Appendix to Are Call Center and Hospital Arrivals Well Modeled by Nonhomogeneous Poisson Processes?

Song-Hee Kim and Ward Whitt

Industrial Engineering and Operations Research
Columbia University
New York, NY, 10027
{sk3116, ww2040} @columbia.edu

January 6, 2014

Abstract

Service systems such as call centers and hospitals typically have strongly time-varying arrivals. A natural model for such an arrival process is a nonhomogeneous Poisson process (NHPP), but that should be tested by applying appropriate statistical tests to arrival data. Assuming that the NHPP has a rate that is piecewise-constant, a Kolmogorov-Smirnov (KS) statistical test of a Poisson process (PP) can be applied to test for a NHPP, by combining data from separate subintervals, exploiting the classical conditional-uniform property. In this paper we apply KS tests to call center and hospital arrival data and show that they are consistent with the NHPP property, but only if that data is analyzed carefully. Initial testing rejected the NHPP null hypothesis, because it failed to take account of three common features of arrival data: (i) data rounding, e.g., to seconds, (ii) over-dispersion caused by combining data from multiple days that do not have the same arrival rate, and (iii) choosing subintervals over which the rate varies too much. In the main paper we investigate how to address each of these three problems. This appendix provides additional details for the main paper.

Keywords: nonhomogeneous Poisson process, Kolmogorov-Smirnov statistical test, rounding, over-dispersion,

Contents

1 Overview

We present supporting material in this appendix to the main paper. The main paper is a sequel to our previous paper Kim and Whitt [2014] in which we studied the performance of alternative Kolmogorov-Smirnov (KS) statistical tests of a Poisson process (PP) (in other words, an nonhomogeneous Poisson process (NHPP) with constant arrival rate). The KS tests we considered exploit the conditional-uniform (CU) property. The CU property states that, given an observation of n arrivals of a PP over an interval $[0, t]$, the unordered arrival times divided by t are distributed as n independent and identically distributed (i.i.d.) random variables, uniformly distributed over $[0, 1]$. The CU KS test tests whether the observations are consistent with this property. Given the CU property, we can test if arrival data are consistent with an NHPP with a piecewise-constant arrival rate by combining data from separate intervals. The combined data should again be a sample of i.i.d. random variables uniformly distributed on $[0, 1]$. Following Brown et al. [2005], in Kim and Whitt [2014] we found that it is important to transform the data before applying the KS test; KS tests without any data transformation (which we called the *CU* test) had little power. We suggested a KS test first proposed by Lewis [1965], using a transformation proposed by Durbin [1961], as the best way to do so (which we called the *Lewis* test). In the main paper, we focus on these two tests, the CU test and the Lewis tests, to illustrate three important issues that arise when testing whether real arrival data are from an NHPP.

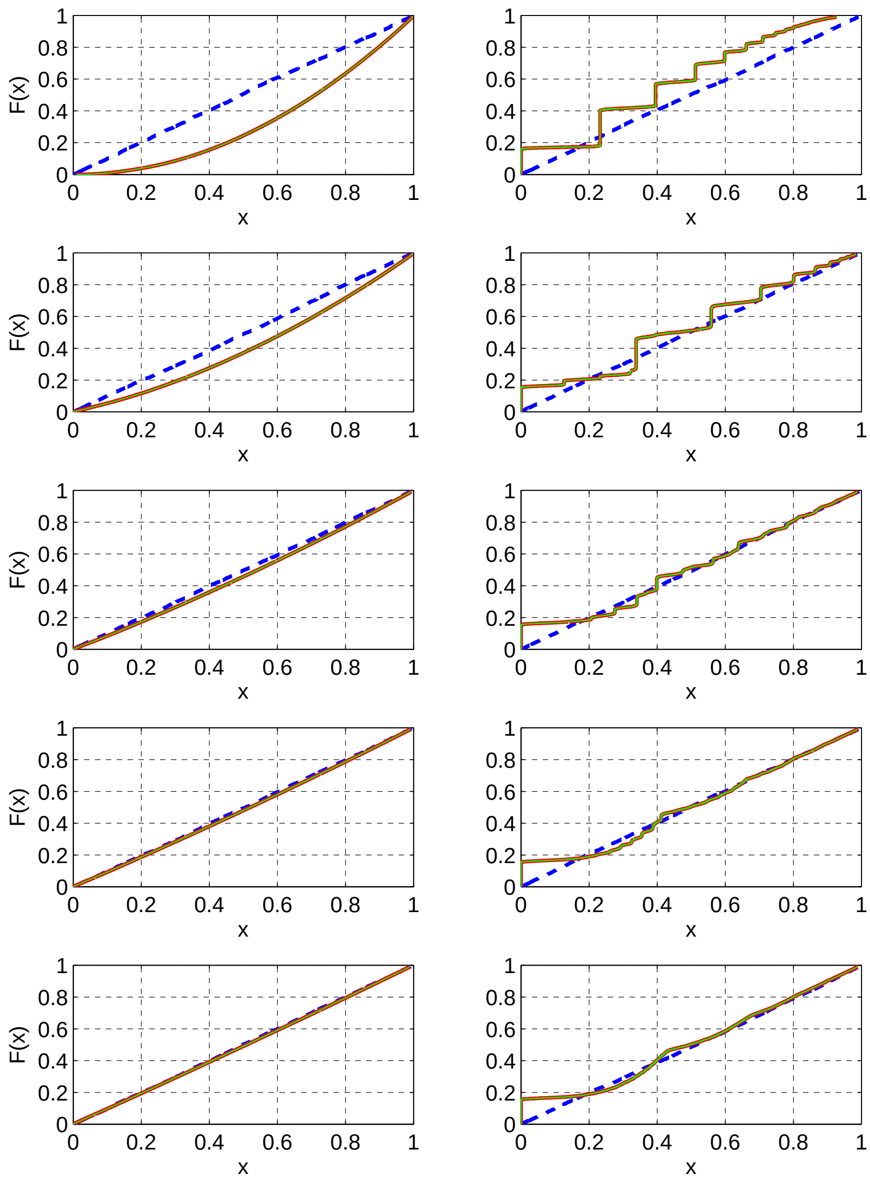
Here is how this appendix is organized: In §2 we present additional results to supplement Section 3.1 of the main paper. Supplementary material for the practical guidelines provided in Sections 3.4 and 3.6 of the main paper are in §3. Supplementary material for Section 4.3 of the main paper, on the role of relative slope, is given in §4. In §5, we provide supplementary material for Section 4 of the online supplement, on the issue of un-rounding. §6 provides details and additional results on our call center and hospital data, discussed in Section 5 of the main paper.

2 Subintervals - Supplementary Material for Section 3.1

Since the arrival rate function can often be regarded as piecewise-linear, it is often appropriate to regard the arrival rate as linear over subintervals. Given a piecewise-linear arrival rate function, the key is to choose a subinterval length so that piecewise-constant approximations are appropriate (i.e., the rates can be regarded as constat in each subinterval). To illustrate this issue, we simulated 1000 replications of an NHPP with linear arrival rate function $\lambda(t) = 1000t/3$ on the interval $[0, 6]$.

In addition to the results in Table 3 and Figure 4 of the main paper, Figure 1 shows the effect of subintervals when the data have been *rounded*. As expected, we see that the role of subintervals is significant, and the problem gets worse when the data are rounded.

Figure 1: Comparison of the average ecdf based on 100 replications of an NHPP with arrival rate function $\lambda(t) = 1000t/3$ on the time interval [0,6] with the cdf of the null hypothesis. All arrival times are rounded to the nearest second: From left to right: CU, Lewis test. From top to bottom: L=6, 3, 1, 0.5, 0.25.



3 Examples for Practical Guidelines in Sections 3.4 and 3.6

3.1 Practical Guidelines for a Single Interval

Table 1: Judging when the rate is approximately constant: looking at the ratio $D/\delta(n,\alpha)$ for $\alpha = 0.05$ for $\lambda(t) = a + bt$ with $r = 1$ ($a = 250$ and $b = 250$)

							CU		Lewis	
L	Interval	$ave[n]$	r	D	$ave[\delta(n,\alpha)]$	$D/ave[\delta(n,\alpha)]$	# P	ave[p-value]	# P	ave[p-value]
6	[0,6]	6002.7	1.00	0.188	0.018	10.71	0	0.00	0	0.00
3	[0,3]	1876.9	1.00	0.150	0.031	4.80	0	0.00	10	0.00
	[3,6]	4125.8	0.25	0.068	0.021	3.23	0	0.00	750	0.29
1	[0,1]	374.3	1.00	0.083	0.070	1.19	86	0.02	916	0.44
	[1,2]	626.4	0.50	0.050	0.054	0.93	251	0.06	945	0.49
	[2,3]	876.2	0.33	0.036	0.046	0.78	370	0.10	947	0.51
	[3,4]	1125.3	0.25	0.028	0.040	0.69	492	0.14	943	0.50
	[4,5]	1375.6	0.20	0.023	0.037	0.62	554	0.18	941	0.50
	[5,6]	1624.9	0.17	0.019	0.034	0.57	604	0.21	936	0.49
0.5	[0,0.5]	156.6	1.00	0.050	0.108	0.46	735	0.28	944	0.49
	[0.5,1]	217.8	0.67	0.036	0.091	0.39	810	0.33	941	0.49
	[1,1.5]	282.3	0.50	0.028	0.080	0.35	835	0.37	951	0.49
	[1.5,2]	344.1	0.40	0.023	0.073	0.31	860	0.38	947	0.51
	[2,2.5]	406.3	0.33	0.019	0.067	0.29	872	0.41	945	0.50
	[2.5,3]	469.9	0.29	0.017	0.062	0.27	888	0.42	939	0.51
	[3,3.5]	531.1	0.25	0.015	0.059	0.25	887	0.41	961	0.50
	[3.5,4]	594.3	0.22	0.013	0.055	0.24	913	0.43	940	0.50
	[4,3.5]	656.4	0.20	0.012	0.053	0.23	902	0.43	945	0.48
	[4.5,5]	719.2	0.18	0.011	0.050	0.22	915	0.44	955	0.50
	[5,5.5]	780.7	0.17	0.010	0.048	0.21	906	0.43	962	0.51
	[5.5,6]	844.2	0.15	0.009	0.047	0.20	901	0.44	950	0.51
0.25	[0,0.25]	70.3	1.000	0.028	0.160	0.17	893	0.44	942	0.49
	[0.25,0.5]	86.3	0.80	0.023	0.145	0.16	932	0.47	948	0.50
	[0.5,0.75]	101.1	0.67	0.019	0.134	0.14	934	0.47	938	0.49
	[0.75,1]	116.7	0.57	0.017	0.125	0.13	944	0.48	950	0.49
	[1,1.25]	133.5	0.50	0.015	0.117	0.13	934	0.48	950	0.48
	[1.25,1.5]	148.8	0.44	0.013	0.110	0.12	921	0.49	947	0.49
	[1.5,1.75]	164.3	0.40	0.012	0.105	0.11	939	0.49	938	0.51
	[1.75,2]	179.9	0.36	0.011	0.101	0.11	942	0.50	958	0.50
	[2,2.25]	195.1	0.33	0.010	0.097	0.10	957	0.49	948	0.49
	[2.25,2.5]	211.2	0.31	0.009	0.093	0.10	945	0.48	955	0.49
	[2.5,2.75]	227.1	0.29	0.009	0.089	0.10	957	0.50	957	0.51
	[2.75,3]	242.8	0.27	0.008	0.087	0.09	934	0.48	941	0.52
	[3,3.25]	257.6	0.25	0.008	0.084	0.09	932	0.49	948	0.51
	[3.25,3.5]	273.5	0.24	0.007	0.082	0.09	941	0.50	960	0.50
	[3.5,3.75]	289.4	0.22	0.007	0.079	0.09	948	0.48	953	0.50
	[3.75,4]	304.8	0.21	0.006	0.077	0.08	947	0.51	937	0.50
	[4,4.25]	320.6	0.20	0.006	0.075	0.08	937	0.48	953	0.50
	[4.25,4.5]	335.7	0.19	0.006	0.074	0.08	962	0.49	942	0.49
	[4.5,4.75]	351.2	0.18	0.006	0.072	0.08	936	0.48	945	0.51
	[4.75,5]	368.0	0.17	0.005	0.070	0.08	945	0.50	943	0.50
	[5,5.25]	382.3	0.17	0.005	0.069	0.07	936	0.49	940	0.50
	[5.25,5.5]	398.4	0.16	0.005	0.068	0.07	943	0.48	949	0.50
	[5.5,5.75]	413.0	0.15	0.005	0.066	0.07	940	0.49	953	0.50
	[5.75,6]	431.2	0.15	0.005	0.065	0.07	942	0.50	947	0.50

Table 2: Judging when the rate is approximately constant: looking at the ratio $D/\delta(n, \alpha)$ for $\alpha = 0.05$ for $\lambda(t) = a + bt$ with $r = 0.33$ ($a = 500$ and $b = 166.7$)

							CU		Lewis	
L	Interval	$ave[n]$	r	D	$ave[\delta(n,\alpha)]$	$D/ave[\delta(n,\alpha)]$	# P	ave[p-value]	# P	ave[p-value]
6	[0,6]	5999.2	0.33	0.125	0.018	7.14	0	0.00	0	0.00
3	[0,3]	2248.6	0.33	0.083	0.029	2.92	0	0.00	707	0.25
	[3,6]	3750.6	0.17	0.050	0.022	2.26	0	0.00	889	0.42
1	[0,1]	582.4	0.33	0.036	0.056	0.64	572	0.18	945	0.50
	[1,2]	750.5	0.25	0.028	0.049	0.56	676	0.23	943	0.50
	[2,3]	915.7	0.20	0.023	0.045	0.51	686	0.24	948	0.49
	[3,4]	1081.9	0.17	0.019	0.041	0.47	741	0.29	953	0.49
	[4,5]	1250.4	0.14	0.017	0.038	0.44	780	0.31	941	0.48
	[5,6]	1418.3	0.13	0.015	0.036	0.41	773	0.31	947	0.50
0.5	[0,0.5]	270.8	0.33	0.019	0.082	0.23	902	0.44	951	0.48
	[0.5,1]	311.6	0.29	0.017	0.076	0.22	912	0.44	952	0.50
	[1,1.5]	354.9	0.25	0.015	0.072	0.21	908	0.44	947	0.49
	[1.5,2]	395.5	0.22	0.013	0.068	0.19	914	0.46	955	0.49
	[2,2.5]	436.3	0.20	0.012	0.065	0.18	915	0.44	957	0.50
	[2.5,3]	479.4	0.18	0.011	0.062	0.18	933	0.45	928	0.49
	[3,3.5]	520.3	0.17	0.010	0.059	0.17	926	0.46	954	0.50
	[3.5,4]	561.6	0.15	0.009	0.057	0.16	941	0.47	951	0.50
	[4,3.5]	604.6	0.14	0.009	0.055	0.16	933	0.47	941	0.49
	[4.5,5]	645.8	0.13	0.008	0.053	0.15	936	0.47	951	0.49
	[5,5.5]	687.4	0.13	0.008	0.052	0.15	929	0.48	957	0.51
	[5.5,6]	730.9	0.12	0.007	0.050	0.14	937	0.49	951	0.50
0.25	[0,0.25]	130.1	0.33	0.010	0.118	0.08	932	0.49	945	0.50
	[0.25,0.5]	140.7	0.31	0.009	0.114	0.08	944	0.50	945	0.49
	[0.5,0.75]	150.8	0.29	0.009	0.110	0.08	954	0.49	961	0.51
	[0.75,1]	160.8	0.27	0.008	0.106	0.08	938	0.50	946	0.49
	[1,1.25]	172.3	0.25	0.008	0.103	0.07	944	0.48	941	0.50
	[1.25,1.5]	182.6	0.24	0.007	0.100	0.07	941	0.49	951	0.50
	[1.5,1.75]	192.8	0.22	0.007	0.097	0.07	954	0.49	949	0.50
	[1.75,2]	202.7	0.21	0.006	0.095	0.07	948	0.51	953	0.50
	[2,2.25]	212.8	0.20	0.006	0.092	0.07	934	0.49	954	0.52
	[2.25,2.5]	223.5	0.19	0.006	0.090	0.06	941	0.50	945	0.50
	[2.5,2.75]	234.6	0.18	0.006	0.088	0.06	938	0.49	942	0.49
	[2.75,3]	244.8	0.17	0.005	0.086	0.06	935	0.49	948	0.50
	[3,3.25]	254.9	0.17	0.005	0.084	0.06	942	0.50	954	0.51
	[3.25,3.5]	265.4	0.16	0.005	0.083	0.06	947	0.51	954	0.51
	[3.5,3.75]	276.1	0.15	0.005	0.081	0.06	936	0.49	948	0.52
	[3.75,4]	285.6	0.15	0.005	0.080	0.06	948	0.49	949	0.50
	[4,4.25]	296.8	0.14	0.004	0.078	0.06	933	0.50	957	0.50
	[4.25,4.5]	307.8	0.14	0.004	0.077	0.06	958	0.50	922	0.48
	[4.5,4.75]	317.9	0.13	0.004	0.076	0.05	945	0.48	942	0.49
	[4.75,5]	327.9	0.13	0.004	0.075	0.05	942	0.51	957	0.49
	[5,5.25]	338.2	0.13	0.004	0.073	0.05	948	0.51	958	0.52
	[5.25,5.5]	349.3	0.12	0.004	0.072	0.05	952	0.52	952	0.49
	[5.5,5.75]	360.4	0.12	0.004	0.071	0.05	948	0.50	960	0.51
	[5.75,6]	370.5	0.11	0.004	0.070	0.05	960	0.51	945	0.50

Table 3: Judging when the rate is approximately constant: looking at the ratio $D/\delta(n,\alpha)$ for $\alpha = 0.05$ for $\lambda(t) = a + bt$ with $r = 0.11$ ($a = 750$ and $b = 83.3$)

							CU		Lewis	
L	Interval	$ave[n]$	r	D	$ave[\delta(n,\alpha)]$	$D/ave[\delta(n,\alpha)]$	# P	ave[p-value]	# P	ave[p-value]
6	[0,6]	5997.3	0.11	0.062	0.018	3.57	0	0.00	754	0.29
3	[0,3]	2623.4	0.11	0.036	0.026	1.35	23	0.01	945	0.49
	[3,6]	3373.9	0.08	0.028	0.023	1.19	69	0.02	957	0.50
1	[0,1]	789.7	0.11	0.013	0.048	0.27	875	0.40	961	0.50
	[1,2]	876.3	0.10	0.012	0.046	0.26	874	0.40	952	0.51
	[2,3]	957.4	0.09	0.011	0.044	0.25	881	0.42	959	0.49
	[3,4]	1040.2	0.08	0.010	0.042	0.24	918	0.45	946	0.50
	[4,5]	1125.4	0.08	0.009	0.040	0.23	908	0.45	940	0.50
	[5,6]	1208.3	0.07	0.009	0.039	0.22	890	0.44	959	0.51
0.5	[0,0.5]	384.4	0.11	0.007	0.069	0.10	947	0.49	951	0.50
	[0.5,1]	405.4	0.11	0.006	0.067	0.10	938	0.47	943	0.48
	[1,1.5]	427.7	0.10	0.006	0.065	0.09	936	0.48	953	0.52
	[1.5,2]	448.6	0.10	0.006	0.064	0.09	931	0.48	953	0.50
	[2,2.5]	468.1	0.09	0.006	0.062	0.09	935	0.49	948	0.50
	[2.5,3]	489.3	0.09	0.005	0.061	0.09	945	0.48	947	0.49
	[3,3.5]	509.8	0.08	0.005	0.060	0.09	948	0.49	947	0.48
	[3.5,4]	530.4	0.08	0.005	0.059	0.08	947	0.50	958	0.51
	[4,3.5]	552.3	0.08	0.005	0.058	0.08	946	0.49	944	0.49
	[4.5,5]	573.1	0.07	0.005	0.056	0.08	946	0.50	952	0.50
	[5,5.5]	594.1	0.07	0.004	0.055	0.08	949	0.50	956	0.50
	[5.5,6]	614.2	0.07	0.004	0.055	0.08	943	0.48	953	0.51
0.25	[0,0.25]	189.8	0.11	0.003	0.098	0.03	938	0.49	945	0.50
	[0.25,0.5]	194.6	0.11	0.003	0.097	0.03	948	0.51	951	0.50
	[0.5,0.75]	199.4	0.11	0.003	0.095	0.03	946	0.50	939	0.48
	[0.75,1]	206.0	0.10	0.003	0.094	0.03	939	0.50	957	0.50
	[1,1.25]	211.4	0.10	0.003	0.093	0.03	948	0.50	955	0.51
	[1.25,1.5]	216.4	0.10	0.003	0.092	0.03	949	0.50	965	0.50
	[1.5,1.75]	221.7	0.10	0.003	0.091	0.03	934	0.50	958	0.50
	[1.75,2]	226.9	0.09	0.003	0.090	0.03	952	0.48	958	0.49
	[2,2.25]	231.1	0.09	0.003	0.089	0.03	942	0.51	946	0.49
	[2.25,2.5]	237.0	0.09	0.003	0.088	0.03	961	0.51	947	0.50
	[2.5,2.75]	241.9	0.09	0.003	0.087	0.03	940	0.49	942	0.51
	[2.75,3]	247.4	0.09	0.003	0.086	0.03	942	0.49	954	0.48
	[3,3.25]	252.4	0.08	0.003	0.085	0.03	946	0.51	929	0.50
	[3.25,3.5]	257.4	0.08	0.003	0.084	0.03	956	0.49	959	0.48
	[3.5,3.75]	263.3	0.08	0.002	0.083	0.03	947	0.50	944	0.49
	[3.75,4]	267.1	0.08	0.002	0.083	0.03	941	0.51	966	0.51
	[4,4.25]	274.0	0.08	0.002	0.082	0.03	943	0.50	944	0.50
	[4.25,4.5]	278.3	0.08	0.002	0.081	0.03	964	0.51	951	0.51
	[4.5,4.75]	284.3	0.07	0.002	0.080	0.03	960	0.49	936	0.49
	[4.75,5]	288.8	0.07	0.002	0.079	0.03	952	0.50	948	0.49
	[5,5.25]	294.6	0.07	0.002	0.079	0.03	953	0.51	959	0.50
	[5.25,5.5]	299.5	0.07	0.002	0.078	0.03	940	0.49	950	0.50
	[5.5,5.75]	304.1	0.07	0.002	0.077	0.03	935	0.49	951	0.50
	[5.75,6]	310.2	0.07	0.002	0.077	0.03	951	0.50	949	0.50

