Contests for Experimentation

Marina Halac    Navin Kartik    Qingmin Liu

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Introduction

- Principal wants to obtain an innovation whose feasibility is uncertain
- Agents can work on or experiment with this project
- Probability of success depends on state and agents’ hidden efforts

→ How should principal incentivize agents to experiment?

→ This paper: What is the optimal contest for experimentation?
Contests for experimentation

- Long tradition of using contests to achieve specific innovations
  - More broadly, intellectual property and patent policy debates

- Increased use in last two decades
  - Accounts for 78% of new prize money since 1991 (McKinsey)
  - America Competes Reauthorization Act signed by Obama in 2011

- Many examples
  - British Parliament’s longitude prize
  - Napoleon’s food preservation prize
  - Orteig prize
  - X Prizes: Ansari, Google Lunar, Progressive Automotive
  - Methuselah Foundation: Mouse Prize, NewOrgan Liver Prize
Contest design

- Netflix contest: $1M to improve recommendation accuracy by 10%
  - Not initially known if target attainable; contestants learn over time
  - Contestants’ effort is unobservable $\implies$ learning is private
  - Contest architecture affects contestants’ incentives to exert effort

- What contest design should be used?
  - Given a prize, principal aims to maximize probability of success
  - Propose tractable model based on exponential-bandit framework
Contest design: Payments and info disclosure

■ Should Netflix award full prize to first successful contestant?
  • Intuitive: Yes (under risk neutrality), sharing lowers expected reward

■ Should Netflix publicly announce when a first success is obtained?
  • Intuitive: Yes, values only one success, hiding lowers expected reward

→ Intuition says “public winner-takes-all” contest is optimal

→ Indeed, dominates any other public and any other winner-takes-all
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→ Indeed, dominates *any other public* and *any other winner-takes-all*

But will show that it is often dominated by “hidden shared-prize”
Main results

- Optimal info disclosure policy and prize scheme

- Conditions for optimality of hidden shared-prize and public WTA
  - Tradeoff: \( \uparrow \) agent’s reward for success vs \( \uparrow \) his belief he will succeed

- More generally, sharing the prize with cutoff disclosure is optimal
Related literature

Contest design without learning

- **Research**: Taylor 95, Fullerton-McAfee 99, Moldovanu-Sela 01, Che-Gale 03
- **Innovation**: Bhattacharya et al. 90, Moscarini-Smith 11, Judd et al. 12

Innovation contests with learning

- **WTA**: Choi 91, Malueg-Tsutsui 97, Mason-Välimäki 10, Moscarini-Squintani 10
- **Contest design**: Bimpikis et al. 14, Moroni 15

Multi-agent strategic experimentation

- **Games**: Keller et al. 05, Keller-Rady 10, Bonatti-Hörner 11, Cripps-Thomas 14
- **Info. disclosure**: Bimpikis-Drakopoulos 14, Che-Hörner 14, Heidhues et al. 14, Kremer et al. 14, Akcigit-Liu 14
Model (1)

Build on exponential bandit model (Keller, Rady, and Cripps, 2005):

- Innovation feasibility or state is either good or bad
  - Persistent but (initially) unknown; prior on good is $p_0 \in (0, 1)$

- At each $t \in [0, T]$, agent $i \in \mathcal{N}$ covertly chooses effort $a_{i,t} \in [0, 1]$
  - Instantaneous cost of effort is $ca_{i,t}$, where $c > 0$
  - $\mathcal{N} \equiv \{1, \ldots, N\}$ is given; $T \geq 0$ will be chosen by principal

- If state is good and $i$ exerts $a_{i,t}$, succeeds with inst. prob. $\lambda a_{i,t}$
  - No success if state is bad
  - Successes are conditionally independent given state
Model (2)

- Project success yields principal a payoff $v > 0$
  - Agents do not intrinsically care about success
  - Principal values only one success (specific innovation)

- Success is observable only to agent who succeeds and principal
  - Extensions: only agent or only principal observes success

- All parties are risk neutral and have quasi-linear preferences
  - Assume no discounting
Belief updating

