REAL-TIME DELAY ESTIMATION BASED ON DELAY HISTORY SUPPLEMENTARY MATERIAL

by

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Abstract

Motivated by interest in making delay announcements to arriving customers who must wait in call centers and related service systems, we study the performance of alternative real-time delay estimators for the delay before entering service of an arriving customer based on recent customer delay experience. We characterize performance by the bias and the mean squared error (MSE) We do analysis and conduct simulations for the standard GI/M/s multi-server queueing model, emphasizing the case of large s. The main estimators considered are: (i) the delay of the last customer to enter service (LES), (ii) the delay experienced so far by the customer at the head of the line (HOL), and (iii) the delay experienced by the customer to have arrived most recently among those who have already completed service (RCS). We compare these delay-history estimators to the estimator based on the queue length (QL), which requires knowledge of the mean interval between successive service completions in addition to the queue length. We obtain analytical results for the conditional distribution of the delay given the observed HOL delay. An approximation for the mean value of that conditional distribution, which is not the observed delay, serves as a refined estimator, yielding lower MSE than the direct estimator. We show that the MSE relative to the square of the mean is asymptotically negligible for all three candidate delay estimators in the many-server and classical heavy-traffic limiting regimes.

Keywords: delay estimation, real-time delay estimation, delay prediction, delay announcements, many-server queues, call centers, heavy traffic.

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1. Introduction

This is a supplement to the main paper, Ibrahim and Whitt (2006), with the same title. Here, we report the results of simulation experiments supporting some of the theoretical results of the main paper. But, we also go beyond that and use simulation to gain insight into the performance of candidate delay estimators for which we have not obtained closed mathematical expressions.

The Main Paper. In the main paper, we present and analyze several real-time delay estimators based on recent customer delay history, in the standard GI/M/s model. In the framework of the GI/M/s model, we were able to obtain tractable expressions for some of our candidate delay estimators. Our research is motivated by applications to call centers but could be applied to virtually any service system where customers are subject to delays prior to receiving service, especially when they are unable to approximate the waiting time themselves (e.g., service systems with invisible queues).

We propose seven different delay estimators. A detailed description of these estimators can be found in section 2 of the main paper. In a nutshell, these estimators can be grouped into three categories. The first category holds the two useful reference estimators, QL and NI. QL (Full-Information Queue-Length Delay Estimator) is expected to perform better than all other partial-information delay estimators. NI (No-Information Steady-State Estimator) is expected to perform worse than the other estimators since they all assume some state information beyond the model. Any estimator performing consistently worse than NI is not worth serious consideration.

The second category holds LES and HOL which both announce, as estimates, delays of customers who haven't yet completed service. We therefore put them in the same category. HOL (Head-Of-The-Line Estimator) can be used as an approximation to LES (Last Customer to Enter Service). The reader interested in the validity of this approximation is referred to section 4 of the main paper. Here, we report simulation results that support this approximation.

The third category holds the LCS, RCS and RCS $-c\sqrt{s}$ estimators. These estimators all announce delays of customers who have already completed service. That is why we put them in the same category. LCS (Last Customer to Complete Service), RCS (Most Recent Arrival to Complete Service) and RCS $-c\sqrt{s}$ (Most Recent Arrival Among the Last $c\sqrt{s}$ Customers to Complete Service) are hard to analyze mathematically. Their analysis remains incomplete at

this point; simulation thus plays a key role in studying the performance of these alternative estimators.

Organization of this Supplement. We start in section 2 by describing our simulation experiments. In section 3, we report simulation results comparing the performances of our delay estimators in the GI/M/s model for three different interarrival time distributions. We measure the performance of a delay estimator by computing the corresponding average squared *error* (ASE). In section 4, we go beyond measuring the overall performance of a given estimator and report average square errors conditional on the level of actual delay observed. By actual delay, we mean the measured delay of a customer who has been given a delay estimate upon arrival. The computation of these conditional errors is detailed in that section. There, we see interesting results that weren't captured in the overall comparisons of section 3. In section 5, we focus on the efficiency of the LES estimator. In particular, we report simulation results supporting the approximation for the MSE of the LES estimator, which can be found in section 4 of the main paper. In section 6, we report simulation results that support another theoretical result of the main paper: the relative efficiency of HOL as compared to QL. This result can be found in section 4 of the main paper. In section 7, we study the effect of using the delay history of customers who have completed service. That is, we vary the amount of information used and make observations. We consider a family of delay estimators RCS - f(s) for different functions f of the number of servers s and compare these estimators' efficiencies. Our aim here is to show that, consistent with heavy-traffic analysis, it is enough to use the delay history of the last $c\sqrt{s}$ customers, for some constant c which we determine by simulation. In section 8, we look more carefully at the distribution of our delay estimators, studying in particular the distribution of $W_{HOL}(w)$ and testing approximations in the main paper. In section 9, we draw conclusions. In section 10, we produce all tables and figures relevant to the sections of this supplement.