Table 4: Judging when the rate is approximately constant: looking at the ratio $D/\delta(n,\alpha)$ for $\alpha = 0.05$ for $\lambda(t) = a + bt$ with $r = 0$ ($a = 1000$ and $b = 0$)

							CU		Lewis	
L	Interval	$ave[n]$	r	D	$ave[\delta(n,\alpha)]$	$D/ave[\delta(n,\alpha)]$	# P	ave[p-value]	# P	ave[p-value]
6	[0,6]	5996.3	0	0	0.018	0	938	0.50	948	0.51
3	[0,3]	2998.0	0	0	0.025	0	954	0.48	955	0.50
	[3,6]	2998.3	0	0	0.025	0	952	0.50	955	0.51
1	[0,1]	999.2	0	0	0.043	0	957	0.50	960	0.50
	[1,2]	1000.3	0	0	0.043	0	950	0.50	955	0.49
	[2,3]	998.6	0	0	0.043	0	945	0.50	958	0.50
	[3,4]	999.7	0	0	0.043	0	949	0.51	949	0.49
	[4,5]	999.0	0	0	0.043	0	942	0.50	954	0.51
	[5,6]	999.6	0	0	0.043	0	946	0.50	957	0.51
0.5	[0,0.5]	499.7	0	0	0.060	0	948	0.50	949	0.51
	[0.5,1]	499.5	0	0	0.060	0	937	0.50	946	0.51
	[1,1.5]	499.2	0	0	0.060	0	941	0.49	949	0.49
	[1.5,2]	501.1	0	0	0.060	0	957	0.51	949	0.49
	[2,2.5]	499.1	0	0	0.060	0	940	0.51	950	0.50
	[2.5,3]	499.4	0	0	0.060	0	947	0.51	949	0.50
	[3,3.5]	500.2	0	0	0.060	0	951	0.52	957	0.52
	[3.5,4]	499.5	0	0	0.060	0	949	0.49	941	0.49
	[4,3.5]	500.0	0	0	0.060	0	943	0.51	947	0.51
	[4.5,5]	499.0	0	0	0.060	0	938	0.51	944	0.49
	[5,5.5]	499.3	0	0	0.060	0	950	0.52	958	0.50
	[5.5,6]	500.3	0	0	0.060	0	946	0.48	962	0.51
0.25	[0,0.25]	250.1	0	0	0.085	0	933	0.50	957	0.50
	[0.25,0.5]	249.6	0	0	0.085	0	951	0.50	946	0.50
	[0.5,0.75]	250.5	0	0	0.085	0	946	0.51	951	0.53
	[0.75,1]	249.0	0	0	0.085	0	947	0.50	951	0.51
	[1,1.25]	249.7	0	0	0.085	0	947	0.50	954	0.49
	[1.25,1.5]	249.5	0	0	0.085	0	943	0.51	950	0.50
	[1.5,1.75]	250.6	0	0	0.085	0	961	0.51	953	0.51
	[1.75,2]	250.5	0	0	0.085	0	956	0.51	950	0.50
	[2,2.25]	249.8	0	0	0.085	0	959	0.50	961	0.50
	[2.25,2.5]	249.3	0	0	0.085	0	956	0.50	954	0.50
	[2.5,2.75]	249.1	0	0	0.085	0	947	0.49	947	0.50
	[2.75,3]	250.3	0	0	0.085	0	955	0.49	960	0.51
	[3,3.25]	249.9	0	0	0.085	0	958	0.51	947	0.51
	[3.25,3.5]	250.3	0	0	0.085	0	952	0.52	970	0.52
	[3.5,3.75]	249.6	0	0	0.085	0	942	0.50	944	0.50
	[3.75,4]	249.9	0	0	0.085	0	952	0.50	936	0.50
	[4,4.25]	249.5	0	0	0.085	0	949	0.51	966	0.51
	[4.25,4.5]	250.5	0	0	0.085	0	952	0.51	939	0.50
	[4.5,4.75]	249.9	0	0	0.085	0	954	0.51	942	0.49
	[4.75,5]	249.1	0	0	0.085	0	947	0.50	942	0.49
	[5,5.25]	250.3	0	0	0.085	0	953	0.52	949	0.50
	[5.25,5.5]	249.0	0	0	0.085	0	949	0.51	955	0.49
	[5.5,5.75]	250.4	0	0	0.085	0	942	0.48	957	0.50
	[5.75,6]	249.9	0	0	0.085	0	953	0.51	954	0.51

3.2 Practical Guidelines for Dividing an Interval into Equal Subintervals

Table 5: Judging when the rate is approximately constant: Looking at the ratio $D/\delta(n,\alpha)$ for $\lambda(t) = a + bt$ with $r = 1$ ($a = 250$ and $b = 250$)

			CU		Lewis	
L	D	$D/ave[\delta(n,\alpha)]$	# P	ave[p-value]	# P	ave[p-value]
6	0.1875	10.714	0	0.00	0	0.00
3	0.0938	5.357	0	0.00	68	0.01
1	0.0313	1.786	1	0.00	933	0.47
0.5	0.0156	0.893	279	0.06	943	0.50
0.25	0.0078	0.446	749	0.31	948	0.50
0.1	0.0031	0.179	925	0.46	948	0.50
0.09	0.0028	0.159	919	0.46	954	0.50
0.08	0.0025	0.143	933	0.48	947	0.50
0.07	0.0022	0.124	940	0.47	941	0.49
0.06	0.0019	0.107	949	0.48	942	0.50
0.05	0.0016	0.089	936	0.49	952	0.50
0.01	0.0003	0.018	949	0.50	938	0.49
0.005	0.0002	0.009	949	0.50	961	0.50
0.001	0.00003	0.002	946	0.51	948	0.50

Table 6: Judging when the rate is approximately constant: Looking at the ratio $D/\delta(n,\alpha)$ for $\lambda(t) = a + bt$ with $r = 0.33$ ($a = 500$ and $b = 166.7$)

			CU		Lewis	
L	D	$D/ave[\delta(n,\alpha)]$	# P	ave[p-value]	# P	ave[p-value]
6	0.1250	7.143	0	0.00	0	0.00
3	0.0625	3.571	0	0.00	712	0.26
1	0.0208	1.190	66	0.01	938	0.49
0.5	0.0104	0.595	610	0.20	948	0.49
0.25	0.0052	0.298	864	0.38	952	0.50
0.1	0.0021	0.119	936	0.47	946	0.50
0.09	0.0019	0.106	931	0.48	940	0.49
0.08	0.0017	0.095	943	0.48	950	0.49
0.07	0.0014	0.083	937	0.49	950	0.50
0.06	0.0013	0.071	948	0.49	947	0.49
0.05	0.0010	0.060	945	0.50	948	0.49
0.01	0.0002	0.012	948	0.49	950	0.50
0.005	0.0001	0.006	952	0.49	951	0.49
0.001	0.00002	0.001	954	0.50	938	0.49

Table 7: Judging when the rate is approximately constant: Looking at the ratio $D/\delta(n, \alpha)$ for $\lambda(t) = a + bt$ with $r = 0.11$ ($a = 750$ and $b = 83.3$)

			CU		Lewis	
L	D	$D/ave[\delta(n, \alpha)]$	# P	ave[p-value]	# P	ave[p-value]
6	0.0625	3.571	0	0.00	754	0.29
3	0.0313	1.786	0	0.00	945	0.47
1	0.0104	0.595	613	0.20	964	0.49
0.5	0.0052	0.298	879	0.40	960	0.49
0.25	0.0026	0.149	924	0.47	959	0.49
0.1	0.0010	0.060	956	0.51	956	0.49
0.09	0.0009	0.053	957	0.50	957	0.48
0.08	0.0008	0.048	955	0.51	961	0.49
0.07	0.0007	0.041	957	0.50	953	0.49
0.06	0.0006	0.036	954	0.51	958	0.49
0.05	0.0005	0.030	948	0.50	951	0.50
0.01	0.0001	0.006	944	0.49	949	0.50
0.005	0.0001	0.003	948	0.49	951	0.50
0.001	0.00001	0.001	953	0.50	953	0.50

Table 8: Judging when the rate is approximately constant: Looking at the ratio $D/\delta(n, \alpha)$ for $\lambda(t) = a + bt$ with $r = 0$ ($a = 1000$ and $b = 0$)

			CU		Lewis	
L	D	$D/ave[\delta(n, \alpha)]$	# P	ave[p-value]	# P	ave[p-value]
6	0	0	938	0.50	948	0.51
3	0	0	956	0.50	952	0.51
1	0	0	959	0.50	947	0.51
0.5	0	0	941	0.50	945	0.51
0.25	0	0	959	0.50	947	0.51
0.1	0	0	958	0.50	952	0.51
0.09	0	0	952	0.50	942	0.51
0.08	0	0	963	0.53	950	0.51
0.07	0	0	957	0.52	945	0.51
0.06	0	0	949	0.51	954	0.51
0.05	0	0	960	0.48	946	0.50
0.01	0	0	949	0.51	953	0.51
0.005	0	0	954	0.50	956	0.49
0.001	0	0	927	0.50	942	0.49

3.3 Using the Anderson-Darling Tests

Given transformed interarrival times, many standard statistical tests can be applied to see whether they are from a PP. Brown et al. [2005] elected to apply the standard *Kolmogorov-Smirnov* (KS) test, so we have focused on the KS test in this work. There are other tests of fit that are based on the ecdf defined in equation (1) of the main paper. Some of the well-known ones are the one-sided Kolmogorov-Smirnov test, Kuiper test, Cramer-von Mises test, Anderson-Darling test and Watson test, all of which are based on the vertical differences between $F_n(x)$ and $F(x)$ [D'Agostino and Stephens 1986]. In this section, we provide results of one other test: the Anderson-Darling test, e.g., see Stephens [1974].

The Anderson-Darling (AD) statistic is defined as

$$A^2 \equiv n \int_{-\infty}^{\infty} \{F_n(x) - F(x)\}^2 \psi(x) dF(x), \tag{3.1}$$

with the weight function $\psi(x) \equiv [\{F(x)\}\{1 - F(x)\}]^{-1}$. That is, the AD distance places more weight on observations in the tails of the distribution. The AD test compares the observed value of A^2 to the *critical value*, $P(A^2 > \alpha)$ under the null hypothesis, where α is the significance level of the test, which we always take to be $\alpha = 0.05$. When $F(x)$ is completely known (our case), this critical value is 2.492 for all $n \geq 5$. Hence, in contrast to $\delta(n, \alpha)$ of the KS test, the AD critical values do not provide much insight.

Tables 9-11 show the AD test results on the same example cases in §3.4 and 3.6 of the main paper and §3.1 and 3.2 of this appendix. The AD test results are very similar to those of the KS tests. Hence, the practice guidelines provided in §3.4 and 3.6 of the main paper can be used when one wants to test a NHPP with the AD test.

Table 9: AD test results for each subinterval for $\lambda(t) = a + bt$ with $r = b/a$.

		$r = \infty$					$r = 1$					$r = 0.33$				
			CU		Lewis			CU		Lewis			CU		Lewis	
L	Interval	ave[n]	# P	$\bar{p}$	# P	$\bar{p}$	ave[n]	# P	$\bar{p}$	# P	$\bar{p}$	ave[n]	# P	$\bar{p}$	# P	$\bar{p}$
6	[0,6]	5995	0	0.00	0	0.00	6003	0	0.00	0	0.00	5999	0	0.00	0	0.00
3	[0,3]	1498	0	0.00	0	0.00	1877	0	0.00	3	0.00	2249	0	0.00	657	0.22
	[3,6]	4497	0	0.00	410	0.12	4126	0	0.00	695	0.26	3751	0	0.00	877	0.41
1	[0,1]	167	0	0.00	16	0.00	374	47	0.01	904	0.43	582	519	0.17	946	0.50
	[1,2]	499	8	0.00	900	0.43	626	198	0.05	942	0.49	750	622	0.21	939	0.51
	[2,3]	833	85	0.02	940	0.48	876	302	0.08	954	0.51	916	633	0.22	954	0.50
	[3,4]	1164	216	0.05	947	0.51	1125	414	0.12	951	0.50	1082	713	0.27	960	0.49
	[4,5]	1502	309	0.08	938	0.49	1376	491	0.16	941	0.49	1250	748	0.30	936	0.48
	[5,6]	1830	401	0.11	956	0.49	1625	550	0.19	939	0.49	1418	745	0.31	954	0.51
0.5	[0,0.5]	42	26	0.01	546	0.17	157	680	0.26	930	0.49	271	899	0.44	957	0.48
	[0.5,1]	125	418	0.13	929	0.47	218	786	0.32	944	0.49	312	910	0.44	951	0.51
	[1,1.5]	208	650	0.23	953	0.50	282	817	0.36	959	0.49	355	907	0.44	953	0.49
	[1.5,2]	291	723	0.29	950	0.50	344	849	0.38	947	0.51	396	908	0.46	949	0.49
	[2,2.5]	375	788	0.32	956	0.50	406	857	0.40	940	0.50	436	907	0.45	951	0.51
	[2.5,3]	458	788	0.34	948	0.51	470	876	0.41	944	0.51	479	926	0.44	933	0.50
	[3,3.5]	540	829	0.37	956	0.51	531	875	0.41	946	0.50	520	923	0.46	946	0.50
	[3.5,4]	624	844	0.38	957	0.51	594	901	0.43	949	0.50	562	921	0.47	954	0.50
	[4,3.5]	709	863	0.39	946	0.49	656	892	0.43	943	0.48	605	913	0.46	947	0.49
	[4.5,5]	793	860	0.41	950	0.49	719	898	0.44	954	0.51	646	926	0.48	948	0.50
	[5,5.5]	873	860	0.41	959	0.49	781	902	0.43	952	0.51	687	926	0.47	951	0.50
	[5.5,6]	957	892	0.43	954	0.49	844	895	0.44	951	0.51	731	932	0.48	959	0.50
0.25	[0,0.25]	10	595	0.18	891	0.43	70	888	0.43	949	0.49	130	932	0.50	954	0.50
	[0.25,0.5]	31	833	0.34	931	0.50	86	925	0.46	950	0.49	141	954	0.50	950	0.48
	[0.5,0.75]	52	882	0.40	949	0.48	101	931	0.45	945	0.50	151	950	0.50	961	0.51
	[0.75,1]	73	885	0.42	965	0.51	117	935	0.48	947	0.49	161	941	0.50	941	0.49
	[1,1.25]	94	892	0.44	943	0.49	133	922	0.48	955	0.48	172	934	0.48	952	0.50
	[1.25,1.5]	114	925	0.47	961	0.51	149	929	0.49	942	0.50	183	941	0.49	944	0.50
	[1.5,1.75]	135	921	0.46	954	0.50	164	940	0.48	936	0.50	193	953	0.49	957	0.50
	[1.75,2]	156	939	0.49	959	0.49	180	944	0.50	952	0.51	203	955	0.50	951	0.51
	[2,2.25]	177	913	0.46	962	0.50	195	952	0.49	952	0.50	213	935	0.48	948	0.51
	[2.25,2.5]	197	930	0.48	951	0.50	211	934	0.47	947	0.50	224	935	0.50	946	0.51
	[2.5,2.75]	218	931	0.47	949	0.50	227	945	0.50	952	0.51	235	946	0.50	938	0.49
	[2.75,3]	240	934	0.48	958	0.51	243	941	0.48	954	0.52	245	945	0.49	948	0.50
	[3,3.25]	260	933	0.47	937	0.49	258	941	0.49	948	0.50	255	938	0.49	959	0.51
	[3.25,3.5]	281	945	0.48	954	0.49	273	944	0.50	956	0.50	265	948	0.51	956	0.51
	[3.5,3.75]	301	943	0.48	951	0.52	289	948	0.48	952	0.49	276	940	0.49	948	0.51
	[3.75,4]	323	949	0.49	941	0.50	305	956	0.50	943	0.50	286	947	0.49	950	0.49
	[4,4.25]	344	945	0.49	952	0.51	321	935	0.48	956	0.50	297	945	0.50	953	0.50
	[4.25,4.5]	365	947	0.49	952	0.51	336	957	0.49	950	0.49	308	954	0.49	940	0.49
	[4.5,4.75]	387	940	0.48	948	0.50	351	931	0.49	947	0.51	318	945	0.48	944	0.50
	[4.75,5]	406	923	0.48	948	0.49	368	941	0.51	956	0.51	328	943	0.51	952	0.50
	[5,5.25]	426	941	0.49	956	0.49	382	941	0.49	941	0.50	338	946	0.50	956	0.51
	[5.25,5.5]	447	947	0.49	943	0.48	398	939	0.49	946	0.50	349	951	0.51	946	0.50
	[5.5,5.75]	470	941	0.48	944	0.50	413	940	0.49	942	0.50	360	948	0.50	956	0.51
	[5.75,6]	487	941	0.49	947	0.50	431	942	0.49	947	0.50	370	955	0.51	953	0.50

Table 10: (continued) AD test results for each subinterval for $\lambda(t) = a + bt$ with $r = b/a$.

		$r = 0.11$					$r = 0$				
			CU		Lewis			CU		Lewis	
L	Interval	ave[n]	# P	$\bar{p}$	# P	$\bar{p}$	ave[n]	# P	$\bar{p}$	# P	$\bar{p}$
6	[0,6]	5999	0	0.00	0	0.00	5997	0	0.00	686	0.25
3	[0,3]	2249	0	0.00	657	0.22	2623	10	0.00	948	0.49
	[3,6]	3751	0	0.00	877	0.41	3374	39	0.01	950	0.49
1	[0,1]	582	519	0.17	946	0.50	790	868	0.39	958	0.50
	[1,2]	750	622	0.21	939	0.51	876	870	0.40	951	0.52
	[2,3]	916	633	0.22	954	0.50	957	871	0.41	952	0.49
	[3,4]	1082	713	0.27	960	0.49	1040	917	0.45	949	0.50
	[4,5]	1250	748	0.30	936	0.48	1125	896	0.44	953	0.50
	[5,6]	1418	745	0.31	954	0.51	1208	886	0.44	959	0.50
0.5	[0,0.5]	271	899	0.44	957	0.48	384	934	0.49	943	0.50
	[0.5,1]	312	910	0.44	951	0.51	405	925	0.46	951	0.48
	[1,1.5]	355	907	0.44	953	0.49	428	925	0.48	952	0.51
	[1.5,2]	396	908	0.46	949	0.49	449	923	0.48	950	0.50
	[2,2.5]	436	907	0.45	951	0.51	468	939	0.49	942	0.49
	[2.5,3]	479	926	0.44	933	0.50	489	942	0.49	957	0.48
	[3,3.5]	520	923	0.46	946	0.50	510	952	0.50	948	0.49
	[3.5,4]	562	921	0.47	954	0.50	530	956	0.51	952	0.51
	[4,3.5]	605	913	0.46	947	0.49	552	946	0.49	953	0.49
	[4.5,5]	646	926	0.48	948	0.50	573	947	0.50	950	0.50
	[5,5.5]	687	926	0.47	951	0.50	594	948	0.50	953	0.50
	[5.5,6]	731	932	0.48	959	0.50	614	948	0.48	955	0.51
0.25	[0,0.25]	130	932	0.50	954	0.50	190	943	0.49	943	0.50
	[0.25,0.5]	141	954	0.50	950	0.48	195	955	0.52	948	0.51
	[0.5,0.75]	151	950	0.50	961	0.51	199	955	0.49	945	0.48
	[0.75,1]	161	941	0.50	941	0.49	206	938	0.50	960	0.50
	[1,1.25]	172	934	0.48	952	0.50	211	944	0.50	953	0.51
	[1.25,1.5]	183	941	0.49	944	0.50	216	952	0.51	949	0.51
	[1.5,1.75]	193	953	0.49	957	0.50	222	937	0.50	954	0.50
	[1.75,2]	203	955	0.50	951	0.51	227	939	0.48	948	0.50
	[2,2.25]	213	935	0.48	948	0.51	231	946	0.51	940	0.49
	[2.25,2.5]	224	935	0.50	946	0.51	237	951	0.51	951	0.49
	[2.5,2.75]	235	946	0.50	938	0.49	242	946	0.49	951	0.51
	[2.75,3]	245	945	0.49	948	0.50	247	948	0.50	954	0.49
	[3,3.25]	255	938	0.49	959	0.51	252	946	0.50	932	0.50
	[3.25,3.5]	265	948	0.51	956	0.51	257	960	0.50	945	0.48
	[3.5,3.75]	276	940	0.49	948	0.51	263	945	0.50	945	0.50
	[3.75,4]	286	947	0.49	950	0.49	267	951	0.51	956	0.51
	[4,4.25]	297	945	0.50	953	0.50	274	948	0.49	946	0.49
	[4.25,4.5]	308	954	0.49	940	0.49	278	967	0.51	954	0.51
	[4.5,4.75]	318	945	0.48	944	0.50	284	947	0.49	948	0.49
	[4.75,5]	328	943	0.51	952	0.50	289	952	0.50	947	0.50
	[5,5.25]	338	946	0.50	956	0.51	295	947	0.50	957	0.50
	[5.25,5.5]	349	951	0.51	946	0.50	299	949	0.50	953	0.50
	[5.5,5.75]	360	948	0.50	956	0.51	304	940	0.49	949	0.50
	[5.75,6]	370	955	0.51	953	0.50	310	943	0.50	952	0.50

Table 11: AD test results for $\lambda(t) = a + bt$ with $r = b/a$.