- Given effort profile \( \{a_{i,t}\}_{i,t} \), let \( p_t \) be the public belief at \( t \), i.e. posterior on good state when no-one succeeds by \( t \):

\[
p_t = \frac{p_0 e^{-\int_0^t \lambda A_z dz}}{p_0 e^{-\int_0^t \lambda A_z dz} + 1 - p_0}
\]

where \( A_t \equiv a_{1,t} + \ldots + a_{N,t} \)

- Evolution of \( p_t \) governed by familiar differential equation:

\[
\dot{p}_t = -p_t (1 - p_t) \lambda A_t
\]
First best

- Efficient to stop after success; hence, social optimum maximizes

\[ \int_0^\infty (p_t\lambda v - c) A_t \left( e^{-\int_0^t p_z\lambda A_z dz} \right) dt \]

- \( p_t \) decreasing \( \implies \) an efficient effort profile is, for all \( i \in \mathcal{N} \),

\[ a_{i,t} = \begin{cases} 
1 & \text{if } p_t\lambda v \geq c \text{ and no success by } t \\
0 & \text{otherwise} 
\end{cases} \]

- Assume \( p_0\lambda v > c \). First-best stopping belief is

\[ p_{FB} = \frac{c}{\lambda v} \]
Contests

A contest specifies:

1. **Deadline:** $T \geq 0$

2. **Prizes:** $\overline{w}$ and prize-sharing scheme $(w_i(s))_{i \in \mathcal{N}}$ such that
   
   (i) $w_i(s) = w(s_i, s_{-i})$, where $w(s_i, s_{-i}) = w(s_i, \sigma(s_{-i}))$ for any perm. $\sigma$

   (ii) $w(\emptyset, \cdot) = 0$

   (iii) $s \neq (\emptyset, \ldots, \emptyset) \implies \sum_{i=1}^{N} w_i(s) = \overline{w}$

   → **Salient cases:** WTA and equal-sharing

3. **Disclosure:** $(M_t, \mu_t)_{t \in [0, T]}$, at each $t$ agents observe $m_t = \mu_t(o^t) \in M_t$

   → **Salient cases:** public and hidden
Principal’s problem

Principal designs contest to maximize her expected payoff gain

\[(v - \bar{w})p_0 \left(1 - e^{-\lambda A^T}\right)\]

where \(A^T \equiv \int_0^T A_z \, dz\)
Principal’s problem

- Principal designs contest to maximize her expected payoff gain

\[(v - \bar{w})p_0 \left(1 - e^{-\lambda A_T}\right)\]

where \(A_T \equiv \int_0^T A_z dz\)

- Decompose problem into two steps
  1. For any given \(\bar{w}\), solve for optimal contest
  2. Use solution to step 1. to solve for optimal prize \(\bar{w}\)

- Strategies & Equilibrium:
  - Wlog, \(a_{i,t}\) is \(i\)'s effort at \(t\) conditional on \(i\) not succeeding by \(t\)
  - Symmetric Nash equilibria; refinements would not alter analysis
Principal’s problem: Step 1

- For any given \( \overline{w} \), solve for optimal prize scheme and info disclosure
- Given \( \overline{w} \leq v \), principal’s objective is to maximize prob. of a success
- Study public and hidden contests, then general info disclosure
Let $A_{-i,z}$ be (i’s conjecture of) total effort by agents $-i$ at $z$ given no success by $z$.

Then i’s problem reduces to

$$\max_{(a_{i,t})_{t\in[0,T]}} \int_0^T \left( p_{i,t} \lambda \bar{w} - c \right) a_{i,t} e^{-\int_0^t p_{i,z} \lambda (a_{i,z} + A_{-i,z}) dz} dt$$

where

$$p_{i,t} = \frac{p_0 e^{-\int_0^t \lambda (a_{i,z} + A_{-i,z}) dz}}{p_0 e^{-\int_0^t \lambda (a_{i,z} + A_{-i,z}) dz} + 1 - p_0}$$
Public winner-takes-all contest