2. Description of the Simulation Experiments

In this section we describe our simulation experiments. Our adopted model in the main paper is the standard GI/M/s queueing model with s homogeneous servers working in parallel, an unlimited waiting room and the first-come first-served service discipline. We thus let the service times V_n be independent and identically distributed (i.i.d.) exponential random variables with mean E[V] = 1. Fixing the mean service time as such is done without loss of generality since the service rate can be made equal to 1 by a proper choice of time units. We let the interarrival times U_n be i.i.d. positive random variables with a non-lattice cumulative distribution function (cdf) F. We measure the variability of a distribution by computing its squared coefficient of variation (SCV) defined as $c_a^2 = Var[U]/E[U]^2$.

In this supplement to the main paper, we use simulation to validate some of the theoretical results of the main paper. We also use simulation to gain insight into the performance of candidate delay estimators that are hard to analyze mathematically, even within the framework of the relatively simple GI/M/s model. In all of our experiments, we measure the performance of a delay estimator by computing the *average squared error* (ASE), which is defined by:

$$ASE \equiv \frac{1}{n} \sum_{i=1}^{n} (a_i - p_i)^2,$$
 (2.1) {ASEEq}

where a_i is the actual delay observed, p_i is the estimated delay announced according to the given scheme and n is the number of customers that were given a delay announcement. Throughout, we only give delay announcements to customers that had to wait in line for a positive amount of time before receiving service. In other words, we will always have $a_i > 0$ in (2.1) above. For each delayed customer i, we record the delay estimate p_i , announced according to a certain delay estimator and the actual delay observed a_i . We then average the square of the differences between these two over all customers who were given a delay estimation. The ASE computed as such approximates the *mean squared error* (MSE) of the estimators especially when the samples are sufficiently large. This motivates our using simulation experiments to further validate the closed form expressions we obtained for the MSE of some delay estimators in the main paper.

We now describe our simulation experiments. Our simulations are *steady-state discrete*event simulations since the performance measures we aim to calculate are steady-state performance measures. Therefore, we could potentially face two problems: (1) *Initial Bias* and (2) *Autocorrelations*. The initial bias is the difference between the expected value of the statistical

estimator, the ASE, and the quantity it is estimating, the MSE. This happens because the system is not started in steady-state. A partial solution to this problem is to delete some initial segment of the data, i.e., to have a warmup period which we later discard. To assess the impact of the initial bias in our simulations, we simulated the M/M/900, $H_2/M/900$ and D/M/900 queuing models and varied the length of the warmup period reporting, in each case, the computed point estimates of the ASEs for the alternative delay estimators. We define the length of a simulation run in terms of number of events. We define an event to be either an arrival or a service completion. That is, a simulation run length of 5 million events is terminated when the sum of the number of arrivals and the number of service completions in that run is equal to 5 million. To assess the importance of the initial bias, we simulated the three models above with $\rho = 0.95$. We chose a large number of servers, s, and a large traffic intensity, ρ , because it is known that the initial bias effect is likely to be more important for large s and high ρ . We simulated our models four times with: (1) 5×10^6 events and no warmup period, (2) 5×10^6 events and a warmup period of 5×10^5 events, (3) 5×10^6 events and a warmup period of 1×10^6 events and (4) 5×10^6 events and a warmup period of 2×10^6 events. We compared the point estimates of the ASEs obtained in each case and found that these estimates do not differ by a significant amount. Indeed, not having an initial warmup period caused a bias of at most 8 percent, reported for the $H_2/M/900$ model. Since we didn't detect significant initial transient effect, our simulation replications in this supplement are done without deleting an initial portion of the data, i.e., with no warmup period.

Autocorrelations are the correlations between the successive values of the process. The effect of autocorrelations can be reduced by using the method of independent replications. We use the method of independent replications to generate point and confidence interval estimates for the ASE of an estimator in a given model (for a chosen ρ). We simulate each model (for each ρ) for 10 independent replications. The length of these runs depends on the particular model and the traffic intensity at hand. In general, when the traffic intensity ρ is high (but still, $\rho < 1$), there tend to be large fluctuations and thus high variability in which case the simulation run need be longer. Also, simulation runs need be longer as the variability in the arrival process increases, especially for larger values of s. We determine whether simulation run lengths are roughly appropriate by comparing our simulation estimates to exact analytical values we have for the model at hand. For example, in our simulations for the M/M/s queue, we compared our point estimates for the ASEs of QL and NIE to the well-known exact values of the MSEs of these two estimators. As a result, we varied our simulation run lengths from 5

million events when both ρ and s are small (For example, $\rho = 0.85$, s = 10) to 7, 10 and 15 million as s, ρ and the variability in the arrival process increase (for example, we considered a length of 15 million for each replication of the $H_2/M/900$ model with $\rho = 0.98$). For more on the estimation of simulation run lengths in queueing simulations, see Whitt (1989). In each replication k, we return the value of

$$\overline{ASE_k} = \frac{1}{n_k} \sum_{i=1}^{n_k} (a_i - p_i)^2 , \qquad (2.2) \quad \{\texttt{est1}\}$$

where n_k is the size of our sample in replication k (k = 1, 2, ..., 10). We then average $\overline{ASE_k}$ over all 10 replications :

$$\widehat{ASE} = \sum_{k=1}^{10} \overline{ASE_k}$$
(2.3) {est2}

Everywhere in this supplement, we report \widehat{ASE} as our point estimate to the ASE of the given estimator in that particular model (for a chosen ρ). The corresponding interval estimate for \widehat{ASE} is constructed as follows: given our sample { $\overline{ASE_k}$, k = 1, 2, ..., 10}, we compute the sample variance:

$$s^{2} = \frac{1}{9} \sum_{k=1}^{10} (\overline{ASE_{k}} - \widehat{ASE})^{2}$$
 (2.4) {est3}