	$r = \infty$				$r = 1$				$r = 0.33$				$r = 0.11$				$r = 0$			
	CU		Lewis		CU		Lewis		CU		Lewis		CU		Lewis		CU		Lewis	
L	# P	$\bar{p}$	# P	$\bar{p}$	# P	$\bar{p}$	# P	$\bar{p}$	# P	$\bar{p}$	# P	$\bar{p}$	# P	$\bar{p}$	# P	$\bar{p}$	# P	$\bar{p}$	# P	$\bar{p}$
6	0	0.00	0	0.00	0	0.00	0	0.00	0	0.00	0	0.00	0	0.00	686	0.25	946	0.50	952	0.50
3	0	0.00	0	0.00	0	0.00	30	0.01	0	0.00	654	0.23	0	0.00	942	0.47	951	0.50	951	0.50
1	0	0.00	788	0.31	1	0.00	927	0.47	42	0.01	945	0.49	542	0.18	958	0.50	959	0.50	951	0.50
0.5	48	0.01	941	0.48	210	0.05	945	0.50	545	0.18	953	0.50	849	0.39	957	0.50	949	0.50	951	0.50
0.25	538	0.17	947	0.49	717	0.30	946	0.51	852	0.38	954	0.50	922	0.47	957	0.49	960	0.50	953	0.51
0.1	892	0.43	959	0.49	907	0.46	948	0.50	934	0.48	947	0.50	946	0.51	960	0.49	960	0.49	948	0.50
0.09	896	0.44	947	0.49	921	0.46	948	0.50	934	0.48	942	0.50	956	0.50	963	0.50	948	0.50	948	0.50
0.08	905	0.45	956	0.49	939	0.48	950	0.50	943	0.48	947	0.50	952	0.51	957	0.49	963	0.53	955	0.50
0.07	934	0.47	960	0.50	935	0.47	944	0.49	933	0.49	953	0.50	959	0.50	960	0.49	955	0.52	949	0.51
0.06	917	0.46	948	0.50	941	0.48	945	0.50	948	0.49	946	0.49	960	0.51	960	0.49	938	0.50	946	0.50
0.05	938	0.49	960	0.49	937	0.49	951	0.50	948	0.50	948	0.50	954	0.50	955	0.49	958	0.49	945	0.50
0.01	947	0.49	945	0.49	956	0.50	939	0.49	949	0.50	950	0.51	948	0.50	943	0.50	952	0.51	950	0.50
0.005	944	0.51	950	0.51	952	0.50	957	0.51	955	0.50	955	0.49	944	0.49	950	0.50	946	0.49	949	0.48
0.001	957	0.52	944	0.50	952	0.51	953	0.50	953	0.51	942	0.49	951	0.50	954	0.50	943	0.50	947	0.49

4 Relative Slope - Supplementary Material for Section 3.3

In the main paper, given an NHPP with linear arrival rate function $\lambda(t) = a + bt$, we discuss how the KS test results are independent of the scale parameter a, but are dependent of the relative slope, $r \equiv b/a$ and rT where T is the interval length. We provide additional supporting tests results in this section. We again use 1000 replications of an NHPP with linear arrival rate function $\lambda(t) = 1000t/3$ on the interval $[0, 6]$. We then test each subinterval with five different subinterval lengths, L=6, 3, 1, 0.5, 0.25. Table 12 shows that with equal subinterval length, the null NHPP hypothesis is rejected more at the beginning of the interval with higher r values. Given these results, we apply the KS tests to the interval without the first segment in Table 14. Comparing the results to those in Table 4 of the main paper, we see that the result has improved much just by not including the very first segment.

In contrast to the bad case of $\lambda(t) = 1000t/3$ with one end point at 0, we consider another example with $\lambda(t) = 1000 + 1000t/6$. Now the relative slopes are much smaller and the results in Table 13 and Table 15 reflect this.

Table 12: Performance of the alternative KS test of a Nonhomogeneous Poisson process with arrival rate function $\lambda(t) = 1000t/3$ on the time interval [0,6], with and without rounding effects. Different subinterval lengths (denoted by L) are used: Number of KS tests passed (denoted by #P) at significance level 0.05 out of 1000 replications, the average p-values (denoted by ave[p-value]), and the average percentage of 0 interarrival times (denoted by ave[% 0]).

				CU		Lewis	
L	Interval	$r \equiv b/a$	$ave[n]$	# P	ave[p-value]	# P	ave[p-value]
6	[0,6]	∞	5997.3	0	0.00	0	0.00
3	[0,3]	∞	1498.8	0	0.00	0	0.00
	[3,6]	0.33	4498.5	0	0.00	481	0.15
1	[0,1]	∞	166.8	0	0.00	46	0.01
	[1,2]	1.00	499.7	22	0.01	896	0.43
	[2,3]	0.50	832.4	145	0.03	928	0.48
	[3,4]	0.33	1166.9	300	0.08	931	0.49
	[4,5]	0.25	1501.0	358	0.09	949	0.49
	[5,6]	0.20	1830.6	453	0.13	948	0.49
0.5	[0,0.5]	∞	42.0	46	0.01	562	0.18
	[0.5,1]	2.00	124.8	479	0.14	918	0.48
	[1,1.5]	1.00	207.7	684	0.25	935	0.49
	[1.5,2]	0.67	292.0	766	0.29	945	0.50
	[2,2.5]	0.50	375.4	783	0.33	935	0.49
	[2.5,3]	0.40	456.9	833	0.35	960	0.51
	[3,3.5]	0.33	543.6	822	0.36	949	0.48
	[3.5,4]	0.29	623.3	865	0.38	938	0.51
	[4,3.5]	0.25	708.9	861	0.40	957	0.51
	[4.5,5]	0.22	792.1	882	0.41	936	0.50
	[5,5.5]	0.20	873.9	873	0.42	941	0.49
	[5.5,6]	0.18	956.6	893	0.42	951	0.50
0.25	[0,0.25]	∞	10.4	588	0.17	888	0.42
	[0.25,0.5]	4.00	31.6	841	0.37	946	0.49
	[0.5,0.75]	2.00	51.8	885	0.41	943	0.49
	[0.75,1]	1.33	73.0	907	0.44	947	0.50
	[1,1.25]	1.00	93.7	902	0.45	938	0.49
	[1.25,1.5]	0.80	114.0	920	0.47	951	0.50
	[1.5,1.75]	0.67	135.5	936	0.46	939	0.50
	[1.75,2]	0.57	156.5	924	0.48	940	0.49
	[2,2.25]	0.50	177.6	916	0.46	968	0.51
	[2.25,2.5]	0.44	197.9	925	0.48	939	0.48
	[2.5,2.75]	0.40	218.2	934	0.48	946	0.50
	[2.75,3]	0.36	238.7	931	0.48	956	0.50
	[3,3.25]	0.33	261.1	929	0.48	941	0.49
	[3.25,3.5]	0.31	282.6	941	0.47	948	0.49
	[3.5,3.75]	0.29	301.3	942	0.48	949	0.50
	[3.75,4]	0.27	322.0	941	0.47	946	0.50
	[4,4.25]	0.25	344.2	948	0.48	952	0.50
	[4.25,4.5]	0.24	364.7	932	0.47	957	0.52
	[4.5,4.75]	0.22	385.5	952	0.47	946	0.50
	[4.75,5]	0.21	406.7	937	0.48	953	0.50
	[5,5.25]	0.20	426.8	938	0.48	951	0.50
	[5.25,5.5]	0.19	447.2	943	0.49	942	0.48
	[5.5,5.75]	0.18	467.5	945	0.47	952	0.50
	[5.75,6]	0.17	489.2	941	0.50	943	0.50

Table 13: Performance of the alternative KS test of a Nonhomogeneous Poisson process with arrival rate function $\lambda(t) = 1000 + 1000t/6$ on the time interval [0,6], with and without rounding effects. Different subinterval lengths (denoted by L) are used: Number of KS tests passed (denoted by #P) at significance level 0.05 out of 1000 replications, the average p-values (denoted by ave[p-value]), and the average percentage of 0 interarrival times (denoted by ave[% 0]).

				CU		Lewis	
L	Interval	$r \equiv b/a$	$ave[n]$	# P	ave[p-value]	# P	ave[p-value]
6	[0,6]	0.17	9000.8	0	0.00	219	0.05
3	[0,3]	0.17	3749.5	0	0.00	905	0.44
	[3,6]	0.11	5251.4	1	0.00	930	0.48
1	[0,1]	0.17	1085.9	728	0.27	946	0.50
	[1,2]	0.14	1249.6	786	0.31	959	0.50
	[2,3]	0.13	1413.9	784	0.33	947	0.50
	[3,4]	0.11	1584.7	809	0.34	953	0.50
	[4,5]	0.10	1749.2	826	0.35	947	0.50
	[5,6]	0.09	1917.4	840	0.35	945	0.49
0.5	[0,0.5]	0.17	522.0	913	0.45	954	0.48
	[0.5,1]	0.15	563.9	929	0.46	942	0.50
	[1,1.5]	0.14	604.4	917	0.47	950	0.50
	[1.5,2]	0.13	645.2	917	0.46	948	0.50
	[2,2.5]	0.13	686.4	939	0.48	948	0.50
	[2.5,3]	0.12	727.5	940	0.47	941	0.50
	[3,3.5]	0.11	771.8	933	0.46	959	0.51
	[3.5,4]	0.11	812.9	938	0.47	960	0.50
	[4,3.5]	0.10	853.1	940	0.49	956	0.49
	[4.5,5]	0.10	896.2	949	0.49	962	0.49
	[5,5.5]	0.09	937.7	942	0.48	936	0.51
	[5.5,6]	0.09	979.7	934	0.47	954	0.50
0.25	[0,0.25]	0.17	255.6	931	0.49	954	0.49
	[0.25,0.5]	0.16	266.4	929	0.47	954	0.49
	[0.5,0.75]	0.15	276.5	945	0.51	951	0.50
	[0.75,1]	0.15	287.5	940	0.49	957	0.50
	[1,1.25]	0.14	296.5	947	0.48	932	0.49
	[1.25,1.5]	0.14	307.9	945	0.50	943	0.50
	[1.5,1.75]	0.13	317.9	955	0.50	962	0.51
	[1.75,2]	0.13	327.3	950	0.50	943	0.49
	[2,2.25]	0.13	338.1	954	0.52	950	0.49
	[2.25,2.5]	0.12	348.3	947	0.51	947	0.50
	[2.5,2.75]	0.12	358.7	948	0.49	954	0.51
	[2.75,3]	0.11	368.8	951	0.50	953	0.51
	[3,3.25]	0.11	380.2	941	0.49	954	0.49
	[3.25,3.5]	0.11	391.6	948	0.49	947	0.50
	[3.5,3.75]	0.11	401.3	939	0.48	955	0.49
	[3.75,4]	0.10	411.6	959	0.51	954	0.51
	[4,4.25]	0.10	421.4	955	0.51	959	0.49
	[4.25,4.5]	0.10	431.6	960	0.51	944	0.50
	[4.5,4.75]	0.10	442.8	946	0.49	953	0.50
	[4.75,5]	0.09	453.4	950	0.51	960	0.49
	[5,5.25]	0.09	463.3	945	0.49	945	0.50
	[5.25,5.5]	0.09	474.4	947	0.51	943	0.50
	[5.5,5.75]	0.09	484.7	952	0.50	960	0.48
	[5.75,6]	0.09	495.1	942	0.50	947	0.51

Table 14: Performance of the alternative KS test of a Nonhomogeneous Poisson process with arrival rate function $\lambda(t) = 1000t/3$ on the time interval [L,6]. Different subinterval lengths (denoted by L) are used: Number of KS tests passed (denoted by #P) at significance level 0.05 out of 1000 replications and the average p-values (denoted by ave[p-value]).

	CU		Lewis	
L	# P	ave[p-value]	# P	ave[p-value]
3	0	0.00	481	0.15
1	0	0.00	927	0.44
0.5	100	0.02	958	0.48
0.25	596	0.20	951	0.48
0.1	896	0.43	956	0.48
0.09	904	0.43	954	0.48
0.08	913	0.45	947	0.48
0.07	923	0.47	960	0.49
0.06	929	0.47	942	0.49
0.05	941	0.50	959	0.49
0.01	953	0.50	948	0.48
0.005	944	0.49	943	0.48
0.001	952	0.50	959	0.49

Table 15: Performance of the alternative KS test of a Nonhomogeneous Poisson process with arrival rate function $\lambda(t) = 1000 + 1000t/6$ on the time interval [0,6] and [L,6] with different subinterval lengths (denoted by L): Number of KS tests passed (denoted by #P) at significance level 0.05 out of 1000 replications and the average p-values (denoted by ave[p-value]).

		CU		Lewis	
Interval	L	# P	ave[p-value]	# P	ave[p-value]
[0,6]	6	0	0.00	219	0.05
	3	0	0.00	886	0.43
	1	191	0.05	952	0.50
	0.5	718	0.27	952	0.49
	0.25	894	0.42	951	0.50
[3,6]	3	1	0.00	930	0.48
[1,6]	1	293	0.07	954	0.50
[0.5,6]	0.5	723	0.29	951	0.50
[0.25,6]	0.25	901	0.42	957	0.50

5 More on Un-Rounding - Supplementary Material for Section 4 of the Online Supplement

In this section, we provide the full experiment results for Section 4 of the online supplement.

Table 16: Summary statistics of a rate-1000 renewal process on $[0, 6]$, in which the interarrival times are 0 with probability p and an exponential random variable with probability $1-p$. Results over 10000 iterations.

p	Type	ave[X]	ave[c_X^2]	ave[n]	min[X]	max[X]
0.1	Raw	0.0009	1.2218	6666.4	0	0.0093
	Rounded	0.0009	1.2361	6666.2	0	0.0093
	Un-rounded	0.0009	1.2015	6666.2	7.8×10^{-8}	0.0093
0.05	Raw	0.0009	1.1052	6316.1	0	0.0093
	Rounded	0.0009	1.1187	6316.0	0	0.0093
	Un-rounded	0.0009	1.0957	6316.0	1.1×10^{-7}	0.0093
0.01	Raw	0.0010	1.0200	6061.2	0	0.0093
	Rounded	0.0010	1.0330	6061.1	0	0.0093
	Un-rounded	0.0010	1.0182	6061.1	1.5×10^{-7}	0.0093

Table 17: Test results of a rate-1000 renewal process on $[0, 6]$, in which the interarrival times are 0 with probability p and an exponential random variable with probability $1-p$. Results over 10000 iterations.

		CU		Log		Lewis	
p	Type	# P	ave[p-value]	# P	ave[p-value]	# P	ave[p-value]
0.1	Raw	9015	0.41	0	0.00	0	0.00
	Rounded	9015	0.41	0	0.00	0	0.00
	Un-rounded	9018	0.41	0	0.00	0	0.00
0.05	Raw	9306	0.46	0	0.00	0	0.00
	Rounded	9304	0.46	0	0.00	0	0.00
	Un-rounded	9308	0.46	2	0.00	0	0.00
0.01	Raw	9453	0.49	8798	0.26	7879	0.22
	Rounded	9454	0.49	0	0.00	0	0.00
	Un-rounded	9453	0.49	8877	0.37	8175	0.32

Table 18: Summary statistics of batch Poisson processes on $[0, 6]$ in which every kth point comes in pairs; the total rate is kept the same. Results over 10000 iterations.

	Type	ave[X]	ave[c_X^2]	ave[n]	min[X]	max[X]
k=1	Raw	0.0010	2.9982	5998.0	0	0.0172
	Rounded	0.0010	3.0047	5997.9	0	0.0172
	Un-rounded	0.0010	2.8237	5997.9	3.6×10^{-8}	0.0171
k=3	Raw	0.0010	1.6662	5998.2	0	0.0120
	Rounded	0.0010	1.6759	5998.1	0	0.0120
	Un-rounded	0.0010	1.6076	5998.1	1.2×10^{-9}	0.0120
k=6	Raw	0.0010	1.3328	5998.4	0	0.0106
	Rounded	0.0010	1.3439	5998.3	0	0.0106
	Un-rounded	0.0010	1.3037	5998.3	1.7×10^{-9}	0.0106
k=9	Raw	0.0010	1.2217	5997.3	0	0.0102
	Rounded	0.0010	1.2333	5997.2	0	0.0102
	Un-rounded	0.0010	1.2023	5997.2	1.9×10^{-9}	0.0102

Table 19: Test results of batch Poisson processes on $[0, 6]$ in which every kth point comes in pairs; the total rate is kept the same. Results over 10000 iterations.

		CU		Log		Lewis	
	Type	# P	ave[p-value]	# P	ave[p-value]	# P	ave[p-value]
k=1	Raw	6801	0.21	0	0.00	0	0.00
	Rounded	6796	0.21	0	0.00	0	0.00
	Un-rounded	6802	0.21	0	0.00	0	0.00
k=3	Raw	8671	0.36	0	0.00	0	0.00
	Rounded	8669	0.36	0	0.00	0	0.00
	Un-rounded	8670	0.36	0	0.00	0	0.00
k=6	Raw	9179	0.43	0	0.00	0	0.00
	Rounded	9178	0.43	0	0.00	0	0.00
	Un-rounded	9181	0.43	0	0.00	0	0.00
k=9	Raw	9196	0.45	0	0.00	0	0.00
	Rounded	9194	0.45	0	0.00	0	0.00
	Un-rounded	9195	0.45	0	0.00	0	0.00

Figure 2: Comparison of the average ecdf of a rate-1000 renewal process on $[0, 6]$, in which the interarrival times are 0 with probability $p = 0.1$ and an exponential random variable with probability $1 - p$. From top to bottom: CU, Lewis test. From left to right: Raw, Rounded, and Un-rounded.

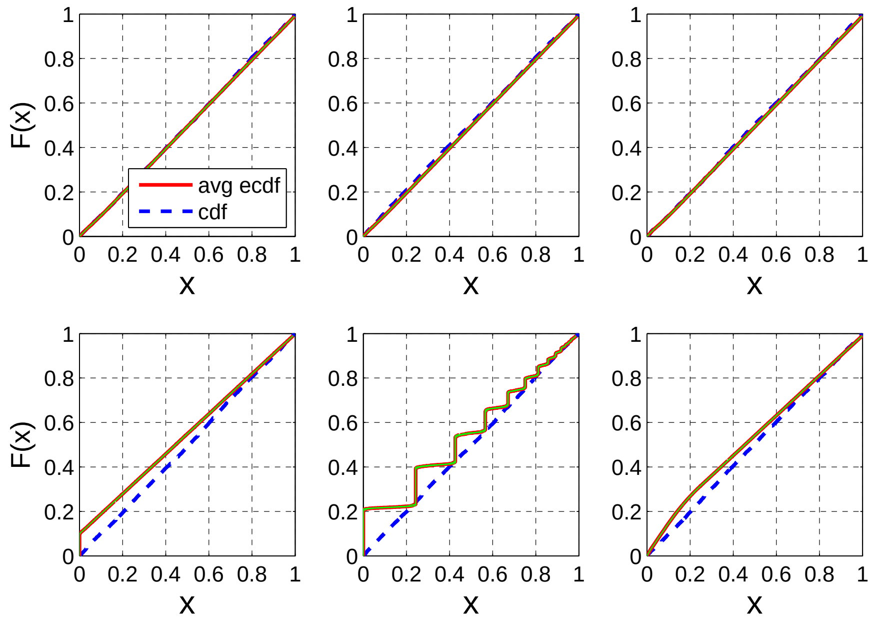


Figure 3: Comparison of the average ecdf of a rate-1000 renewal process on $[0, 6]$, in which the interarrival times are 0 with probability $p = 0.05$ and an exponential random variable with probability $1 - p$. From top to bottom: CU, Lewis test. From left to right: Raw, Rounded, and Un-rounded.

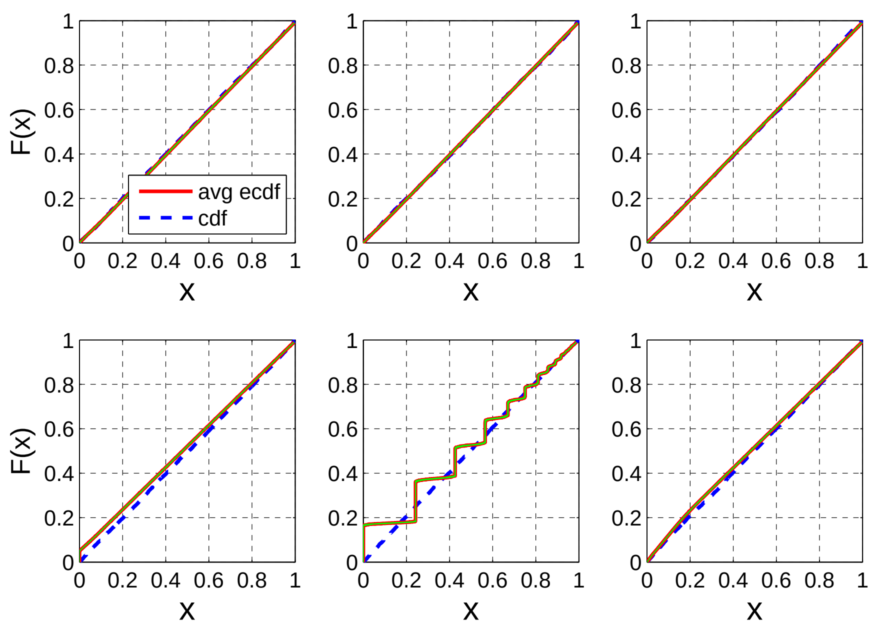


Figure 4: Comparison of the average ecdf of a rate-1000 renewal process on $[0, 6]$, in which the interarrival times are 0 with probability $p = 0.01$ and an exponential random variable with probability $1 - p$. From top to bottom: CU, Lewis test. From left to right: Raw, Rounded, and Un-rounded.