- Unique equilibrium is symmetric: for all $i \in \mathcal{N}$,

$$a_{i,t} = \begin{cases} 
1 & \text{if } p_{i,t} \geq \frac{c}{\lambda w} \equiv p^{PW} \text{ and no success by } t \\
0 & \text{otherwise}
\end{cases}$$

- Implies deadline $T$ optimal iff $T \geq T^{PW}$, where

$$\frac{p_0 e^{-N\lambda T^{PW}}}{p_0 e^{-N\lambda T^{PW}} + 1 - p_0} = \frac{c}{\lambda w}$$

Remark 1: Implements first-best solution iff $\bar{w} = v$

Remark 2: Probability of success is invariant to $\mathcal{N}$
Hidden winner-takes-all contest

Now $i$’s problem is

$$
\max_{(a_i,t)_{t \in [0,T]}} \int_0^T \left( p_{i,t}^{(1)} \lambda w e^{-\int_0^t \lambda A_{-i,z} dz} - c \right) a_{i,t} e^{-\int_0^t p_{i,z}^{(1)} \lambda a_{i,z} dz} dt,
$$

where $p_{i,t}^{(1)}$ is $i$’s private belief given he did not succeed by $t$:

$$
p_{i,t}^{(1)} = \frac{p_0 e^{-\int_0^t \lambda a_{i,z} dz}}{p_0 e^{-\int_0^t \lambda a_{i,z} dz} + 1 - p_0}
$$
Hidden winner-takes-all contest

- Unique equilibrium is symmetric: for all $i \in \mathcal{N}$,

$$a_{i,t} = \begin{cases} 
1 & \text{if } p_{i,t}^{(1)} \lambda \overline{w} e^{-\int_0^t \lambda A_{-i,s} ds} \geq c \\
0 & \text{otherwise}
\end{cases}$$

- Under non-binding $T$, stopping time $T^{HW}$ is then given by

$$\frac{p_0 e^{-N\lambda T^{HW}}}{p_0 e^{-\lambda T^{HW}} + 1 - p_0} = \frac{c}{\lambda w} = \frac{p_0 e^{-N\lambda T^{PW}}}{p_0 e^{-N\lambda T^{PW}} + 1 - p_0}$$

- Hence, $T^{HW} < T^{PW} \rightarrow$ Strictly dominated by public WTA
Public shared-prize contest

Now $i$’s problem is

$$\max_{(a_{i,t})_{t\in[0,T]}} \int_0^T \left[ (p_{i,t} \lambda w_{i,t} - c) a_{i,t} + p_{i,t} \lambda A_{-i,t} u_{i,t} \right] e^{-\int_0^t p_{i,z} \lambda (a_{i,z} + A_{-i,z}) dz} dt$$

where (suppressing dependence on strategies):

- $w_{i,t} \equiv i$’s expected reward if he succeeds at $t$
- $u_{i,t} \equiv i$’s continuation payoff if some $-i$ succeeds at $t$

Since $u_{i,t} \geq 0$ and $w_{i,t} \leq \bar{w}$,

$$a_{i,t} > 0 \implies p_{i,t} \geq \frac{c}{\lambda w_{i,t}} \geq \frac{c}{\lambda \bar{w}} = p^{PW}$$

→ Dominated by public WTA (strictly if different)
Hidden shared-prize contest

Proposition

Among hidden contests, an optimal prize scheme is equal sharing:
for any number of successful agents $n \in \mathcal{N}$, $w_i = \frac{w}{n} \forall i \in \{1, \ldots, n\}$.

- Idea of Proof:
  - Wlog to consider prize scheme that induces full effort from 0 to $T$
  - Equal sharing $\implies$ constant sequence of expected rewards
  - Stopping time $T^{HS}$ s.t. each agent’s IC binds at each $t \in [0, T^{HS}]$
  - Thus, no hidden contest can induce more experimentation
    - If $T > T^{HS}$, IC violated at some $t \leq T$
Hidden equal-sharing contest

Under equal sharing, $i$’s problem is

$$
\max_{(a_{i,t})_{t \in [0,T]}} \int_0^T \left( p_{i,t}^{(1)} \lambda w_i - c \right) a_{i,t} \ dt - \int_0^t p_{i,z}^{(1)} \lambda a_{i,z} \ dz
$$

prob. $i$ does not succeed by $t$

An optimal strategy is

$$
a_{i,t} = \begin{cases} 
1 & \text{if } p_{i,t}^{(1)} \lambda w_i \geq c \\
0 & \text{otherwise}
\end{cases}
$$

