio determines his preferences over  $\Delta$ , the principal cares about the levels
- ▶ Reduce problem to one-dimensional, with agent's type  $\theta := \frac{b_1}{b_2}$ 
  - ▶ Can treat agent as if  $u(x, y, \theta) = x\theta + y$
- ▶ Now  $y$  looks similar to a transfer in standard mechanism design
  - ▶ but the analogy is imperfect, because of probability constraint

## Optimal Commitment (4)

Mild regularity condition that suitable “virtual valuation” is piecewise monotone:

$$J(\theta) := -(1 - \theta)b_0f(\theta) + \int_{\theta}^{\bar{\theta}} \left( \mathbb{E} \left[ b_2 \left| \frac{b_1}{b_2} = s \right. \right] - b_0 \right) f(s) ds.$$

### Theorem

*If  $\mathbf{q}^* < \mathbf{1}$ , then the optimal simple mechanism is optimal in the class of all mechanisms.*

- ▶ Optimal mechanism induces pandering
- ▶ Straightforward implementation of the optimal mechanism through intermediary delegation

# Ignorance Can Be Bliss

- ▶ Suppose there are two projects  $A$  and  $B$  that are ex-ante identical, but there will be a public signal  $s \in S$  (finite set) about them after which communication game ensues
- ▶ Say that the signal is **value neutral** if

$$\mathbb{E}[\max\{b_A, b_B\} | s] \text{ is the same for all } s \in S$$

and **non-trivial** if

$$\mathbb{E}[b_A | b_A > b_B, s] \neq \mathbb{E}[b_A | b_A > b_B, s'] \text{ for some } s, s' \in S$$

## Theorem

*If the signal is value neutral, the DM at least weakly prefers not observing the signal. If the signal is also non-trivial, then there is an interval of  $b_0$  in which the DM strictly prefers not observing the signal.*

Implications:

- ▶ If DM cannot commit to anything after observing  $s$ , she would prefer to commit to not observing information (if possible)
- ▶ Can be optimal to appoint a less capable/informed DM

# Endogenous Status Quo

Interpret  $b_0 \in \mathbb{R}_+$  as an alternative developed by the principal at cost  $c(b_0)$ , where  $c'(\cdot) > 0$ . This is done after observing hard information but before soft information is communicated. Assume there is a solution to  $\max[b_0 - c(b_0)]$ .

## Corollary

*Assume that the largest equilibrium of communication game is played. Then depending on  $(F_1, F_2)$ , the principal chooses either*

- 1.  $b_0 = 0$  and rubberstamps the agent's recommendation*
  - 2.  $\hat{b}_0 > 0$  and never accepts agent's recommendation*
- ▶ That is, no soft communication on eqm path, only hard info (reflected by  $F_1, F_2$ )*

# Conclusions

- ▶ New model of strategic communication/persuasion, relevant to wide variety of settings
  - ▶ equilibrium features interaction of hard and soft information:  
**pandering and pitching to persuade**
  - ▶ pandering arises even under full commitment for the DM
- ▶ Important (and potentially destructive) role of verifiable information
  - ▶ can distort and even crowd out the communication of soft information
  - ▶ DM might be better off not observing the verifiable information, or committing to ignore it



# Some Extensions

- ▶ Routine
  - ▶ Variable project size
  - ▶ Private info of DM about outside option
- ▶ More Interesting: conflicts of interest between projects
  - ▶ E.g. agent gets  $a * b_1$  from project 1, for some  $a > 0$
  - ▶ View pandering as distortion of agent's true preference ranking
  - ▶ Agent now has an incentive to pander to counter preference bias
    - ▶ can reinforce or mitigate/reverse informational-pandering
  - ▶ Pandering can be good for the DM, e.g. if  $a < 1$
  - ▶ Delegation may not be good

# Future

- ▶ Multiple agents with distinct projects competing
  - ▶ Intuition: exacerbate pandering distortions
  - ▶ DM may be better off limiting the set of agents
  
- ▶ Information acquisition
  - ▶ Strong incentives to strengthen project distributions
    - ▶ delegation can **demotivate**, contrast to Aghion and Tirole (1997); cf. Che & Kartik (2009)
  - ▶ But may also distort effort incentives towards projects with observable characteristics

Thank you.

# Weak Ordering

◀ Stronger

## Definition

For  $n = 2$ , projects are **weakly ordered** if

$$\forall \alpha \geq 1, \mathbb{E}[b_1 | b_1 > \alpha b_2] > \mathbb{E}[b_2 | \alpha b_2 > b_1].$$

whenever the LHS and RHS are well-defined.

## Theorem

*Assume  $n = 2$  and the projects are weakly ordered. Then, for any equilibrium  $\mathbf{q}$ :*

- 1. If  $q_1 > 0$ , then  $q_1 \geq q_2$ .*
- 2. If  $q_i > 0$  and  $q_{-i} < 1$ , then  $q_i > q_{-i}$ .*
- 3. If  $\mathbb{E}[b_2 | b_2 \geq b_1] < b_0$ , then  $q_2 < 1$ ; otherwise, there is a truthful equilibrium.*

# Weak vs. Strong Ordering

Stronger

- ▶ Weak Ordering can be satisfied even when  $\mathbb{E}[b_1|b_1 \geq \alpha b_2]$  is not increasing in  $\alpha$  (hence Strong Ordering fails)
- ▶ Truncated Normal Example:  $G_1 \sim N(5, 1)$  and  $G_2 \sim N(4.5, 1)$   
Let  $b_1$  and  $b_2$  have support on  $\mathbb{R}^+$  with

$$f_i(x) = g_i(x)/[1 - G_i(0)]$$

$\mathbb{E}[b_1|b_1 > \alpha b_2]$  and  $\mathbb{E}[b_2|\alpha b_2 > b_1]$  :