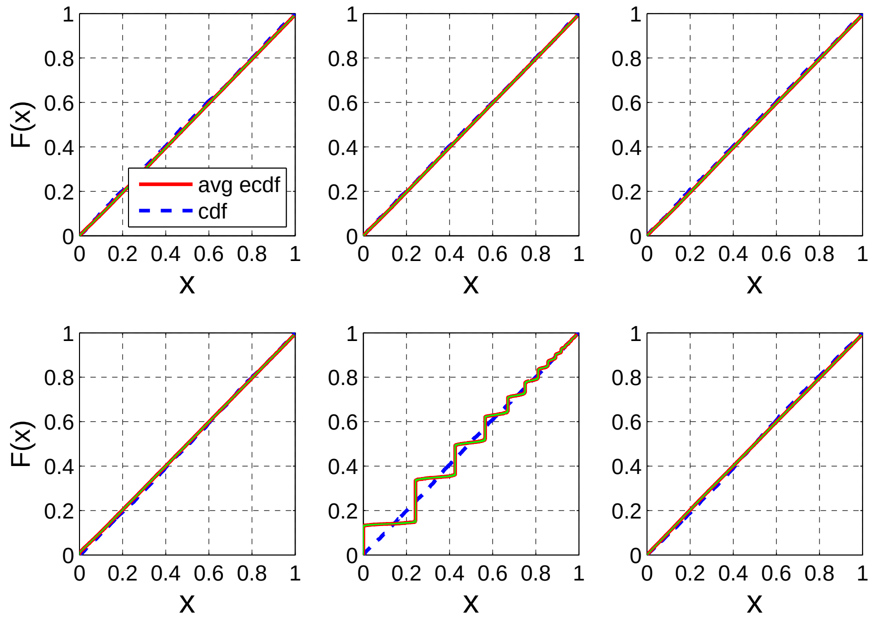


Figure 5: Comparison of the average ecdf of a batch Poisson process on $[0, 6]$ in which every 3rd point comes in pairs. From top to bottom: CU, Lewis test. From left to right: Raw, Rounded, and Un-rounded.

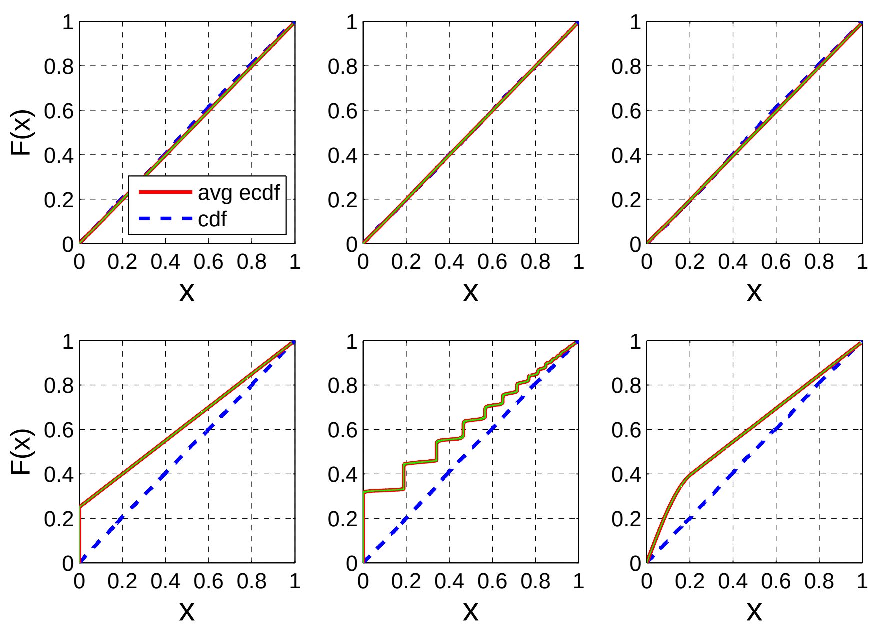


Figure 6: Comparison of the average ecdf of a batch Poisson process on $[0, 6]$ in which every 6th point comes in pairs. From top to bottom: CU, Lewis test. From left to right: Raw, Rounded, and Un-rounded.

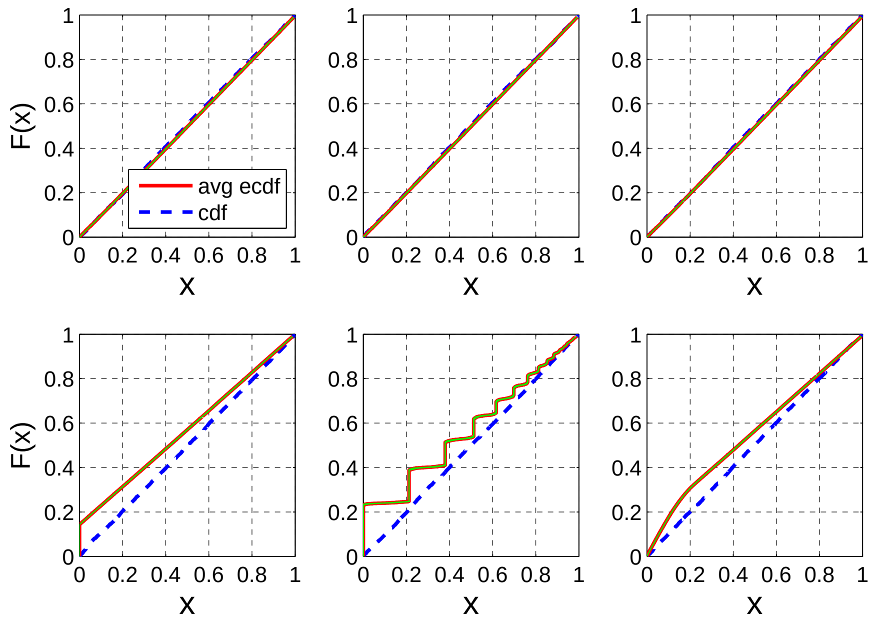


Figure 7: Comparison of the average ecdf of a batch Poisson process on $[0, 6]$ in which every 9th point comes in pairs. From top to bottom: CU, Lewis test. From left to right: Raw, Rounded, and Un-rounded.

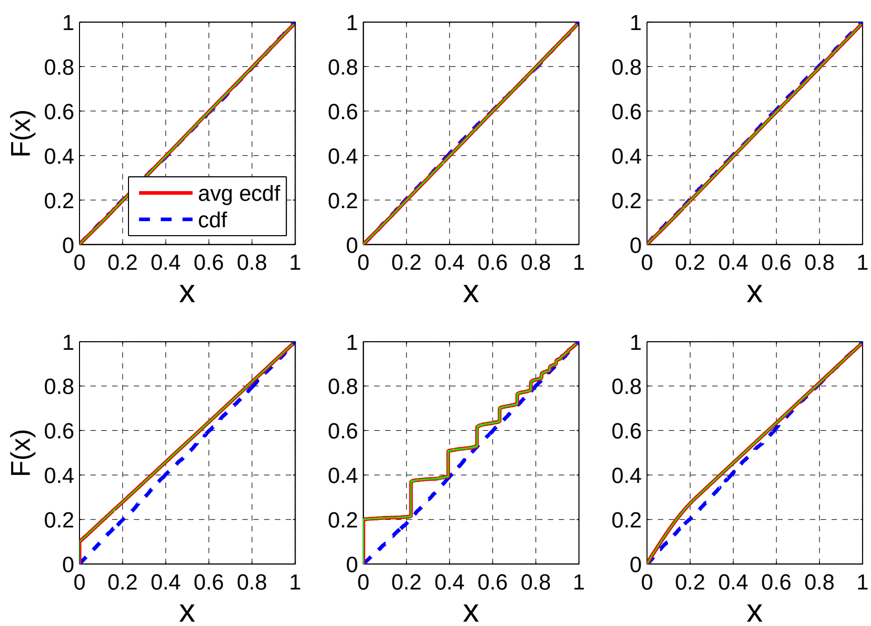


Table 20: [Compare to Tables 1 and 2 of the Main Paper: Rounding done to the nearest minute] Results of the two KS tests with rounding and un-rounding

		CU			Lewis		
Interarrival Times	Type	# P	ave[p-value]	ave[% 0]	# P	ave[p-value]	ave[% 0]
M	Raw	944	0.50	0.0	955	0.50	0.0
	Rounded	927	0.40	0.1	0	0.00	94.0
	Un-rounded	949	0.50	0.0	946	0.51	0.0
H_2	Raw	705	0.21	0.0	0	0.00	0.0
	Rounded	630	0.16	0.1	0	0.00	94.0
	Un-rounded	704	0.22	0.0	42	0.01	0.0

Table 21: [Compare to Table 17: Rounding done to the nearest minute] Test results of a rate-1000 renewal process on $[0, 6]$, in which the interarrival times are 0 with probability p and an exponential random variable with probability $1 - p$. Results over 10000 iterations.

		CU		Log		Lewis	
p	Type	# P	ave[p-value]	# P	ave[p-value]	# P	ave[p-value]
0.1	Raw	9015	0.41	0	0.00	0	0.00
	Rounded	8580	0.32	0	0.00	0	0.00
	Un-rounded	9019	0.41	9156	0.45	8774	0.40
0.05	Raw	9306	0.46	0	0.00	0	0.00
	Rounded	8959	0.36	0	0.00	0	0.00
	Un-rounded	9307	0.46	9421	0.49	9289	0.47
0.01	Raw	9453	0.49	8798	0.26	7879	0.22
	Rounded	9178	0.40	0	0.00	0	0.00
	Un-rounded	9451	0.49	9484	0.50	9480	0.50

Table 22: [Compare to Table 19: Rounding done to the nearest minute] Test results of batch Poisson processes on $[0, 6]$ in which every kth point comes in pairs; the total rate is kept the same. Results over 10000 iterations.

		CU		Log		Lewis	
	Type	# P	ave[p-value]	# P	ave[p-value]	# P	ave[p-value]
k=1	Raw	6801	0.21	0	0.00	0	0.00
	Rounded	6156	0.16	0	0.00	0	0.00
	Un-rounded	6914	0.21	4411	0.13	456	0.01
k=3	Raw	8671	0.36	0	0.00	0	0.00
	Rounded	8224	0.29	0	0.00	0	0.00
	Un-rounded	8685	0.37	8694	0.40	7467	0.28
k=6	Raw	9179	0.43	0	0.00	0	0.00
	Rounded	8809	0.34	0	0.00	0	0.00
	Un-rounded	9141	0.43	9285	0.47	8922	0.43
k=9	Raw	9196	0.45	0	0.00	0	0.00
	Rounded	8875	0.36	0	0.00	0	0.00
	Un-rounded	9196	0.44	9360	0.48	9163	0.45

6 Case Examples - Supplementary Material for Sections 5 and 6

In this section, we work with call center and hospital arrival data to show how our methods work. We first describe call center and hospital arrival datasets we have in §6.1. We start with three examples that illustrate what we need to be careful about when dealing with real data in §6.3. In §6.4, we show an example of call center arrival data that can be well modeled by an NHPP, and similarly for hospital emergency department arrivals in §6.5.

6.1 Call Center and Hospital Arrival Data

We use the same **Call Center** data we used in Kim and Whitt [2013a,b], from a telephone call center of a medium-sized American bank from the data archive of Mandelbaum [2012], collected from March 26, 2001 to October 26, 2003. This banking call center had sites in New York, Pennsylvania, Rhode Island, and Massachusetts, which were integrated to form a single virtual call center. The virtual call center had 900 - 1200 agent positions on weekdays and 200 - 500 agent positions on weekends. The center processed about 300,000 calls per day during weekdays, with about 60,000 (20%) handled by agents, with the rest being served by Voice Response Unit (VRU) technology. In this study, we focus on arrival data during April 2001. There are 4 significant entry points to the system: through VRU ~92%, Announcement ~6%, Message ~1% and Direct group (callers that directly connect to an agent) ~1%; there are a very small number of outgoing and internal calls, and we are not including them. Furthermore, among the customers that arrive to the VRU, there are five customer types: Retail ~91.4%, Premier ~1.9%, Business ~4.4%, Customer Loan ~0.3%, and Summit ~2.0%.

Hospital Emergency Department (ED) data are from one of major teaching hospitals in South Korea, collected from September 1, 2012 to November 15, 2012; we focus on 70 days, from September 1, 2012 to November 9, 2012. There are two major entry groups, walk-ins and ambulance arrivals. On average, there are 138.5 arrivals each day with ~88% walk-ins and ~12% by ambulance.

Figures 8 and 9 show average hourly arrival rate as well as individual hourly arrival rate for each arrival type on Mondays for the call center and hospital ED data, respectively. We observe strong within-day variations in call center arrivals, but not as much for the hospital ED arrivals, especially for the ambulance arrivals. Furthermore, Tables 26 and 27 show the number of arrivals in each day for each arrival type, as well as the estimated values of the mean $\bar{\mu}$, variance $\bar{\sigma^2}$ with its 95% confidence interval, and the dispersion test result. If $x_1, x_2, ..., x_n$ are n observations from a Poisson population, then the index of dispersion $D \equiv \sum_i (x_i - \bar{x})^2/\bar{x}$ is approximately distributed as a χ^2 statistic with $n-1$ degrees of freedom (see Kathirgamatamby [1953] and references therein). The *dispersion test* (also known as Fisher's dispersion test) then uses this fact to test the null hypothesis that $x_1, x_2, ..., x_n$ are independent Poisson distributed variables with mean parameter $\bar{x}$; we report the p-value for this test. We observe that the call center has significant day-to-day variation in its arrivals (the null hypothesis for Poisson distribution is rejected for every arrival type); hospital ED walk-in arrivals also have strong day-to-day variation, whereas the dispersion test for ambulance arrivals fails to reject the null hypothesis for Poisson distribution.

Figure 8: Average hourly arrival rate as well as individual hourly arrival rate for the 17 hours in [6,22] on 5 Mondays in April 2001 from the call center arrival data.

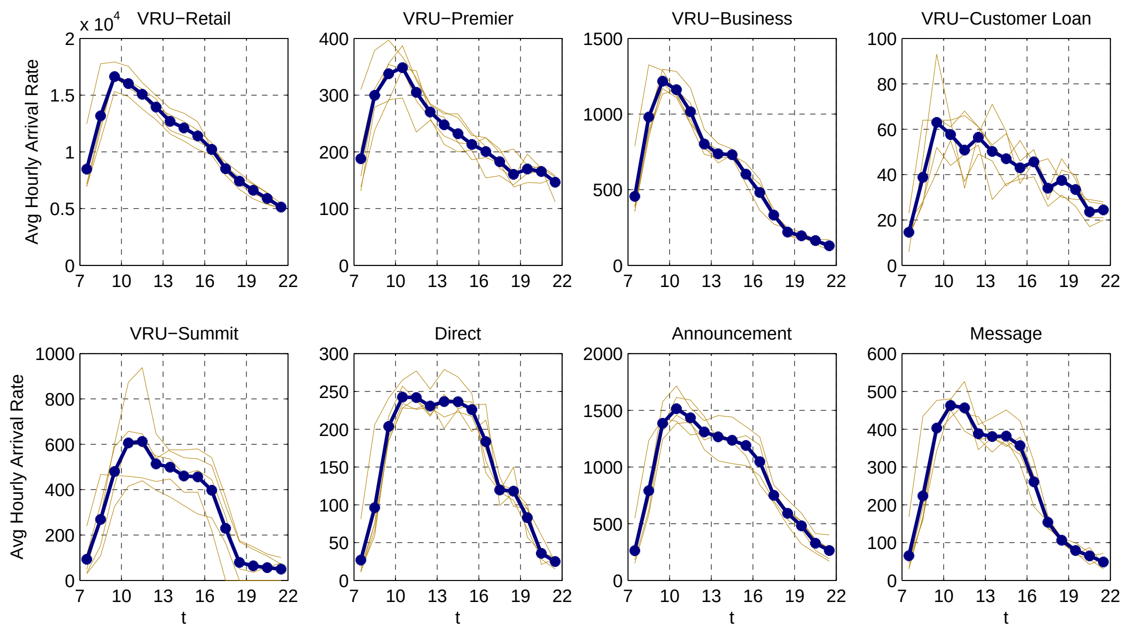


Table 23: Call center arrival data in April 2001: Number of arrivals in each day for each arrival type is shown. The estimated values of the mean $\bar{\mu}$, variance $\bar{\sigma^2}$ with associated 95% confidence interval, index of dispersion $\bar{D} = \sum_i (x_i - \bar{\mu})^2/\bar{\mu}$, and p-value for the dispersion test are also reported.

Day	VRU-Retail	VRU-Premier	VRU-Business	VRU-Customer Loan	VRU-Summit	Direct	Announcement	Message	Total
1	85181	2366	1902	287	842	591	3589	336	95094
2	202118	4281	10283	723	4008	2703	13992	4179	242287
3	201517	4097	10188	588	3238	1970	13083	3680	238361
4	182828	3511	9713	549	2700	1914	11123	3373	215711
5	192109	3434	9697	535	2535	1785	10036	3242	223373
6	189772	3481	8997	540	2697	1684	10088	3270	220529
7	112524	2407	3124	345	1755	652	5401	653	126861
8	77216	2021	1773	229	788	359	2845	309	85540
9	179099	3420	9669	748	3145	2179	12393	4202	214855
10	184241	3530	9711	592	3701	1878	11073	3892	218618
11	178620	3695	9488	496	4500	1802	11756	3568	213925
12	201309	4052	9710	476	4650	1728	12763	3652	238340
13	189446	3955	7696	460	3977	1590	11980	3149	222253
14	106201	2311	2885	220	1914	988	4977	512	120008
15	56001	1426	1656	142	603	323	6665	226	67042
16	174513	3922	9101	694	6417	2249	15352	3844	216092
17	177270	3727	9844	607	4682	2236	12500	3553	214419
18	172025	3623	9577	545	4439	2076	12716	3495	208496
19	181634	3529	9088	516	3868	2022	12089	3352	216098
20	170605	3411	8576	515	4623	1806	13216	2950	205702
21	106633	2169	2930	273	1984	728	5158	527	120402
22	68754	1749	1482	170	932	274	2858	279	76498
23	161338	3339	9117	532	4854	2210	14259	3695	199344
24	166741	3300	9377	541	4838	1842	11612	3444	201695
25	175563	3523	9289	542	4341	1840	10532	3439	209069
26	177882	3679	9294	492	4260	1740	10777	3188	211312
27	151997	3822	9116	409	4753	1536	15478	3651	190762
28	106756	2456	2807	229	2612	703	5050	1129	121742
29	74226	1945	1743	207	1060	305	2930	488	82904
30	175378	4005	10246	579	6118	2361	16284	3637	218608
All days									
$\bar{\mu}$	152649.9	3206.2	7269.3	459.4	3361.1	1535.8	10085.8	2630.5	181198.0
$\bar{\sigma^2}$	2072855181.0	633730.3	11502448.7	28451.4	2558803.5	505719.2	17063574.8	2111259.2	3207849601.0
	[1314737424.0, 3746029268.0]	[401952.3, 1145266.8]	[7295589.2, 20787033.2]	[18045.7, 51416.9]	[1622957.0, 4624227.0]	[320759.5, 913927.3]	[10822811.2, 30837007.4]	[1339095.7, 3815432.3]	[2034623528.0, 5797172232.0]
$\bar{D}$	393795.2	5732.1	45887.6	1796.1	22077.5	9549.3	49063.2	23275.9	513403.2
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Weekdays									
$\bar{\mu}$	180286.0	3682.7	9418.0	556.1	4207.0	1960.0	12528.7	3545.5	216183.3
$\bar{\sigma^2}$	165523099.1	76912.8	345686.3	6965.6	1014163.5	80597.5	3102420.1	100618.6	158831206.0
	[96883232.9, 345171392.0]	[45018.2, 160389.2]	[202335.6, 720872.4]	[4077.1, 14525.7]	[593605.6, 2114872.4]	[47174.9, 168072.8]	[1815894.5, 6469590.5]	[58893.6, 209823.6]	[92966364.2, 331216541.7]
$\bar{D}$	18362.3	417.7	734.1	250.5	4821.5	822.6	4952.5	567.6	14694.1
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Figure 9: Average hourly arrial rate as well as individual hourly arrival rate for the 24 hours in [0,24] on 10 Mondays in September 1, 2012 - November 9, 2012 from from hospital ED arrival data.