In a symmetric equilibrium, expected reward $w^{HS}$, stopping time $T^{HS}$
Hidden equal-sharing contest

- Given $T^{HS}$, the expected reward for success is

$$w^{HS} = \frac{1 - e^{-\lambda NT^{HS}}}{(1 - e^{-\lambda T^{HS}})N}$$

- Under non-binding $T$, stopping time $T^{HS}$ solves

$$\frac{1 - e^{-\lambda NT^{HS}}}{(1 - e^{-\lambda T^{HS}})N} = \frac{p_0 e^{-\lambda T^{HS}}}{p_0 e^{-\lambda T^{HS}} + 1 - p_0}$$

which has a unique solution; hence essentially unique symmetric eqm

Remark: Increase in $N$ can increase or decrease probability of success
Public or hidden?

- Recall $T_{PW}$ and $T_{HS}$ satisfy respectively

\[
\frac{p_0 e^{-N\lambda T_{PW}}}{p_0 e^{-N\lambda T_{PW}} + 1 - p_0} = \frac{c}{\lambda w}
\]

\[
\frac{1 - e^{-\lambda N T_{HS}}}{(1 - e^{-\lambda T_{HS}})N} \cdot \frac{p_0 e^{-\lambda T_{HS}}}{p_0 e^{-\lambda T_{HS}} + 1 - p_0} = \frac{c}{\lambda w}
\]
Public winner-takes-all versus hidden equal-sharing

Contests for Experimentation

Halac, Kartik, Liu
Result for public and hidden contests

Proposition

Among public and hidden contests, if

\[ \frac{p_0 e^{-\lambda T_{PW}^c}}{p_0 e^{-\lambda T_{PW}^c} + 1 - p_0} \cdot \frac{1 - e^{-\lambda N T_{PW}^c}}{(1 - e^{-\lambda T_{PW}^c}) N} > \frac{c}{\lambda w} \]

then a hidden equal-sharing contest is optimal.

Otherwise, a public winner-takes-all contest is optimal.
Intuition: Interpreting the condition

- We can rewrite condition as follows:

\[ \lambda w \sum_{m=1}^{N-1} \frac{\Pr[m \text{ opponents succeed by } T^{PW} | G]}{\Pr[\text{at least one opponent succeeds by } T^{PW} | G]} \left( \frac{1}{m + 1} \right) > c \]

- At \( T^{PW} \), if all \( -i \) failed, \( i \) is indifferent over exerting effort

- So, \( i \) strictly prefers to continue iff he does when some \( -i \) succeeded
Intuition: Necessary and sufficient conditions

- Condition for $N = 2$ is
  \[
  \frac{\bar{w}}{2} \lambda > c
  \]
  \(\rightarrow\) $i$ would experiment to earn half prize if he knew $-i$ succeeded

- If $N > 2$, above condition necessary, and simple sufficient condition is
  \[
  \frac{\bar{w}}{N} \lambda \geq c
  \]
  \(\rightarrow\) HS dominates (is dominated by) PW if $c/\lambda \bar{w}$ sufficiently small (large)
Intuition: Discussion

Why can hidden shared but not hidden WTA/public shared dominate?

- Want to hide info to bolster agent’s belief when no-one succeeded
- But hiding info is counter-productive under WTA
- And public shared can only ↑ effort when not beneficial (+ free-riding)

Hiding information can be beneficial because agents learn from others

- If $p_0 = 1$ or arms uncorrelated $\implies$ public WTA always optimal
- Higher $p_0$ $\implies$ hidden equal-sharing optimal for smaller parameter set
Implication: Number of contestants

- If principal can choose $N$, HS does always at least as well as PW
  - HS can replicate PW by setting $N = 1$

- Our results imply it can be strictly optimal to have multiple agents
  - Despite no exogenous forces such as heterogeneity and discounting