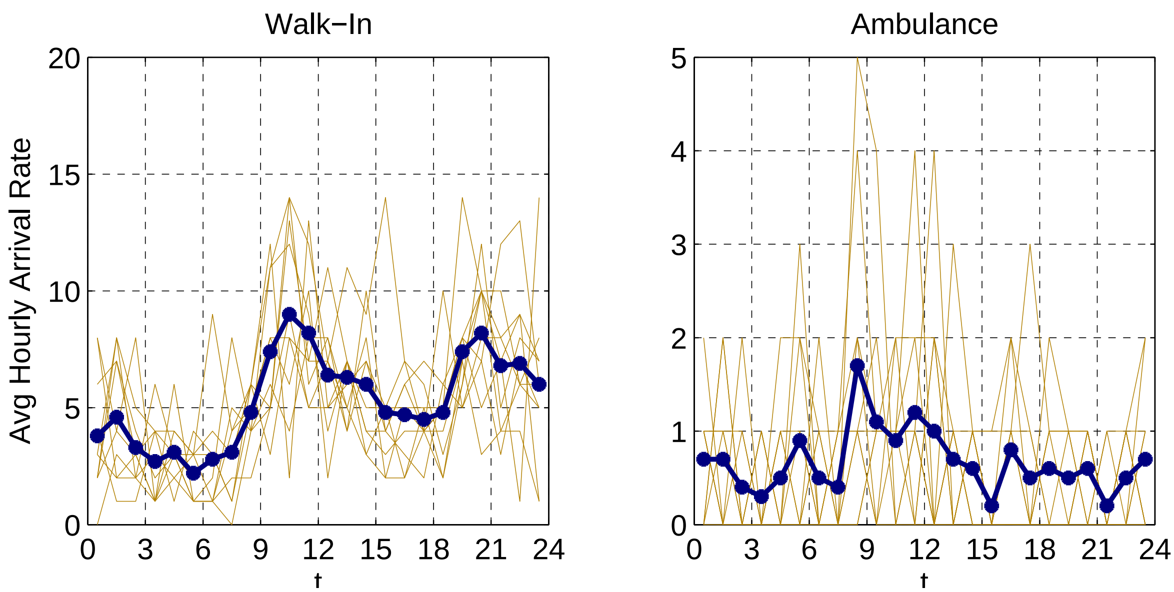


Table 24: Hospital ED arrival data in September 1, 2012 - November 9, 2012: Number of arrivals in each day for each arrival type is shown. The estimated values of the mean $\bar{\mu}$, variance $\bar{\sigma^2}$ with associated 95% confidence interval, index of dispersion $\bar{D} = \sum_i (x_i - \bar{\mu})^2/\bar{\mu}$, and p-value for the dispersion test are also reported.

Day	Walk-In	Ambulance	Total	Day	Walk-In	Ambulance	Total
9/1/12	115	20	135	10/6/12	125	20	145
9/2/12	166	23	189	10/7/12	148	10	158
9/3/12	130	15	145	10/8/12	124	24	148
9/4/12	107	12	119	10/9/12	96	13	109
9/5/12	108	14	122	10/10/12	120	22	142
9/6/12	96	14	110	10/11/12	115	17	132
9/7/12	104	15	119	10/12/12	107	14	121
9/8/12	115	27	142	10/13/12	124	28	152
9/9/12	186	20	206	10/14/12	139	16	155
9/10/12	127	11	138	10/15/12	129	19	148
9/11/12	112	12	124	10/16/12	103	14	117
9/12/12	101	17	118	10/17/12	88	18	106
9/13/12	112	19	131	10/18/12	99	21	120
9/14/12	100	26	126	10/19/12	100	8	108
9/15/12	160	17	177	10/20/12	139	22	161
9/16/12	163	21	184	10/21/12	137	16	153
9/17/12	105	21	126	10/22/12	117	19	136
9/18/12	103	18	121	10/23/12	108	10	118
9/19/12	115	16	131	10/24/12	97	12	109
9/20/12	116	19	135	10/25/12	107	9	116
9/21/12	105	16	121	10/26/12	127	17	144
9/22/12	102	14	116	10/27/12	107	13	120
9/23/12	173	17	190	10/28/12	144	14	158
9/24/12	126	9	135	10/29/12	123	14	137
9/25/12	115	14	129	10/30/12	88	14	102
9/26/12	104	15	119	10/31/12	106	20	126
9/27/12	128	18	146	11/1/12	105	21	126
9/28/12	101	12	113	11/2/12	108	16	124
9/29/12	184	13	197	11/3/12	127	18	145
9/30/12	192	11	203	11/4/12	151	12	163
10/1/12	166	18	184	11/5/12	131	12	143
10/2/12	160	26	186	11/6/12	93	19	112
10/3/12	145	20	165	11/7/12	79	17	96
10/4/12	140	16	156	11/8/12	104	15	119
10/5/12	116	15	131	11/9/12	121	18	139
All days				Weekdays			
$\bar{\mu}$	121.9	16.6	138.5	$\bar{\mu}$	112.7	16.2	129.0
$\bar{\sigma^2}$	615.8	19.2	657.0	$\bar{\sigma^2}$	288.4	16.7	334.7
	[452.7, 886.6]	[14.1, 27.7]	[483.0, 945.9]		[201.3, 447.9]	[11.7, 25.9]	[233.5, 519.7]
$\bar{D}$	348.5	79.8	327.2	$\bar{D}$	125.4	50.5	127.2
p-value	0.00	0.17	0.00	p-value	0.00	0.42	0.00

6.2 Tests for Over-Dispersion

In Section 4.1 of the main paper, we discuss two statistical tests that test for over-dispersion, the dispersion test and the Brown and Zhao [2002] dispersion test. In Tables 25 - 29 below, we provide the test results for both tests; the test results are very similar.

Table 25: Summary Statistics of the Call center arrivals: each hour on 16 Fridays.

Interval	$\bar{\mu}$	$\bar{\sigma}^2$	$\bar{D}$	p-value	$\bar{D}^{bz}$	p-value
[7 , 8]	180.5	12860.1, [7017.6, 30804.5]	1068.7	0.00	889.5	0.00
[8 , 9]	363.7	14078.2, [7682.3, 33722.3]	580.6	0.00	477.5	0.00
[9 , 10]	499.1	7352.8, [4012.3, 17612.5]	221.0	0.00	226.8	0.00
[10 , 11]	496.8	5411.1, [2952.8, 12961.5]	163.4	0.00	176.7	0.00
[11 , 12]	502.8	10428.9, [5690.9, 24980.8]	311.2	0.00	328.8	0.00
[12 , 13]	446.4	7782.0, [4246.5, 18640.6]	261.5	0.00	276.2	0.00
[13 , 14]	444.0	8778.0, [4790.0, 21026.4]	296.6	0.00	314.3	0.00
[14 , 15]	445.4	10669.7, [5822.3, 25557.7]	359.4	0.00	383.1	0.00
[15 , 16]	405.1	8905.1, [4859.4, 21330.9]	329.8	0.00	348.4	0.00
[16 , 17]	353.4	6723.6, [3669.0, 16105.3]	285.4	0.00	376.8	0.00
[17 , 18]	196.3	5239.9, [2859.4, 12551.5]	400.5	0.00	895.0	0.00
[18 , 19]	82.9	1778.9, [970.7, 4261.1]	322.0	0.00	694.9	0.00
[19 , 20]	57.7	752.2, [410.5, 1801.9]	195.6	0.00	446.2	0.00
[20 , 21]	52.0	1174.4, [640.9, 2813.1]	338.8	0.00	535.7	0.00
[21 , 22]	42.3	1027.9, [560.9, 2462.3]	364.9	0.00	562.4	0.00
[22 , 23]	29.2	385.4, [210.3, 923.1]	198.0	0.00	361.9	0.00

Table 26: Call center arrivals: summary statistics by type. (CL = Customer Loan.) All p-values for dispersion tests are 0.

n	Type	$\bar{\mu}$	$\bar{\sigma}^2$	$\bar{D}$	$\bar{D}^{bz}$
All days	VRU-Retail	1.5×10^5	$2.1 \times 10^9, [1.3 \times 10^9, 3.8 \times 10^9]$	3.9×10^5	4.8×10^5
(n=30)	VRU-Premier	3.2×10^3	$6.3 \times 10^5, [4.0 \times 10^5, 1.2 \times 10^6]$	5.7×10^3	6.6×10^3
	VRU-Business	7.3×10^3	$1.2 \times 10^7, [7.3 \times 10^6, 2.1 \times 10^7]$	4.6×10^4	6.5×10^4
	VRU-CL	4.6×10^2	$2.9 \times 10^4, [1.8 \times 10^4, 5.1 \times 10^4]$	1.8×10^3	2.1×10^3
	VRU-Summit	3.4×10^3	$2.6 \times 10^6, [1.6 \times 10^6, 4.6 \times 10^6]$	2.2×10^4	2.8×10^4
	Business	1.5×10^3	$5.1 \times 10^5, [3.2 \times 10^5, 9.1 \times 10^5]$	9.6×10^3	1.3×10^4
	Announcement	1.0×10^4	$1.7 \times 10^7, [1.1 \times 10^7, 3.1 \times 10^7]$	4.9×10^4	6.2×10^4
	Message	2.6×10^3	$2.1 \times 10^6, [1.3 \times 10^6, 3.8 \times 10^6]$	2.3×10^4	3.8×10^4
	Total	1.8×10^5	$3.2 \times 10^9, [2.0 \times 10^9, 5.8 \times 10^9]$	5.1×10^5	6.3×10^5
Weekdays	VRU-Retail	1.8×10^5	$1.7 \times 10^8, [9.7 \times 10^7, 3.5 \times 10^8]$	1.8×10^4	1.8×10^4
(n=21)	VRU-Premier	3.7×10^3	$7.7 \times 10^4, [4.5 \times 10^4, 1.6 \times 10^5]$	4.2×10^2	4.1×10^2
	VRU-Business	9.4×10^3	$3.5 \times 10^5, [2.0 \times 10^5, 7.2 \times 10^5]$	7.3×10^2	7.6×10^2
	VRU-CL	5.6×10^2	$7.0 \times 10^3, [4.1 \times 10^3, 1.2 \times 10^4]$	2.5×10^2	2.4×10^2
	VRU-Summit	4.2×10^3	$1.0 \times 10^6, [5.9 \times 10^5, 2.1 \times 10^6]$	4.8×10^3	4.9×10^3
	Business	2.0×10^3	$8.1 \times 10^4, [4.7 \times 10^4, 1.7 \times 10^5]$	8.2×10^2	7.9×10^2
	Announcement	1.3×10^4	$3.1 \times 10^6, [1.8 \times 10^6, 6.5 \times 10^6]$	5.0×10^3	4.8×10^3
	Message	3.6×10^3	$1.0 \times 10^5, [5.9 \times 10^4, 2.1 \times 10^5]$	5.7×10^2	5.6×10^2
	Total	2.2×10^5	$1.6 \times 10^8, [9.3 \times 10^7, 3.3 \times 10^8]$	1.5×10^4	1.5×10^4

Table 27: Hospital ED arrivals: summary statistics by type.

n	Type	$\bar{\mu}$	$\bar{\sigma}^2$	$\bar{D}$	p-value	$\bar{D}^{bz}$	p-value
All days	Walk-in	121.9	$615.8, [452.7, 886.6]$	348.5	0.00	323.9	0.00
(n=70)	Ambulance	16.6	$19.2, [14.1, 27.7]$	79.8	0.17	77.9	0.22
	Total	138.5	$657.0, [483.0, 945.9]$	327.2	0.00	307.9	0.00
Weekdays	Walk-in	112.7	$288.4, [201.3, 447.9]$	125.4	0.00	118.8	0.00
(n=50)	Ambulance	16.2	$16.7, [11.7, 25.9]$	50.5	0.42	50.1	0.43
	Total	129.0	$334.7, [233.5, 519.7]$	127.2	0.00	119.8	0.00

Table 28: Summary statistics for the samples of each case.

	Day of Week	# Sample	$\bar{\mu}$	$\bar{\sigma}^2$	$\bar{D}$	p-value	$\bar{D}^{bz}$	p-value
Case 1	Sun	5	460.6	10447.3, [3750.2, 86266.7]	90.7	0.00	93.6	0.00
	Mon	4	373.5	4083.0, [1310.3, 56762.1]	32.8	0.00	35.3	0.00
	Tues	4	371.5	11955.0, [3836.5, 166199.2]	96.5	0.00	98.6	0.00
	Wed	4	354.3	9830.3, [3154.6, 136660.8]	83.2	0.00	81.4	0.00
	Thurs	4	387.8	10204.3, [3274.7, 141860.1]	78.9	0.00	83.0	0.00
	Fri	4	139.0	2531.3, [812.3, 35190.8]	54.6	0.00	59.5	0.00
	Sat	5	81.6	270.3, [97.0, 2232.0]	13.3	0.01	14.9	0.00
	ALL	30	307.2	24583.1, [15592.2, 44426.2]	2320.9	0.00	2884.7	0.00
Case 2	Sun	5	841.4	87927.8, [31562.6, 726048.2]	418.0	0.00	455.1	0.00
	Mon	4	919.8	48630.3, [15606.0, 676060.9]	158.6	0.00	179.2	0.00
	Tues	4	823.3	22344.9, [7170.7, 310640.5]	81.4	0.00	84.6	0.00
	Wed	4	769.3	1864.3, [598.3, 25916.9]	7.3	0.06	7.1	0.07
	Thurs	4	833.8	17912.3, [5748.2, 249017.3]	64.5	0.00	64.4	0.00
	Fri	4	516.3	29938.3, [9607.5, 416203.5]	174.0	0.00	174.4	0.00
	Sat	5	134.8	2222.7, [797.9, 18353.5]	66.0	0.00	70.5	0.00
	ALL	30	677.7	99470.7, [63090.7, 179761.8]	4256.7	0.00	6041.2	0.00
Case 3	Sun	10	24.6	40.7, [19.3, 135.7]	14.9	0.09	14.3	0.11
	Mon	10	19.4	20.3, [9.6, 67.5]	9.4	0.40	8.4	0.50
	Tues	10	17.6	30.9, [14.6, 103.1]	15.8	0.07	15.7	0.07
	Wed	10	21.6	25.6, [12.1, 85.3]	10.7	0.30	10.6	0.30
	Thurs	10	17.9	5.9, [2.8, 19.6]	3.0	0.97	3.0	0.96
	Fri	10	20.2	56.8, [26.9, 189.5]	25.3	0.00	24.8	0.00
	Sat	10	31.3	50.2, [23.8, 167.4]	14.4	0.11	16.0	0.07
	ALL	70	21.8	50.2, [36.9, 72.3]	159.0	0.00	149.8	0.00

6.3 Illustrative Examples

We consider three cases in this section. Suppose we are interested in testing for an NHPP in the following settings:

Case 1 We observe that the VRU - Summit arrival rate at the call center is nearly constant in the interval $[14, 15]$ (i.e., from 2pm to 3pm). We want to test whether the arrival process in $[14, 15]$ is a PP.

Case 2 We observe that the VRU - Summit arrival rate at the call center is nearly linear and increasing in the interval $[7, 10]$. We want to test whether the arrival process in $[7, 10]$ is an NHPP.

Case 3 We want to test whether the walk-in arrival process in the hospital ED data in the interval $[9, 12]$ is an NHPP.

Before proceeding to test whether the arrival data in each day come from a NHPP, we can first test whether there is over-dispersion over multiple days in the interval in interest (i.e., whether data from different days in the same interval have variable arrival rate). Table 29 provides the estimated values of mean (μ), variance σ^2 with its 95% confidence interval, and the dispersion test result for arrival data from 30 days for **Case 1** and **2** and from 70 days for **Case 3**. We observe that in three cases, there exist over-dispersion (in other words, day-to-day variation).

Table 29: Estimated values of the mean $\bar{\mu}$, variance $\bar{\sigma^2}$ with associated 95% confidence interval, index of dispersion $\bar{D} = \sum_i (x_i - \bar{\mu})^2/\bar{\mu}$, and p-value for the dispersion test for each case.

Case	# Obs	$\bar{\mu}$	$\bar{\sigma^2}$	$\bar{D} = \sum_i (x_i - \mu)^2/\mu$	p-value for dispersion test
Case 1	30	307.2	24583.1, [15592.2, 44426.2]	2320.9	0.00
Case 2	30	677.7	99470.7, [63090.7, 179761.8]	4256.7	0.00
Case 3	70	21.8	50.2, [36.9, 72.3]	159.0	0.00

In tackling **Case 1**, we do not need subintervals because we observe that the arrival rate is nearly constant in the interval, so we can apply the Lewis test right away. Table 30 provides such test results under 'Raw'. When the significance level $\alpha = 0.05$ is used, we see that the arrival data pass the Lewis test on 19 days out of 30 days in April. We then find that the arrival times have been rounded to the nearest seconds. As discussed in Section 2 of the main paper this can lead us to reject the NHPP null hypothesis more, so we *unround* the arrival data by adding uniform random variables divided by 3600 to the arrival times. The new results are under 'Unrounded' in Table 30; we see that now 29 days out of 30 days pass the Lewis test, and the average p-value has increased from 0.20 to 0.49. We can conclude that the arrival processes in $[14, 15]$

do come from NHPPs, with the understanding that the rates on different days vary, an! d should be regarded as random.

Table 30: Case 1 (The effect of rounding): Performance of the alternative KS test of a Nonhomogeneous Poisson process for Summit customer call arrivals at a banking call center on the time interval $[14, 15]$ in April 2001.

		Raw		Unrounded	
Day	n	CU	$Lewis$	CU	$Lewis$
1	90	0.85	0.81	0.85	0.86
2	389	0.03	0.20	0.03	0.84
3	282	0.96	0.08	0.96	0.34
4	243	0.67	0.18	0.66	0.42
5	247	0.19	0.15	0.19	0.31
6	266	0.82	0.32	0.82	0.66
7	75	0.24	0.39	0.24	0.49
8	85	0.23	0.17	0.23	0.19
9	330	0.22	0.06	0.22	0.82
10	385	0.75	0.01	0.75	0.06
11	507	0.94	0.02	0.94	0.78
12	487	0.30	0.00	0.30	0.21
13	344	0.00	0.19	0.00	0.86
14	140	0.77	0.73	0.78	0.73
15	54	0.94	0.51	0.94	0.53
16	541	0.95	0.00	0.96	0.00
17	430	0.49	0.02	0.49	0.63
18	390	0.13	0.07	0.14	0.63
19	337	0.67	0.04	0.67	0.25
20	480	0.88	0.01	0.88	0.20
21	143	0.19	0.38	0.19	0.63
22	82	0.89	0.20	0.89	0.19
23	468	0.71	0.01	0.71	0.52
24	397	0.23	0.00	0.24	0.33
25	346	0.91	0.27	0.91	0.43
26	346	0.90	0.15	0.89	0.52
27	461	0.60	0.01	0.60	0.79
28	198	0.19	0.70	0.19	0.76
29	97	0.31	0.24	0.31	0.27
30	575	0.31	0.00	0.32	0.47
Average	307.17	0.54	0.20	0.54	0.49
# Pass ($\alpha = 0.05$)		$28/30$	$19/30$	$28/30$	$29/30$

To answer our second question in **Case 2**, we first *unround* the arrival times following **Case 1**. Because the arrival rate is nearly linear and increasing in the interval $[7, 10]$, we want to use subintervals as discussed in §2. Table 31 shows the result of using different subinterval lengths, L = 3, 1.5, 1, and 0.5 hours. We observe that as we decrease the subinterval lengths, and hence make the piecewise-constant approximation more appropriate in each subinterval, more days pass the Lewis test. When we use L=0.5 hours, all 30 days in April pass the Lewis test. In contrast, suppose we did not *unround* the arrival times and applied the tests; the results are in Table 32. We observe that much fewer days pass the tests. For example, with L=0.5 hours, only 18 days (instead 30 days) in April pass the Lewis test. This again emphasizes the importance of *unrounding*.

Table 31: Case 2 (The role of subintervals): Performance of the alternative KS test of a Nonhomogeneous Poisson process for Summit customer call arrivals at a banking call center on the time interval $[7, 10]$ in April 2001. Arrival times are unrounded.

		$L = 3$		$L = 1.5$		$L = 1$		$L = 0.5$	
Day	n	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$
1	177	0.00	0.04	0.00	0.98	0.00	0.46	0.02	0.12
2	1168	0.00	0.14	0.00	0.77	0.02	0.69	0.01	0.67
3	999	0.00	0.08	0.00	0.03	0.18	0.15	0.71	0.12
4	964	0.00	0.00	0.03	0.24	0.31	0.05	0.76	0.31
5	830	0.00	0.03	0.00	0.09	0.22	0.45	0.57	0.61
6	862	0.00	0.67	0.00	0.54	0.33	0.24	0.07	0.15
7	711	0.00	0.31	0.03	0.73	0.29	0.98	0.96	0.91
8	94	0.00	0.00	0.02	0.14	0.08	0.43	0.09	0.93
9	463	0.00	0.00	0.00	0.02	0.00	0.91	0.16	0.92
10	600	0.00	0.00	0.00	0.43	0.00	0.83	0.16	0.29
11	635	0.00	0.00	0.00	0.07	0.01	0.58	0.20	0.90
12	733	0.00	0.00	0.00	0.10	0.00	0.58	0.00	0.48
13	680	0.00	0.00	0.00	0.00	0.00	0.01	0.10	0.19
14	366	0.00	0.00	0.00	0.00	0.09	0.43	0.10	0.38
15	78	0.00	0.00	0.04	0.42	0.39	0.86	0.33	0.99
16	934	0.00	0.00	0.00	0.00	0.00	0.10	0.00	0.14
17	975	0.00	0.00	0.00	0.00	0.00	0.73	0.01	0.37
18	774	0.00	0.00	0.00	0.61	0.00	0.82	0.56	0.69
19	745	0.00	0.00	0.00	0.20	0.00	0.71	0.06	0.96
20	793	0.00	0.00	0.00	0.01	0.01	0.70	0.01	0.77
21	375	0.00	0.00	0.00	0.00	0.13	0.91	0.07	0.76
22	144	0.00	0.00	0.01	0.06	0.17	0.22	0.19	0.50
23	606	0.00	0.00	0.00	0.00	0.00	0.23	0.01	0.46
24	1105	0.00	0.00	0.00	0.00	0.01	0.59	0.01	0.70
25	920	0.00	0.00	0.00	0.03	0.00	0.08	0.00	0.41
26	769	0.00	0.00	0.00	0.00	0.00	0.11	0.28	0.14
27	1000	0.00	0.00	0.00	0.71	0.14	0.54	0.15	0.29
28	613	0.00	0.00	0.00	0.24	0.06	0.49	0.70	0.64
29	181	0.00	0.00	0.43	0.62	0.01	0.27	0.34	0.23
30	1036	0.00	0.00	0.00	0.87	0.00	0.30	0.12	0.14
Average	677.67	0.00	0.04	0.02	0.26	0.08	0.48	0.23	0.51
# Pass ($\alpha = 0.05$)		0/30	4/30	1/30	18/30	12/30	29/30	21/30	30/30

Now we move on to **Case 3** in which we consider hospital ED walk-in arrival data. We find that the arrival times have been rounded to the nearest minutes, so unround them by adding uniform random variables divided by 60. We then apply the CU and the Lewis test to the arrival process in $[9, 12]$ for each of the 70 days as in Table 33. When the arrival times are *unrounded*, 67 days out of 70 days pass the Lewis test. However, even when we use the raw arrival times (before unrounding), we observe that 66 days out of 70

Table 32: Case 2 (The role of subintervals): Performance of the alternative KS test of a Nonhomogeneous Poisson process for Summit customer call arrivals at a banking call center on the time interval $[7, 10]$ in April 2001. Before unrounding the arrival times.

		$L = 3$		$L = 1.5$		$L = 1$		$L = 0.5$	
Day	n	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$
1	177	0.00	0.04	0.00	0.99	0.00	0.49	0.02	0.12
2	1168	0.00	0.00	0.00	0.00	0.02	0.00	0.01	0.00
3	999	0.00	0.00	0.00	0.01	0.18	0.01	0.69	0.01
4	964	0.00	0.00	0.03	0.00	0.31	0.00	0.76	0.02
5	830	0.00	0.00	0.00	0.02	0.22	0.05	0.58	0.09
6	862	0.00	0.03	0.00	0.03	0.34	0.03	0.07	0.03
7	711	0.00	0.06	0.03	0.23	0.29	0.29	0.96	0.29
8	94	0.00	0.00	0.02	0.17	0.08	0.44	0.09	0.94
9	463	0.00	0.00	0.00	0.01	0.00	0.25	0.16	0.34
10	600	0.00	0.00	0.00	0.03	0.00	0.41	0.16	0.16
11	635	0.00	0.00	0.00	0.02	0.02	0.19	0.21	0.32
12	733	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.01
13	680	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.13
14	366	0.00	0.00	0.00	0.00	0.09	0.29	0.11	0.33
15	78	0.00	0.00	0.04	0.42	0.39	0.88	0.33	0.96
16	934	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
17	975	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
18	774	0.00	0.00	0.00	0.05	0.00	0.01	0.54	0.06
19	745	0.00	0.00	0.00	0.01	0.00	0.12	0.06	0.12
20	793	0.00	0.00	0.00	0.00	0.01	0.03	0.01	0.09
21	375	0.00	0.00	0.00	0.00	0.13	0.57	0.07	0.57
22	144	0.00	0.00	0.01	0.06	0.17	0.25	0.20	0.44
23	606	0.00	0.00	0.00	0.00	0.00	0.17	0.01	0.17
24	1105	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.00
25	920	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
26	769	0.00	0.00	0.00	0.00	0.00	0.01	0.29	0.03
27	1000	0.00	0.00	0.00	0.00	0.15	0.00	0.16	0.00
28	613	0.00	0.00	0.00	0.02	0.06	0.09	0.72	0.35
29	181	0.00	0.00	0.43	0.61	0.01	0.26	0.34	0.25
30	1036	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00
Average	677.67	0.00	0.00	0.02	0.09	0.08	0.16	0.23	0.20
# Pass ($\alpha = 0.05$)		$0/30$	$1/30$	$1/30$	$7/30$	$12/30$	$15/30$	$21/30$	$18/30$

days pass the Lewis test. The very small sample sizes evidently cause the rounding not to matter (Note that rounding matters when it produces 0 interarrival times, which do not occur in NHPPs). However, we note that the sample size is small (21.8 observations on average), suggesting that the power of these tests are weak. In order to ensure our test results, we increase the sample size by combining data from multiple days; we group a! rrival data into seven groups, each group with 10 consecutive days. For example, in the first group, we subtract 9 from arrivals in $[9, 12]$ on September 1 will so that they happen in the interval $[0, 3]$, 9 from arrivals in $[9, 12]$ on September 2 and add 3 so that they happen in the interval $[3, 6]$, and so on until the 10th day. Table 34 shows the result of applying the CU and the Lewis tests to these seven groups of multiple days. We first apply the test assuming that the arrival rate is constant and the same in $[9, 12]$ in all of the days and hence use subinterval length of $L = 30$, the entire interval. We see that four of the seven groups pass the Lewis test. However, as discussed in §4 of the main paper, there can be day-to-day variation (as supported by Table 33) which causes over-dispersion. So we use $L = 3$, which means one subinterval for each day. The test result improves and now six out of seven groups pass the Lewis test. We c! onclude that the hospital emergency department walk-in arrival! s in $[9, 12]$ in our example are from NHPPs.

Table 33: Case 3 (Small sample size): Performance of the alternative KS test of a Nonhomogeneous Poisson process for hospital ED walk-in arrivals on the time interval $[9, 12]$ in Sept 1, 2012 - Nov 9, 2012.

		Raw		Unrounded	
Date	n	CU	$Lewis$	CU	$Lewis$
9/1/12	40	0.67	0.01	0.62	0.37
9/2/12	25	0.72	0.17	0.67	0.12
9/3/12	18	0.03	0.07	0.03	0.11
9/4/12	17	0.05	0.72	0.05	0.81
9/5/12	22	0.15	0.28	0.16	0.82
9/6/12	17	0.12	0.45	0.12	0.53
9/7/12	11	0.04	0.03	0.04	0.03
9/8/12	31	0.12	0.31	0.13	0.45
9/9/12	24	0.84	0.78	0.82	0.76
9/10/12	18	0.29	0.48	0.30	0.22
9/11/12	17	0.95	0.26	0.93	0.30
9/12/12	29	0.99	0.06	0.99	0.29
9/13/12	14	0.15	0.64	0.14	0.73
9/14/12	25	0.30	0.11	0.29	0.40
9/15/12	38	0.06	0.08	0.06	0.30
9/16/12	19	0.62	0.09	0.65	0.14
9/17/12	20	0.92	0.07	0.92	0.06
9/18/12	17	0.86	0.08	0.89	0.08
9/19/12	20	0.46	0.39	0.43	0.38
9/20/12	18	0.11	0.26	0.12	0.22
9/21/12	13	0.43	0.99	0.43	0.98
9/22/12	31	0.69	0.17	0.64	0.24
9/23/12	18	0.26	0.48	0.25	0.89
9/24/12	23	0.46	0.48	0.46	0.84
9/25/12	13	0.93	0.95	0.91	0.97
9/26/12	23	0.05	0.06	0.05	0.12
9/27/12	18	0.29	0.56	0.28	0.58
9/28/12	35	0.75	0.26	0.69	0.61
9/29/12	37	0.74	0.37	0.75	0.73
9/30/12	37	0.97	0.17	0.96	0.57
10/1/12	30	0.72	0.18	0.70	0.15
10/2/12	24	0.26	0.22	0.24	0.45
10/3/12	30	0.42	0.27	0.41	0.18
10/4/12	21	0.69	0.33	0.68	0.21
10/5/12	23	0.19	0.62	0.21	0.50
10/6/12	30	0.27	0.61	0.24	0.75
10/7/12	32	0.23	0.11	0.24	0.75
10/8/12	14	0.94	0.40	0.94	0.34
10/9/12	27	0.87	0.23	0.87	0.32
10/10/12	22	0.18	0.75	0.18	0.85
10/11/12	18	0.04	0.22	0.04	0.21
10/12/12	19	0.76	0.16	0.76	0.14
10/13/12	18	0.13	0.41	0.12	0.38
10/14/12	16	0.57	0.62	0.54	0.73
10/15/12	19	0.56	0.90	0.58	0.77
10/16/12	9	0.39	0.30	0.37	0.26
10/17/12	18	0.29	0.13	0.28	0.20
10/18/12	19	0.07	0.27	0.07	0.22
10/19/12	21	0.14	0.35	0.14	0.59
10/20/12	32	0.03	0.85	0.03	0.92
10/21/12	25	0.30	0.29	0.27	0.55
10/22/12	16	0.20	0.31	0.20	0.24
10/23/12	13	0.62	0.60	0.61	0.59
10/24/12	13	0.56	0.86	0.55	0.85
10/25/12	19	0.41	0.72	0.38	0.57
10/26/12	11	0.21	0.38	0.23	0.43
10/27/12	21	0.60	0.73	0.59	0.88
10/28/12	23	0.44	0.01	0.42	0.02
10/29/12	16	0.23	0.76	0.24	0.59
10/30/12	23	0.72	0.37	0.74	0.69
10/31/12	21	0.26	0.05	0.24	0.04
11/1/12	14	0.97	0.48	0.97	0.54
11/2/12	18	0.52	0.92	0.49	0.97
11/3/12	35	0.07	0.34	0.06	0.31
11/4/12	27	0.50	0.18	0.49	0.32
11/5/12	20	0.35	0.46	0.38	0.49
11/6/12	16	0.04	0.87	0.04	0.86
11/7/12	18	0.38	0.76	0.36	0.46
11/8/12	21	0.27	0.85	0.26	0.74
11/9/12	26	0.59	0.84	0.55	0.99
Average	21.8	0.43	0.41	0.42	0.48
# Pass ($\alpha = 0.05$)		$64/70$	$66/70$	$64/70$	$67/70$

Table 34: Case 3 (Combining data over multiple days): Performance of the alternative KS test of a Non-homogeneous Poisson process for hospital ED walk-in arrivals on the time interval $[9, 12]$ in Sept 1, 2012 - Nov 9, 2012.

		Raw				Unrounded			
		$L = 30$		$L = 3$		$L = 30$		$L = 3$	
Day	n	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$
9/1/12-9/10/12	223	0.02	0.01	0.27	0.43	0.02	0.01	0.32	0.39
9/11/12-9/20/12	217	0.18	0.15	0.20	0.50	0.18	0.39	0.24	0.90
9/21/12-9/30/13	248	0.00	0.01	0.34	0.04	0.00	0.23	0.29	0.80
10/1/12-10/10/12	253	0.17	0.00	0.39	0.04	0.18	0.03	0.47	0.05
10/11/12-10/20/12	189	0.02	0.42	0.06	0.77	0.02	0.44	0.05	0.81
109/21/12-10/30/13	180	0.33	0.40	0.61	0.16	0.32	0.50	0.62	0.24
10/31/12-11/9/15	216	0.18	0.01	0.02	0.07	0.18	0.05	0.02	0.11
Average	218	0.13	0.14	0.27	0.29	0.13	0.23	0.29	0.47
# Pass ($\alpha = 0.05$)		$4/7$	$3/7$	$6/7$	$5/7$	$4/7$	$5/7$	$6/7$	$6/7$

6.4 Are Call Center Arrivals Well Modeled by NHPPs?

Having examine what we need to account for in testing whether real data are from NHPPs, we now apply our tests to our call center data to examine whether the arrival process in each day (from 7 am to 10 pm) is from a NHPP. Tables 35 - 42 support that when we treat the call center data appropriately by 'unrounding' and using appropriate subinterval lengths ($L = 1hr$ in this case), we can conclude that the arrival process for each arrival type is from a NHPP. Table 35, due to is huge sample size, provides the weakest result. To reduce the sample size, we further divided the time intervals into 3-hour long subintervals. Tables 43 - 47 provide the results and we see that more tests pass with reduced sample size.

Table 35: VRU-Retail Arrivals at a banking call center in April 2001.

		Raw				Unrounded			
		$L = 24$		$L = 1$		$L = 24$		$L = 1$	
Day	n	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$
1	76048	0.00	0.00	0.32	0.00	0.00	0.00	0.31	0.87
2	182726	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06
3	171595	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	156916	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	161903	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	159132	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	97733	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	68511	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	164794	0.00	0.00	0.06	0.00	0.00	0.00	0.10	0.03
10	165800	0.00	0.00	0.04	0.00	0.00	0.00	0.06	0.13
11	159364	0.00	0.00	0.01	0.00	0.00	0.00	0.02	0.02
12	177961	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.56
13	167795	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
14	95587	0.00	0.00	0.17	0.00	0.00	0.00	0.24	0.01
15	46940	0.00	0.00	0.28	0.00	0.00	0.00	0.34	0.90
16	161402	0.00	0.00	0.02	0.00	0.00	0.00	0.04	0.06
17	160566	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.38
18	154288	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
19	161406	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.14
20	149807	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
21	95954	0.00	0.00	0.01	0.00	0.00	0.00	0.02	0.03
22	59615	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.59
23	148114	0.00	0.00	0.31	0.00	0.00	0.00	0.29	0.39
24	149044	0.00	0.00	0.16	0.00	0.00	0.00	0.24	0.02
25	156386	0.00	0.00	0.02	0.00	0.00	0.00	0.03	0.00
26	156333	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
27	128859	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
28	95439	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
29	64942	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16
30	159624	0.00	0.00	0.22	0.00	0.00	0.00	0.34	0.00
Average	135152.8	0.00	0.00	0.05	0.00	0.00	0.00	0.07	0.15
# Pass ($\alpha = 0.05$)		0/30	0/30	7/30	0/30	0/30	0/30	8/30	11/30

Table 36: VRU-Premier Arrivals at a banking call center in April 2001.

		Raw				Unrounded			
		$L = 24$		$L = 1$		$L = 24$		$L = 1$	
Day	n	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$
1	2169	0.00	0.12	0.21	0.18	0.00	0.49	0.22	0.20
2	3915	0.00	0.00	0.06	0.00	0.00	0.00	0.06	0.59
3	3605	0.00	0.00	0.70	0.00	0.00	0.00	0.67	0.11
4	3076	0.00	0.00	0.05	0.01	0.00	0.00	0.05	0.15
5	2935	0.00	0.00	0.24	0.02	0.00	0.00	0.24	0.81
6	2946	0.00	0.00	0.22	0.00	0.00	0.00	0.24	0.74
7	2124	0.00	0.00	0.58	0.13	0.00	0.00	0.58	0.28
8	1826	0.00	0.01	0.81	0.28	0.00	0.02	0.82	0.84
9	3144	0.00	0.00	0.14	0.00	0.00	0.00	0.14	0.36
10	3160	0.00	0.00	0.86	0.01	0.00	0.00	0.85	0.71
11	3326	0.00	0.00	0.98	0.00	0.00	0.00	0.99	0.66
12	3635	0.00	0.00	0.82	0.00	0.00	0.00	0.82	0.31
13	3610	0.00	0.00	0.51	0.00	0.00	0.00	0.52	0.50
14	2085	0.00	0.00	0.29	0.42	0.00	0.00	0.28	0.65
15	1194	0.03	0.45	0.33	0.25	0.03	0.59	0.33	0.28
16	3624	0.00	0.00	0.33	0.00	0.00	0.01	0.34	0.25
17	3389	0.00	0.00	0.34	0.01	0.00	0.00	0.33	0.20
18	3305	0.00	0.00	0.58	0.00	0.00	0.00	0.58	0.76
19	3140	0.00	0.00	0.69	0.00	0.00	0.00	0.69	0.11
20	3065	0.00	0.00	0.73	0.00	0.00	0.00	0.73	0.29
21	1994	0.00	0.00	0.83	0.05	0.00	0.00	0.84	0.88
22	1540	0.00	0.10	0.44	0.77	0.00	0.26	0.45	0.73
23	3039	0.00	0.00	0.68	0.01	0.00	0.02	0.70	0.99
24	2954	0.00	0.00	0.07	0.01	0.00	0.00	0.08	0.55
25	3141	0.00	0.00	0.46	0.02	0.00	0.00	0.44	0.47
26	3239	0.00	0.00	0.96	0.00	0.00	0.00	0.97	0.29
27	3420	0.00	0.00	0.39	0.00	0.00	0.00	0.37	0.23
28	2211	0.00	0.00	0.38	0.03	0.00	0.00	0.39	0.12
29	1714	0.00	0.09	0.01	0.50	0.00	0.40	0.01	0.80
30	3612	0.00	0.00	0.45	0.00	0.00	0.00	0.45	0.77
Average	2871.2	0.00	0.03	0.47	0.09	0.00	0.06	0.47	0.49
# Pass ($\alpha = 0.05$)		0/30	4/30	28/30	7/30	0/30	4/30	28/30	30/30

Table 37: VRU-Business Arrivals at a banking call center in April 2001.

		Raw				Unrounded			
		$L = 24$		$L = 1$		$L = 24$		$L = 1$	
Day	n	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$
1	1735	0.00	0.06	0.33	0.01	0.00	0.21	0.34	0.01
2	9619	0.00	0.00	0.10	0.00	0.00	0.00	0.10	0.43
3	9217	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.70
4	8736	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	8780	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.69
6	8118	0.00	0.00	0.32	0.00	0.00	0.00	0.34	0.50
7	2730	0.00	0.00	0.98	0.01	0.00	0.00	0.99	0.28
8	1601	0.00	0.21	0.84	0.28	0.00	0.26	0.84	0.29
9	9245	0.00	0.00	0.46	0.00	0.00	0.00	0.48	0.04
10	9177	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.81
11	8919	0.00	0.00	0.22	0.00	0.00	0.00	0.21	0.90
12	9184	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.30
13	7233	0.00	0.00	0.45	0.00	0.00	0.00	0.46	0.63
14	2646	0.00	0.00	0.97	0.00	0.00	0.00	0.97	0.03
15	1397	0.00	0.00	0.24	0.00	0.00	0.00	0.24	0.01
16	8705	0.00	0.00	0.77	0.00	0.00	0.00	0.76	0.24
17	9322	0.00	0.00	0.11	0.00	0.00	0.00	0.12	0.79
18	9082	0.00	0.00	0.13	0.00	0.00	0.00	0.15	0.45
19	8575	0.00	0.00	0.05	0.00	0.00	0.00	0.05	0.63
20	8082	0.00	0.00	0.31	0.00	0.00	0.00	0.33	0.30
21	2679	0.00	0.00	0.55	0.01	0.00	0.00	0.54	0.85
22	1288	0.00	0.06	0.29	0.88	0.00	0.15	0.28	0.93
23	8753	0.00	0.00	0.92	0.00	0.00	0.00	0.95	0.49
24	8795	0.00	0.00	0.35	0.00	0.00	0.00	0.37	0.37
25	8744	0.00	0.00	0.42	0.00	0.00	0.00	0.44	0.28
26	8724	0.00	0.00	0.89	0.00	0.00	0.00	0.91	1.00
27	8599	0.00	0.00	0.23	0.00	0.00	0.00	0.25	0.90
28	2496	0.00	0.00	0.45	0.01	0.00	0.00	0.43	0.02
29	1542	0.00	0.02	0.79	0.67	0.00	0.06	0.80	0.79
30	9803	0.00	0.00	0.49	0.00	0.00	0.00	0.52	0.98
Average	6784.2	0.00	0.01	0.39	0.06	0.00	0.02	0.40	0.49
# Pass ($\alpha = 0.05$)		$0/30$	$3/30$	$24/30$	$3/30$	$0/30$	$4/30$	$25/30$	$24/30$

Table 38: VRU-Customer Loan Arrivals at a banking call center in April 2001.

		Raw				Unrounded			
		$L = 24$		$L = 1$		$L = 24$		$L = 1$	
Day	n	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$
1	273	0.00	0.53	0.94	0.94	0.00	0.54	0.94	0.95
2	679	0.00	0.01	0.35	0.56	0.00	0.01	0.35	0.62
3	546	0.00	0.06	0.95	0.90	0.00	0.06	0.94	0.90
4	511	0.00	0.00	0.01	0.03	0.00	0.00	0.01	0.03
5	497	0.00	0.13	0.90	0.04	0.00	0.16	0.89	0.04
6	495	0.00	0.04	0.23	0.84	0.00	0.04	0.24	0.82
7	323	0.00	0.00	0.88	0.08	0.00	0.00	0.88	0.10
8	206	0.23	0.08	0.88	0.23	0.23	0.08	0.88	0.24
9	705	0.00	0.00	0.15	0.67	0.00	0.00	0.14	0.76
10	550	0.00	0.00	0.01	0.31	0.00	0.00	0.01	0.36
11	463	0.02	0.04	0.06	0.67	0.02	0.04	0.06	0.74
12	442	0.00	0.09	0.43	0.94	0.00	0.08	0.43	0.95
13	429	0.00	0.00	0.99	0.03	0.00	0.00	0.99	0.03
14	201	0.00	0.00	0.06	0.02	0.00	0.00	0.06	0.02
15	114	0.06	0.02	0.15	0.06	0.06	0.02	0.15	0.06
16	667	0.00	0.00	0.08	0.29	0.00	0.00	0.08	0.34
17	568	0.00	0.08	0.57	0.23	0.00	0.08	0.57	0.30
18	501	0.00	0.00	0.04	0.11	0.00	0.00	0.04	0.11
19	480	0.00	0.08	0.67	0.37	0.00	0.07	0.67	0.39
20	483	0.00	0.00	0.78	0.69	0.00	0.00	0.78	0.70
21	259	0.00	0.00	0.84	0.18	0.00	0.00	0.84	0.18
22	157	0.37	0.35	0.29	0.25	0.37	0.35	0.29	0.27
23	500	0.00	0.02	0.02	0.60	0.00	0.02	0.02	0.67
24	506	0.00	0.00	0.13	0.30	0.00	0.00	0.13	0.26
25	501	0.00	0.06	0.59	0.61	0.00	0.07	0.60	0.62
26	448	0.00	0.00	0.20	0.29	0.00	0.00	0.19	0.31
27	375	0.00	0.02	0.90	0.89	0.00	0.02	0.91	0.92
28	222	0.00	0.00	0.12	0.37	0.00	0.00	0.12	0.38
29	188	0.01	0.00	0.41	0.98	0.01	0.00	0.41	0.99
30	548	0.03	0.00	0.09	0.05	0.03	0.00	0.09	0.04
Average	427.9	0.02	0.05	0.42	0.42	0.02	0.06	0.42	0.44
# Pass ($\alpha = 0.05$)		$3/30$	$9/30$	$26/30$	$25/30$	$3/30$	$9/30$	$26/30$	$25/30$

Table 39: VRU-Summit Arrivals at a banking call center in April 2001.