- $N > 1$ allows to harness benefits from hiding info and sharing prize
General disclosure policies

- **Rank monotonicity**: for any $s$, $s_i < s_j \implies w(s_i, s_{-i}) \geq w(s_j, s_{-j})$
- **Cutoff disclosure**: $M_t = \{0, 1\}$, $\mu_t(o^t) = 1$ iff $n$ or more succeed by $t$
- **Define**

  $$n^* \equiv \max \left\{ n \in \{1, \ldots, N\} : \lambda \frac{\overline{w}}{n} \geq c \right\}$$

**Proposition**

A *cutoff-disclosure equal-sharing contest with cutoff $n^*$ is optimal among rank-monotonic contests.*

- **Intuition:**
  - Rank monotonicity $\implies$ reward for success bounded by equal share
  - Exert effort given $G$ and equal-sharing iff share $w/less$ than $n^*$ agents
  - Increase (reduce) effort incentive if reveal $n \geq n^*$ ($n < n^*$) successes
Optimal cutoff disclosure equal-sharing contest

- Note that $n^* = 1$ when $\lambda \overline{w} / 2 < c$, whereas $n^* = N$ when $\lambda \overline{w} / N > c$

- Since agents stop exerting effort when $n^*$ successes are announced,
  - Cutoff-disclosure equal-sharing with $n^* = 1$ is equivalent to PW
  - Cutoff-disclosure equal-sharing with $n^* = N$ is equivalent to HS

**Corollary**

*Among rank-monotonic contests, public WTA is optimal if* $\lambda \overline{w} / 2 < c$ *and hidden equal-sharing is optimal if* $\lambda \overline{w} / N > c$.

- Finally, given salience and widespread use of WTA contests, we note:

**Proposition**

*A public contest is optimal among WTA contests.*
Principal’s problem: Step 2

- Given optimal contest as function of $\overline{w}$, principal solves for optimal $\overline{w}$

**Proposition**

Fix any parameters $(p_0, \lambda, c, N)$ and consider rank-monotonic contests.

- $v$ large enough $\implies$ principal chooses $\overline{w} \in (0, v)$ and hidden equal-sharing
- $v$ small enough $\implies$ principal chooses $\overline{w} \in (0, v)$ and public WTA

- Intuition: For $v$ large (small) enough, optimal $\overline{w}$ s.t. $\frac{\lambda w}{N} > c \ (\frac{\lambda w}{2} < c)$
Extensions and discussion

- **Social planner**: May also prefer hidden equal-sharing over public WTA
  - If budget constrained ($\overline{w} < v$), which is likely if value of discovery high
  - Ex post, planner induces wasteful experimentation after discovery made

- **Observability of success**: Results robust to different assumptions
  - If only P observes success, no reason to hide it from successful A
  - If only A observes success and can verifiably reveal it, main result holds
  - If A can verifiably reveal success to opponents, main result holds

- **Discounting, convex costs**: Main insight is robust
  - With discounting, benefit of public WTA: can use success immediately
  - But hidden equal-sharing can yield higher probability of success
Applications

First-to-file vs. first-to-invent rules in patent law
- FTF seen as beneficial because it induces earlier filing, more disclosure (e.g. Scotchmer-Green 90)
- Our results: FTI beneficial because it limits disclosure! (and induces sharing)

Optimal task allocation in organizations
- Principal assigns two tasks of uncertain and indep. difficulty to two agents
- Our results: benefits of making agents jointly responsible for the two tasks

Design of contract awards in government procurement
- In challenge-based acquisitions, no disclosure until evaluation date
- Moreover, multiple contractors, often to have stable supply and competition
- Our results: contract sharing beneficial beyond these considerations
Conclusions

■ Tradeoff in incentivizing experimentation:
  \[ \uparrow \text{agent’s reward for success} \text{ vs } \uparrow \text{his belief that he will succeed} \]

■ Hiding info and sharing prize often dominates public WTA
  - Only hiding info or dividing prize hurts, but both together can help

■ Broader contributions
  - Contest design in an environment with learning
  - Mechanism design approach—over payments and info disclosure—to multi-agent strategic experimentation