		Raw				Unrounded			
		$L = 24$		$L = 1$		$L = 24$		$L = 1$	
Day	n	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$
1	842	0.00	0.00	0.73	0.66	0.00	0.00	0.73	0.77
2	3961	0.00	0.00	0.55	0.00	0.00	0.00	0.56	0.28
3	3153	0.00	0.00	0.87	0.00	0.00	0.00	0.88	0.00
4	2650	0.00	0.00	0.34	0.00	0.00	0.00	0.36	0.09
5	2510	0.00	0.00	0.43	0.00	0.00	0.00	0.44	0.01
6	2661	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.65
7	1755	0.00	0.00	0.75	0.14	0.00	0.00	0.77	0.70
8	788	0.00	0.00	0.85	0.75	0.00	0.00	0.85	0.82
9	3145	0.00	0.00	0.02	0.00	0.00	0.00	0.02	0.59
10	3701	0.00	0.00	0.22	0.00	0.00	0.00	0.22	0.24
11	4500	0.00	0.00	0.64	0.00	0.00	0.00	0.65	0.55
12	4650	0.00	0.00	0.44	0.00	0.00	0.00	0.46	0.42
13	3977	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.71
14	1897	0.00	0.00	0.02	0.26	0.00	0.00	0.02	0.44
15	570	0.00	0.00	0.84	0.86	0.00	0.00	0.84	0.85
16	6360	0.00	0.00	0.52	0.00	0.00	0.00	0.55	0.52
17	4628	0.00	0.00	0.20	0.00	0.00	0.00	0.19	0.77
18	4403	0.00	0.00	0.46	0.00	0.00	0.00	0.48	0.31
19	3824	0.00	0.00	0.05	0.00	0.00	0.00	0.06	0.21
20	4612	0.00	0.00	0.06	0.00	0.00	0.00	0.06	0.75
21	1960	0.00	0.00	0.07	0.08	0.00	0.00	0.07	0.40
22	911	0.00	0.00	0.48	0.13	0.00	0.00	0.48	0.14
23	4806	0.00	0.00	0.39	0.00	0.00	0.00	0.38	0.95
24	4811	0.00	0.00	0.10	0.00	0.00	0.00	0.11	0.74
25	4306	0.00	0.00	0.33	0.00	0.00	0.00	0.35	0.21
26	4238	0.00	0.00	0.23	0.00	0.00	0.00	0.24	0.09
27	4709	0.00	0.00	0.14	0.00	0.00	0.00	0.15	0.58
28	2588	0.00	0.00	0.05	0.00	0.00	0.00	0.05	0.09
29	1037	0.00	0.00	0.72	0.61	0.00	0.00	0.71	0.59
30	6057	0.00	0.00	0.09	0.00	0.00	0.00	0.10	0.32
Average	3333.7	0.00	0.00	0.35	0.12	0.00	0.00	0.36	0.46
# Pass ($\alpha = 0.05$)		0/30	0/30	26/30	8/30	0/30	0/30	26/30	28/30

Table 40: Direct Group Arrivals at a banking call center in April 2001.

		Raw				Unrounded			
		$L = 24$		$L = 1$		$L = 24$		$L = 1$	
Day	n	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$
1	547	0.00	0.00	0.82	0.24	0.00	0.00	0.83	0.23
2	2652	0.00	0.00	0.02	0.00	0.00	0.00	0.03	0.06
3	1952	0.00	0.00	0.04	0.30	0.00	0.00	0.05	0.78
4	1888	0.00	0.00	0.42	0.06	0.00	0.00	0.41	0.15
5	1764	0.00	0.00	0.44	0.31	0.00	0.00	0.44	0.32
6	1650	0.00	0.00	0.69	0.56	0.00	0.00	0.70	0.86
7	642	0.00	0.00	0.04	0.00	0.00	0.00	0.04	0.00
8	316	0.00	0.00	0.34	0.00	0.00	0.00	0.34	0.00
9	2156	0.00	0.00	0.01	0.06	0.00	0.00	0.01	0.86
10	1844	0.00	0.00	0.53	0.10	0.00	0.00	0.53	0.65
11	1769	0.00	0.00	0.08	0.18	0.00	0.00	0.07	0.23
12	1701	0.00	0.00	0.08	0.43	0.00	0.00	0.08	0.87
13	1561	0.00	0.00	0.93	0.59	0.00	0.00	0.93	0.60
14	972	0.00	0.00	0.03	0.13	0.00	0.00	0.03	0.23
15	284	0.06	0.19	0.15	0.82	0.06	0.18	0.15	0.84
16	2217	0.00	0.00	0.01	0.01	0.00	0.00	0.01	0.05
17	2197	0.00	0.00	0.45	0.02	0.00	0.00	0.47	0.30
18	2050	0.00	0.00	0.24	0.06	0.00	0.00	0.23	0.14
19	1998	0.00	0.00	0.20	0.02	0.00	0.00	0.20	0.06
20	1790	0.00	0.00	0.27	0.33	0.00	0.00	0.28	0.81
21	719	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
22	232	0.00	0.00	0.68	0.01	0.00	0.00	0.68	0.01
23	2177	0.00	0.00	0.27	0.07	0.00	0.00	0.28	0.11
24	1824	0.00	0.00	0.78	0.19	0.00	0.00	0.80	0.79
25	1814	0.00	0.00	0.75	0.40	0.00	0.00	0.73	0.90
26	1712	0.00	0.00	0.24	0.62	0.00	0.00	0.24	0.88
27	1512	0.00	0.00	0.13	0.09	0.00	0.00	0.13	0.06
28	684	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
29	265	0.00	0.01	0.44	0.80	0.00	0.01	0.44	0.79
30	2324	0.00	0.00	0.18	0.06	0.00	0.00	0.19	0.91
Average	1507.1	0.00	0.01	0.31	0.21	0.00	0.01	0.31	0.42
# Pass ($\alpha = 0.05$)		$1/30$	$1/30$	$22/30$	$21/30$	$1/30$	$1/30$	$22/30$	$25/30$

Table 41: Announcement Arrivals at a banking call center in April 2001.

		Raw				Unrounded			
		$L=24$		$L=1$		$L=24$		$L=1$	
Day	n	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$
1	3308	0.00	0.00	0.02	0.00	0.00	0.00	0.02	0.24
2	13555	0.00	0.00	0.04	0.00	0.00	0.00	0.04	0.37
3	12571	0.00	0.00	0.03	0.00	0.00	0.00	0.03	0.91
4	10694	0.00	0.00	0.22	0.00	0.00	0.00	0.22	0.00
5	9672	0.00	0.00	0.48	0.00	0.00	0.00	0.51	0.95
6	9688	0.00	0.00	0.17	0.00	0.00	0.00	0.19	0.92
7	5115	0.00	0.00	0.88	0.00	0.00	0.00	0.86	0.53
8	2632	0.00	0.00	0.19	0.03	0.00	0.00	0.20	0.61
9	12066	0.00	0.00	0.32	0.00	0.00	0.00	0.32	0.89
10	10681	0.00	0.00	0.40	0.00	0.00	0.00	0.43	0.78
11	11303	0.00	0.00	0.96	0.00	0.00	0.00	0.97	0.51
12	12312	0.00	0.00	0.62	0.00	0.00	0.00	0.59	0.11
13	11550	0.00	0.00	0.65	0.00	0.00	0.00	0.69	0.57
14	4688	0.00	0.00	0.25	0.00	0.00	0.00	0.27	0.03
15	6284	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
16	14050	0.00	0.00	0.10	0.00	0.00	0.00	0.11	0.02
17	12049	0.00	0.00	0.17	0.00	0.00	0.00	0.19	0.35
18	12218	0.00	0.00	0.14	0.00	0.00	0.00	0.13	0.43
19	11596	0.00	0.00	0.10	0.00	0.00	0.00	0.11	0.00
20	12581	0.00	0.00	0.12	0.00	0.00	0.00	0.14	0.18
21	4895	0.00	0.00	0.45	0.00	0.00	0.00	0.45	0.71
22	2584	0.00	0.00	0.25	0.04	0.00	0.00	0.24	0.19
23	13847	0.00	0.00	0.42	0.00	0.00	0.00	0.40	0.58
24	11184	0.00	0.00	0.04	0.00	0.00	0.00	0.04	0.69
25	10220	0.00	0.00	0.02	0.00	0.00	0.00	0.02	0.28
26	10344	0.00	0.00	0.94	0.00	0.00	0.00	0.93	1.00
27	15107	0.00	0.00	0.06	0.00	0.00	0.00	0.06	0.01
28	4725	0.00	0.00	0.45	0.00	0.00	0.00	0.46	0.02
29	2624	0.00	0.00	0.91	0.01	0.00	0.00	0.91	0.02
30	15709	0.00	0.00	0.70	0.00	0.00	0.00	0.74	0.61
Average	9661.7	0.00	0.00	0.34	0.00	0.00	0.00	0.34	0.42
# Pass ($\alpha = 0.05$)		$0/30$	$0/30$	$24/30$	$0/30$	$0/30$	$0/30$	$24/30$	$22/30$

Table 42: Message Arrivals at a banking call center in April 2001.

		Raw				Unrounded			
		$L = 24$		$L = 1$		$L = 24$		$L = 1$	
Day	n	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$
1	309	0.01	0.15	0.44	0.66	0.01	0.17	0.44	0.70
2	4095	0.00	0.00	0.02	0.00	0.00	0.00	0.02	0.99
3	3587	0.00	0.00	0.24	0.00	0.00	0.00	0.25	0.08
4	3281	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.06
5	3152	0.00	0.00	0.28	0.00	0.00	0.00	0.27	0.84
6	3191	0.00	0.00	0.45	0.00	0.00	0.00	0.43	0.83
7	619	0.00	0.00	0.46	0.82	0.00	0.00	0.46	0.89
8	287	0.07	0.35	0.34	0.52	0.07	0.32	0.34	0.52
9	4138	0.00	0.00	0.22	0.00	0.00	0.00	0.23	0.57
10	3818	0.00	0.00	0.22	0.00	0.00	0.00	0.24	0.10
11	3488	0.00	0.00	0.28	0.00	0.00	0.00	0.30	0.31
12	3566	0.00	0.00	0.32	0.00	0.00	0.00	0.31	0.29
13	3076	0.00	0.00	0.58	0.00	0.00	0.00	0.60	0.62
14	490	0.00	0.00	0.24	0.66	0.00	0.00	0.24	0.66
15	186	0.01	0.12	0.49	0.18	0.01	0.12	0.48	0.18
16	3734	0.00	0.00	0.26	0.00	0.00	0.00	0.25	0.52
17	3481	0.00	0.00	0.44	0.00	0.00	0.00	0.42	0.91
18	3398	0.00	0.00	0.82	0.00	0.00	0.00	0.83	0.09
19	3267	0.00	0.00	0.31	0.00	0.00	0.00	0.32	0.44
20	2896	0.00	0.00	0.69	0.00	0.00	0.00	0.70	0.08
21	500	0.00	0.00	0.93	0.67	0.00	0.00	0.94	0.73
22	247	0.00	0.02	0.65	0.52	0.00	0.02	0.65	0.52
23	3618	0.00	0.00	0.27	0.00	0.00	0.00	0.27	0.41
24	3372	0.00	0.00	0.94	0.00	0.00	0.00	0.94	0.26
25	3377	0.00	0.00	0.92	0.00	0.00	0.00	0.92	0.67
26	3118	0.00	0.00	0.28	0.00	0.00	0.00	0.29	0.57
27	3603	0.00	0.00	0.06	0.00	0.00	0.00	0.06	0.66
28	1099	0.00	0.00	0.58	0.10	0.00	0.00	0.56	0.15
29	462	0.00	0.00	0.87	0.31	0.00	0.00	0.87	0.32
30	3568	0.00	0.00	0.24	0.00	0.00	0.00	0.25	0.99
Average	2567.4	0.00	0.02	0.46	0.15	0.00	0.02	0.46	0.50
# Pass ($\alpha = 0.05$)		$1/30$	$3/30$	$29/30$	$9/30$	$1/30$	$3/30$	$29/30$	$30/30$

Table 43: VRU-Retail Arrivals a banking call center in [7,10] in April 2001.

		Raw				Unrounded			
		$L = 24$		$L = 1$		$L = 24$		$L = 1$	
Day	n	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$
1	13429	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.73
2	48154	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.43
3	53720	0.00	0.00	0.40	0.00	0.00	0.59	0.39	0.32
4	43487	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	49034	0.00	0.00	0.53	0.00	0.00	0.04	0.53	0.02
6	50533	0.00	0.00	0.27	0.00	0.00	0.02	0.26	0.00
7	32502	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.83
8	9644	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.23
9	36321	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.24
10	41195	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
11	40143	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.47
12	45032	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.49
13	50466	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30
14	26669	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.43
15	9111	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
16	35650	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.13
17	39915	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.84
18	39111	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.73
19	42629	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.99
20	44518	0.00	0.00	0.00	0.00	0.00	0.08	0.00	0.49
21	25128	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11
22	10932	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.49
23	33302	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31
24	38995	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.92
25	38968	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04
26	42780	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.10
27	33252	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
28	28472	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.02
29	11886	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.47
30	37967	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04
Average	35098.2	0.00	0.00	0.04	0.00	0.00	0.03	0.04	0.37
# Pass ($\alpha = 0.05$)		$0/30$	$0/30$	$3/30$	$0/30$	$0/30$	$2/30$	$3/30$	$22/30$

Table 44: VRU-Retail Arrivals a banking call center in [10,13] in April 2001.

		Raw				Unrounded			
		$L = 24$		$L = 1$		$L = 24$		$L = 1$	
Day	n	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$
1	19460	0.00	0.00	0.05	0.00	0.00	0.47	0.06	0.48
2	48615	0.00	0.00	0.01	0.00	0.00	0.78	0.01	0.45
3	42969	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00
4	41063	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
5	40430	0.00	0.00	0.00	0.00	0.00	0.51	0.00	0.11
6	42211	0.00	0.00	0.00	0.00	0.00	0.21	0.00	0.01
7	28496	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.65
8	18114	0.00	0.00	0.02	0.00	0.00	0.01	0.02	0.01
9	44379	0.00	0.00	0.01	0.00	0.00	0.01	0.02	0.00
10	46740	0.00	0.00	0.00	0.00	0.00	0.62	0.00	0.11
11	42674	0.00	0.00	0.00	0.00	0.00	0.27	0.00	0.05
12	46843	0.00	0.00	0.00	0.00	0.00	0.08	0.01	0.20
13	49623	0.00	0.00	0.00	0.00	0.00	0.13	0.00	0.01
14	30022	0.00	0.00	0.00	0.00	0.00	0.27	0.00	0.16
15	11262	0.00	0.00	0.00	0.00	0.00	0.91	0.00	0.52
16	46448	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.02
17	45748	0.00	0.00	0.00	0.00	0.00	0.85	0.00	0.27
18	40713	0.00	0.00	0.00	0.00	0.00	0.23	0.00	0.44
19	42616	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
20	39488	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
21	30334	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.62
22	15773	0.00	0.00	0.08	0.00	0.00	0.11	0.09	0.02
23	41461	0.00	0.00	0.01	0.00	0.00	0.07	0.02	0.02
24	39089	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	41901	0.00	0.00	0.00	0.00	0.00	0.07	0.00	0.00
26	38562	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
27	29933	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
28	28013	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
29	16879	0.00	0.00	0.10	0.00	0.00	0.40	0.11	0.46
30	44337	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00
Average	36473.2	0.00	0.00	0.01	0.00	0.00	0.21	0.01	0.15
# Pass ($\alpha = 0.05$)		0/30	0/30	2/30	0/30	0/30	16/30	3/30	13/30

Table 45: VRU-Retail Arrivals a banking call center in [13,16] in April 2001.

		Raw				Unrounded			
		$L = 24$		$L = 1$		$L = 24$		$L = 1$	
Day	n	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$
1	15980	0.00	0.00	0.00	0.00	0.00	0.96	0.00	0.97
2	39880	0.00	0.00	0.00	0.00	0.00	0.20	0.00	0.09
3	35064	0.00	0.00	0.00	0.00	0.00	0.31	0.00	0.41
4	33622	0.00	0.00	0.00	0.00	0.00	0.13	0.00	0.14
5	33729	0.00	0.00	0.02	0.00	0.00	0.40	0.03	0.24
6	34801	0.00	0.00	0.00	0.00	0.00	0.55	0.00	0.26
7	17243	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30
8	15357	0.00	0.00	0.07	0.00	0.00	0.89	0.08	0.82
9	37851	0.00	0.00	0.00	0.00	0.00	0.32	0.00	0.28
10	35139	0.00	0.00	0.00	0.00	0.00	0.15	0.00	0.03
11	33672	0.00	0.00	0.25	0.00	0.00	0.17	0.30	0.07
12	39443	0.00	0.00	0.00	0.00	0.00	0.99	0.00	0.96
13	33558	0.00	0.00	0.00	0.00	0.00	0.07	0.00	0.01
14	18036	0.00	0.00	0.00	0.00	0.00	0.95	0.00	0.25
15	8152	0.00	0.00	0.00	0.00	0.00	0.84	0.00	0.95
16	36238	0.00	0.00	0.00	0.00	0.00	0.29	0.00	0.06
17	33985	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
18	33130	0.00	0.00	0.03	0.00	0.00	0.53	0.04	0.29
19	33054	0.00	0.00	0.00	0.00	0.00	0.26	0.00	0.14
20	29615	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
21	18451	0.00	0.00	0.00	0.00	0.00	0.67	0.00	0.48
22	11619	0.00	0.00	0.03	0.00	0.00	0.32	0.04	0.59
23	32763	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.00
24	31389	0.00	0.00	0.22	0.00	0.00	0.00	0.25	0.00
25	33067	0.00	0.00	0.63	0.00	0.00	0.00	0.70	0.00
26	32170	0.00	0.00	0.03	0.00	0.00	0.00	0.04	0.00
27	31209	0.00	0.00	0.00	0.00	0.00	0.39	0.01	0.57
28	17500	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.01
29	12715	0.00	0.00	0.16	0.00	0.00	0.34	0.17	0.18
30	34238	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Average	28422.3	0.00	0.00	0.05	0.00	0.00	0.33	0.06	0.27
# Pass ($\alpha = 0.05$)		$0/30$	$0/30$	$5/30$	$0/30$	$0/30$	$21/30$	$5/30$	$20/30$

Table 46: VRU-Retail Arrivals a banking call center in [16,19] in April 2001.

		Raw				Unrounded			
		$L = 24$		$L = 1$		$L = 24$		$L = 1$	
Day	n	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$
1	14405	0.13	0.00	0.01	0.00	0.13	0.45	0.01	0.41
2	27545	0.00	0.00	0.00	0.00	0.00	0.37	0.00	0.88
3	24476	0.00	0.00	0.00	0.00	0.00	0.27	0.00	0.45
4	23440	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.65
5	23444	0.00	0.00	0.00	0.00	0.00	0.15	0.00	0.65
6	21090	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.23
7	11378	0.00	0.00	0.25	0.00	0.00	0.00	0.24	0.00
8	12650	0.00	0.00	0.61	0.00	0.00	0.16	0.60	0.11
9	27301	0.00	0.00	0.00	0.00	0.00	0.33	0.00	0.19
10	25603	0.00	0.00	0.00	0.00	0.00	0.10	0.00	0.28
11	25902	0.00	0.00	0.00	0.00	0.00	0.32	0.00	0.84
12	29586	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.56
13	21879	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.92
14	12721	0.00	0.00	0.00	0.00	0.00	0.49	0.00	0.54
15	8262	0.00	0.00	0.59	0.00	0.00	0.75	0.58	0.90
16	26010	0.00	0.00	0.00	0.00	0.00	0.21	0.00	0.27
17	24891	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.80
18	25500	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17
19	26354	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.37
20	23740	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.82
21	13187	0.00	0.00	0.00	0.00	0.00	0.26	0.00	0.05
22	10477	0.55	0.00	0.67	0.00	0.54	0.17	0.70	0.15
23	24350	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.21
24	23562	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.66
25	25700	0.00	0.00	0.00	0.00	0.00	0.81	0.00	0.01
26	26191	0.00	0.00	0.00	0.00	0.00	0.09	0.00	0.90
27	21825	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.20
28	12947	0.00	0.00	0.00	0.00	0.00	0.61	0.00	0.25
29	11391	0.00	0.00	0.26	0.00	0.00	0.87	0.25	0.90
30	25598	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29
Average	21046.8	0.02	0.00	0.08	0.00	0.02	0.22	0.08	0.45
# Pass ($\alpha = 0.05$)		$2/30$	$0/30$	$5/30$	$0/30$	$2/30$	$17/30$	$5/30$	$27/30$

Table 47: VRU-Retail Arrivals a banking call center in [19,22] in April 2001.

		Raw				Unrounded			
		$L = 24$		$L = 1$		$L = 24$		$L = 1$	
Day	n	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$
1	12774	0.00	0.00	0.00	0.00	0.00	0.83	0.00	0.22
2	18532	0.00	0.00	0.00	0.00	0.00	0.08	0.00	0.23
3	15366	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.06
4	15304	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.41
5	15266	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.32
6	10497	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.44
7	8114	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00
8	12746	0.00	0.00	0.02	0.00	0.00	0.15	0.02	0.47
9	18942	0.00	0.00	0.00	0.00	0.00	0.43	0.00	0.13
10	17123	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.00
11	16973	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
12	17057	0.00	0.00	0.00	0.00	0.00	0.13	0.00	0.82
13	12269	0.00	0.00	0.00	0.00	0.00	0.07	0.00	0.75
14	8139	0.00	0.00	0.02	0.00	0.00	0.64	0.02	0.27
15	10153	0.00	0.00	0.58	0.00	0.00	0.50	0.61	0.69
16	17056	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.50
17	16027	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.74
18	15834	0.00	0.00	0.00	0.00	0.00	0.47	0.00	0.03
19	16753	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.96
20	12446	0.00	0.00	0.00	0.00	0.00	0.20	0.00	0.27
21	8854	0.00	0.00	0.00	0.00	0.00	0.20	0.00	0.37
22	10814	0.00	0.00	0.87	0.00	0.00	0.47	0.89	0.86
23	16238	0.00	0.00	0.00	0.00	0.00	0.60	0.00	0.74
24	16009	0.00	0.00	0.08	0.00	0.00	0.61	0.09	0.22
25	16750	0.00	0.00	0.01	0.00	0.00	0.59	0.01	0.99
26	16630	0.00	0.00	0.00	0.00	0.00	0.65	0.00	0.33
27	12640	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.84
28	8507	0.00	0.00	0.00	0.00	0.00	0.65	0.00	0.09
29	12071	0.00	0.00	0.92	0.00	0.00	0.83	0.91	0.82
30	17484	0.00	0.00	0.00	0.00	0.00	0.15	0.00	0.18
Average	14112.3	0.00	0.00	0.08	0.00	0.00	0.28	0.08	0.43
# Pass ($\alpha = 0.05$)		$0/30$	$0/30$	$4/30$	$0/30$	$0/30$	$21/30$	$4/30$	$27/30$

6.5 Are Hospital Arrivals Well Modeled by NHPPs?

In this section, we apply our test to test whether our hospital ED arrival data in each day are from NHPPs. Similar to the result in §6.4, the results in Tables 48 and 49 show that unrounding the data improves the results much. However, due to the nature of ED arrivals, there is not much variation in hourly arrival rate, and hence the use of shorter subintervals does not help much. We also observe that the sample size is small, which can be concerning because it is much easier to pass the NHPP test with small sample size. Therefore, we also tried merging the arrival data by day of week. Because we consider data from 70 consecutive days, arrival data from 10 days are merged for each day of week. Table 50 provides the results; we observe that even with much bigger sample size, the arrival processes pass the test all the time, and hence we conclude that the hospital ED arrivals in our data are from NHPPs.

Table 48: Walk-In arrivals at a hospital ED in September 1, 2012 - November 9, 2012.

		Raw				Unrounded			
		$L = 24$		$L = 1$		$L = 24$		$L = 1$	
Day	n	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$
9/1/12	115	0.00	0.00	0.11	0.00	0.00	0.01	0.23	0.56
9/2/12	166	0.00	0.00	0.45	0.00	0.00	0.06	0.73	0.68
9/3/12	130	0.01	0.00	0.51	0.00	0.01	0.59	0.45	0.82
9/4/12	107	0.03	0.00	0.62	0.00	0.03	0.34	0.57	0.51
9/5/12	108	0.00	0.00	0.20	0.00	0.00	0.04	0.19	0.36
9/6/12	96	0.00	0.00	0.73	0.00	0.00	0.14	0.84	0.56
9/7/12	104	0.15	0.00	0.52	0.00	0.14	0.06	0.48	0.24
9/8/12	115	0.00	0.00	0.13	0.00	0.00	0.42	0.20	0.52
9/9/12	186	0.02	0.00	0.59	0.00	0.02	0.99	0.53	0.51
9/10/12	127	0.04	0.00	0.87	0.00	0.04	0.72	0.77	0.54
9/11/12	112	0.06	0.00	0.52	0.00	0.06	0.32	0.42	0.90
9/12/12	101	0.02	0.00	0.72	0.00	0.02	0.97	0.82	0.32
9/13/12	112	0.00	0.00	0.63	0.00	0.00	0.33	0.58	0.11
9/14/12	100	0.00	0.00	0.63	0.00	0.00	0.23	0.47	0.77
9/15/12	160	0.00	0.00	0.46	0.00	0.00	0.07	0.29	0.42
9/16/12	163	0.00	0.00	0.56	0.00	0.00	0.18	0.35	0.91
9/17/12	105	0.00	0.00	0.19	0.00	0.00	0.01	0.30	0.12
9/18/12	103	0.00	0.00	0.62	0.00	0.00	0.23	0.53	0.47
9/19/12	115	0.10	0.00	0.37	0.00	0.09	0.50	0.34	0.84
9/20/12	116	0.00	0.00	0.99	0.00	0.00	0.01	0.88	0.51
9/21/12	105	0.10	0.00	0.31	0.00	0.10	0.48	0.43	0.36
9/22/12	102	0.00	0.00	0.52	0.00	0.00	0.10	0.49	0.16
9/23/12	173	0.00	0.00	0.39	0.00	0.00	0.38	0.61	0.37
9/24/12	126	0.00	0.00	0.32	0.00	0.00	0.22	0.44	0.95
9/25/12	115	0.01	0.00	0.19	0.00	0.01	0.40	0.15	0.17
9/26/12	104	0.15	0.00	0.06	0.00	0.15	0.35	0.09	0.14
9/27/12	128	0.01	0.00	0.05	0.00	0.01	0.01	0.03	0.22
9/28/12	101	0.00	0.00	0.12	0.00	0.00	0.06	0.09	0.66
9/29/12	184	0.00	0.00	0.71	0.00	0.00	0.16	0.64	0.11
9/30/12	192	0.00	0.00	0.75	0.00	0.00	0.20	0.94	0.31
10/1/12	166	0.00	0.00	0.60	0.00	0.00	0.01	0.90	0.66
10/2/12	160	0.09	0.00	0.84	0.00	0.09	0.19	0.93	0.13
10/3/12	145	0.00	0.00	0.93	0.00	0.00	0.70	0.82	0.83
10/4/12	140	0.00	0.00	0.34	0.00	0.00	0.01	0.52	0.73
10/5/12	116	0.00	0.00	0.77	0.00	0.00	0.14	0.96	0.29
10/6/12	125	0.00	0.00	0.56	0.00	0.00	0.04	0.50	0.44
10/7/12	148	0.00	0.00	0.82	0.00	0.00	0.60	0.94	0.67
10/8/12	124	0.03	0.00	0.85	0.00	0.03	0.12	0.75	0.81
10/9/12	96	0.00	0.00	0.21	0.00	0.00	0.77	0.18	0.89
10/10/12	120	0.01	0.00	0.17	0.00	0.01	0.29	0.27	0.93
10/11/12	115	0.00	0.00	0.06	0.00	0.00	0.00	0.04	0.31
10/12/12	107	0.01	0.00	0.53	0.00	0.01	0.72	0.42	0.31
10/13/12	124	0.00	0.00	0.09	0.00	0.00	0.04	0.07	0.01
10/14/12	139	0.00	0.00	0.21	0.00	0.00	0.13	0.37	0.88
10/15/12	129	0.07	0.00	0.61	0.00	0.07	0.58	0.60	0.18
10/16/12	103	0.00	0.00	0.02	0.00	0.00	0.62	0.01	0.84
10/17/12	88	0.09	0.00	0.91	0.00	0.09	0.83	0.80	0.39
10/18/12	99	0.01	0.00	0.24	0.00	0.01	0.08	0.16	0.69
10/19/12	100	0.00	0.00	0.37	0.00	0.00	0.03	0.60	0.60
10/20/12	139	0.01	0.00	0.27	0.00	0.01	0.14	0.44	0.93
10/21/12	137	0.00	0.00	0.56	0.00	0.00	0.00	0.70	0.20
10/22/12	117	0.02	0.00	0.68	0.00	0.02	0.71	0.63	0.00
10/23/12	108	0.01	0.00	0.14	0.00	0.01	0.14	0.18	0.41
10/24/12	97	0.07	0.00	0.71	0.00	0.07	0.48	0.59	0.54
10/25/12	107	0.00	0.00	0.11	0.00	0.00	0.52	0.07	0.82
10/26/12	127	0.06	0.00	0.22	0.00	0.05	0.98	0.17	0.92
10/27/12	107	0.00	0.00	0.69	0.00	0.00	0.28	0.54	0.46
10/28/12	144	0.00	0.00	0.47	0.00	0.00	0.02	0.38	0.07
10/29/12	123	0.03	0.00	0.27	0.00	0.03	0.53	0.17	0.83
10/30/12	88	0.01	0.00	0.66	0.00	0.01	0.68	0.87	0.92
10/31/12	106	0.00	0.00	0.09	0.00	0.00	0.01	0.06	0.12
11/1/12	105	0.07	0.00	0.12	0.00	0.06	0.16	0.19	0.18
11/2/12	108	0.03	0.00	0.25	0.00	0.03	0.43	0.22	0.80
11/3/12	127	0.00	0.00	0.34	0.00	0.00	0.13	0.51	0.05
11/4/12	151	0.00	0.00	0.89	0.00	0.00	0.08	0.82	0.49
11/5/12	131	0.00	0.00	0.78	0.00	0.00	0.15	0.87	0.96
11/6/12	93	0.01	0.00	0.17	0.00	0.01	0.54	0.13	0.90
11/7/12	79	0.00	0.00	0.79	0.00	0.00	0.01	0.78	0.53
11/8/12	104	0.06	0.00	0.53	0.00	0.06	0.65	0.73	0.58
11/9/12	121	0.04	0.00	0.32	0.00	0.04	0.19	0.33	0.42
Average	121.9	0.02	0.00	0.46	0.00	0.02	0.30	0.47	0.51
# Pass ($\alpha = 0.05$)		12/70	0/70	69/70	0/70	12/70	55/70	67/70	68/70

Table 49: Ambulance arrivals at a hospital ED in September 1, 2012 - November 9, 2012.

		Raw				Unrounded			
		$L = 24$		$L = 1$		$L = 24$		$L = 1$	
Day	n	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$
9/1/12	20	0.45	0.00	0.92	0.00	0.45	0.68	0.95	0.66
9/2/12	23	0.40	0.00	0.97	0.00	0.39	0.31	0.96	0.40
9/3/12	15	0.10	0.00	0.74	0.00	0.10	0.98	0.70	0.13
9/4/12	12	0.43	0.00	0.46	0.01	0.42	0.42	0.41	0.53
9/5/12	14	0.82	0.00	0.65	0.09	0.82	0.48	0.73	0.89
9/6/12	14	0.16	0.00	0.56	0.00	0.16	0.98	0.60	0.98
9/7/12	15	0.18	0.00	0.97	0.00	0.18	0.76	0.94	0.83
9/8/12	27	0.35	0.00	0.48	0.00	0.35	0.96	0.57	0.64
9/9/12	20	0.66	0.00	0.70	0.00	0.66	0.29	0.75	0.38
9/10/12	11	0.92	0.00	0.22	0.63	0.92	0.18	0.19	0.58
9/11/12	12	0.34	0.00	0.91	0.00	0.34	0.90	0.94	0.67
9/12/12	17	0.04	0.00	0.70	0.00	0.04	0.13	0.69	0.20
9/13/12	19	0.81	0.00	0.71	0.00	0.81	0.20	0.76	0.02
9/14/12	26	0.73	0.00	0.01	0.00	0.73	0.12	0.01	0.40
9/15/12	17	0.88	0.00	0.13	0.00	0.88	0.36	0.14	0.88
9/16/12	21	0.34	0.00	0.38	0.00	0.34	0.82	0.43	0.12
9/17/12	21	0.61	0.00	0.71	0.00	0.61	0.83	0.66	0.55
9/18/12	18	0.10	0.00	0.68	0.00	0.10	0.33	0.79	0.43
9/19/12	16	0.37	0.00	0.65	0.00	0.37	0.56	0.69	0.18
9/20/12	19	0.69	0.00	0.46	0.01	0.68	0.34	0.46	0.28
9/21/12	16	0.17	0.00	0.71	0.00	0.17	0.33	0.62	0.70
9/22/12	14	0.52	0.00	0.70	0.00	0.52	0.35	0.64	0.76
9/23/12	17	0.63	0.00	0.81	0.00	0.63	0.16	0.79	0.79
9/24/12	9	0.75	0.00	0.32	0.00	0.75	0.51	0.33	0.03
9/25/12	14	0.39	0.00	0.24	0.21	0.39	0.71	0.27	0.32
9/26/12	15	0.69	0.00	0.84	0.00	0.68	0.57	0.83	0.73
9/27/12	18	0.89	0.00	0.96	0.00	0.89	0.32	0.94	0.82
9/28/12	12	0.79	0.00	0.84	0.00	0.79	0.41	0.79	0.34
9/29/12	13	0.31	0.00	0.88	0.58	0.31	0.68	0.87	0.49
9/30/12	11	0.56	0.00	0.11	0.00	0.56	0.84	0.09	0.47
10/1/12	18	0.77	0.00	0.56	0.00	0.77	0.85	0.68	0.90
10/2/12	26	0.59	0.00	0.27	0.00	0.60	0.46	0.37	0.25
10/3/12	20	0.05	0.00	0.26	0.00	0.05	0.13	0.23	0.36
10/4/12	16	0.24	0.00	0.71	0.00	0.24	0.60	0.60	0.27
10/5/12	15	0.23	0.00	0.63	0.00	0.23	0.86	0.64	0.56
10/6/12	20	0.66	0.00	0.58	0.00	0.65	0.59	0.67	0.19
10/7/12	10	0.10	0.00	0.06	0.00	0.10	0.57	0.08	0.57
10/8/12	24	0.20	0.00	0.12	0.00	0.20	0.18	0.11	0.68
10/9/12	13	0.24	0.00	0.08	0.01	0.23	0.42	0.06	0.00
10/10/12	22	0.09	0.00	0.71	0.00	0.09	0.88	0.73	0.40
10/11/12	17	0.27	0.00	0.57	0.12	0.27	0.44	0.65	0.14
10/12/12	14	0.20	0.00	0.41	0.00	0.19	0.53	0.45	0.40
10/13/12	28	0.88	0.00	0.41	0.00	0.88	0.61	0.40	0.90
10/14/12	16	0.18	0.00	0.54	0.00	0.18	0.54	0.58	0.76
10/15/12	19	0.20	0.00	0.56	0.00	0.20	0.15	0.60	0.98
10/16/12	14	0.43	0.00	0.58	0.00	0.43	1.00	0.68	0.23
10/17/12	18	0.30	0.00	0.64	0.00	0.31	0.14	0.61	0.29
10/18/12	21	0.30	0.00	0.15	0.00	0.30	0.16	0.20	0.90
10/19/12	8	0.14	0.00	0.65	0.00	0.14	0.36	0.66	0.78
10/20/12	22	0.04	0.00	0.14	0.00	0.04	0.05	0.10	0.60
10/21/12	16	0.95	0.00	0.21	0.00	0.95	0.53	0.18	0.08
10/22/12	19	0.22	0.00	0.29	0.00	0.22	0.03	0.35	0.87
10/23/12	10	0.10	0.00	0.90	0.00	0.09	0.90	0.87	0.31
10/24/12	12	0.00	0.00	0.30	0.00	0.00	0.09	0.30	0.38
10/25/12	9	0.53	0.00	0.47	0.00	0.53	0.41	0.53	0.62
10/26/12	17	0.53	0.00	0.87	0.00	0.52	0.32	0.92	0.69
10/27/12	13	0.46	0.00	0.07	0.00	0.46	0.66	0.09	0.04
10/28/12	14	0.81	0.00	0.30	0.00	0.81	0.25	0.34	0.55
10/29/12	14	0.24	0.00	0.14	0.00	0.24	0.38	0.12	0.61
10/30/12	14	0.04	0.00	0.91	0.28	0.04	0.39	0.87	0.32
10/31/12	20	0.73	0.00	0.19	0.00	0.73	0.04	0.25	0.41
11/1/12	21	0.10	0.00	0.00	0.00	0.10	0.56	0.01	0.75
11/2/12	16	0.37	0.00	0.30	0.00	0.37	0.63	0.37	0.90
11/3/12	18	0.28	0.00	0.00	0.00	0.28	0.12	0.00	0.02
11/4/12	12	0.53	0.00	0.30	0.00	0.53	0.78	0.33	0.71
11/5/12	12	0.62	0.00	0.03	0.19	0.63	0.69	0.03	0.16
11/6/12	19	0.33	0.00	0.48	0.00	0.33	0.79	0.43	0.16
11/7/12	17	0.17	0.00	0.77	0.00	0.17	0.85	0.84	0.31
11/8/12	15	0.89	0.00	0.52	0.00	0.89	0.34	0.46	0.24
11/9/12	18	0.25	0.00	0.41	0.00	0.24	0.94	0.33	0.34
Average	16.6	0.42	0.00	0.49	0.03	0.42	0.50	0.50	0.48
# Pass ($\alpha = 0.05$)		66/70	0/70	66/70	7/70	66/70	67/70	66/70	65/70

Table 50: Arrivals at a a hospital ED in September 1, 2012 - November 9, 2012. Arrivals in each day of week merged.

			Raw				Unrounded			
			$L = 24$		$L = 1$		$L = 24$		$L = 1$	
Type	Day of Week	n	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$	CU	$Lewis$
Walk-In	Mon	1599	0.00	0.00	0.34	0.00	0.00	0.00	0.97	0.62
	Tues	1278	0.00	0.00	0.32	0.00	0.00	0.00	0.92	0.63
	Wed	1085	0.00	0.00	0.15	0.00	0.00	0.04	0.13	0.94
	Thurs	1063	0.00	0.00	0.58	0.00	0.00	0.02	0.68	0.36
	Fri	1122	0.00	0.00	0.03	0.00	0.00	0.00	0.03	0.54
	Sat	968	0.00	0.00	0.10	0.00	0.00	0.00	0.07	0.95
	Sun	1298	0.00	0.00	0.26	0.00	0.00	0.00	0.90	0.93
	Average	1201.9	0.00	0.00	0.25	0.00	0.00	0.01	0.53	0.71
	# Pass ($\alpha = 0.05$)		0/7	0/7	6/7	0/7	0/7	0/7	6/7	7/7
Ambulance	Mon	160	0.94	0.00	0.16	0.00	0.94	0.34	0.34	0.24
	Tues	162	0.08	0.00	0.19	0.00	0.08	0.69	0.16	0.88
	Wed	152	0.01	0.00	0.85	0.00	0.01	0.93	0.95	0.32
	Thurs	171	0.00	0.00	0.61	0.00	0.00	0.22	0.50	0.34
	Fri	169	0.03	0.00	0.00	0.00	0.03	0.71	0.01	0.28
	Sat	139	0.07	0.00	0.78	0.00	0.07	0.34	0.69	0.75
	Sun	192	0.15	0.00	0.35	0.00	0.15	0.46	0.48	0.08
	Average	163.6	0.18	0.00	0.42	0.00	0.18	0.53	0.45	0.41
	# Pass ($\alpha = 0.05$)		4/7	0/7	6/7	0/7	4/7	7/7	6/7	7/7

References

Brown, L., N. Gans, A. Mandelbaum, A. Sakov, H. Shen, S. Zeltyn, L. Zhao. 2005. Statistical analysis of a telephone call center: a queueing-science perspective. *J. Amer. Stat. Assoc.* **100** 36–50.

Brown, L., L. H. Zhao. 2002. A test for the Poisson distribution. *Sankhya: The Indian Journal of Statistics* **64** 611–625.

D'Agostino, Ralph B, Michael A Stephens. 1986. *Goodness-of-fit Technicques*, vol. 68. CRC press.

Durbin, J. 1961. Some methods for constructing exact tests. *Biometrika* **48**(1) 41–55.

Kathirgamatamby, N. 1953. Note on the Poisson index of dispersion. *Biometrika* **40**(1) 225–228.

Kim, S.-H., W. Whitt. 2013a. Statistical analysis with Little's law. *Operations Research* **61**(4) 1030–1045.

Kim, S.-H., W. Whitt. 2013b. Statistical analysis with Little's law supplementary material: more on call center data. Columbia University, http://www.columbia.edu/∼ww2040/allpapers.html.

Kim, S.-H., W. Whitt. 2014. Choosing arrival process models for service systems: Tests of a nonhomogeneous Poisson process. *Naval Research Logistics,* Available at: http://www.columbia.edu/∼ww2040/allpapers.html.

Lewis, P. A. W. 1965. Some results on tests for Poisson processes. *Biometrika* **52**(1) 67–77.

Mandelbaum, A. 2012. Service Engineering of Stochastic Networks web page: http://iew3.technion.ac.il/serveng/.

Stephens, M. A. 1974. Edf statistics for goodness of fit and some comparisons. *Journal of the American Statistical Association* **69**(374) 730–737.