Our interval estimate is then given by:

$$\widehat{ASE} \pm \frac{s}{\sqrt{10}} t_{\alpha/2,9} , \qquad (2.5) \quad \{\texttt{ci}\}$$

where α is the chosen level of significance and 9 is the number of degrees of freedom for the t-statistic (we assume normality of the data). In our tables for the ASEs, we report 95 percent confidence intervals for our point estimates, which we constructed as explained above.

In our simulations, we vary three different parameters: (1) the number of servers s, (2) the traffic intensity $\rho = E[V]/sE[U] = 1/sE[U]$ and (3) the variability of the arrival process which we measure by the SCV of the interarrival time distribution. We are particularly interested in the case of large s since we are motivated by applying this research to call centers, but we will include smaller values of s as well. In particular, we will always consider the values: s = 1, 10, 100, 400 and 900. Since delay announcements are more relevant when customer delays are long, we will mostly be focusing on high traffic intensities in our simulations. In general, as we increase the number of servers, we need increase the traffic intensity further in order to see large customer delays. For example, in the extreme case of s = 900, we only consider traffic intensities $\rho \ge 0.93$. To study the effect of variability in the arrival process, we

vary the interarrival time distribution F to consider both low variability and high variability distributions. In particular, we consider Deterministic, Poisson and H_2 arrivals with respective SCVs 0, 1 and 4. We consider these three distributions in all of our simulations.

The simulation program was written in C. We used our C code to create Excel add-ins via XLL Plus and generated output in Excel.

3. Efficiency of the Alternative Estimators in GI/M/s Models

{secUncondASE

In the main paper, we presented analytical results comparing the efficiency of some alternative delay estimators in GI/M/s models. These alternative delay estimators, excepting the No-Information Steady-State estimator (NI), are random variables defined by recent customer delay experience in the model at hand. Here, we complement that analysis by presenting the corresponding empirical results. In this section, we use simulation to compare the performances of our alternative delay estimators in GI/M/s models. We study the effect of variability in the arrival process by considering Poisson, Deterministic and H_2 arrivals with respective squared coefficient of variation (SCV) 1, 0 and 4. We vary the number of servers s and the load in the system, which we measure by the traffic intensity $\rho = E[V]/sE[U] = 1/sE[U]$ under our assumption that E[V] = 1. We are interested in studying the differences between these alternative performances. In the main paper, we made a distinction between *direct* and *refined* delay estimators. Specifically, we did this for the Last Customer to Enter Service (LES) and Head-Of-The-Line (HOL) delay estimators. The observed delay is the direct estimator, while the mean of the conditional distribution of the delay to be estimated, given the observed delay, is the refined estimator based on that same observation. For the LES and HOL delay estimators, we found that the mean values of these conditional distributions are approximately equal to ρw , when the observed delay is w, provided that w is not too small. However, since we are mostly interested in heavily loaded systems where ρ is close to 1, we expect that there won't be substantial differences in performances when refining our predictors as such. We thus only consider here the *direct* LES and HOL delay estimators.

We quantify the performance of a delay estimator by computing point and interval estimates of the *average squared error* (ASE). The ASE is defined by

$$ASE \equiv \frac{1}{n} \sum_{i=1}^{n} (a_i - p_i)^2 ,$$
 (3.1) {ASEEqq}

where a_i is the actual delay of the i^{th} customer, p_i is the estimated delay of that customer (the value of the delay estimator at hand) and n is the number of customers in our sample. For large samples, the ASE should agree with the *mean squared error* (MSE) in steady state. Note that we only give delay announcements to delayed customers, i.e., customers that have to wait in line prior to receiving service. Thus, we will always have $a_i > 0$ for every customer *i*. It is possible, however, to have $p_i = 0$ for some customer *i*. For example, consider the LES delay announcement. Suppose that customer i_0 joins the queue at some time *t* and that the last customer to have entered service prior to t could be served immediately upon her arrival. Then, $p_{i_0} = 0$ and $a_{i_0} > 0$ for our customer i_0 . As the number of such customers in our sample increases, we expect the performance of our delay estimator (in this case, LES) to deteriorate. Then, we might want to adjust our delay estimator in such a way so as to avoid this discrepancy around 0. We do not deal with this issue here, however, because we are interested in heavily loaded systems where the proportion of delayed customers is high. What's more, we want to study the performances of our alternative delay estimators specifically when the actual (alternatively, announced) delays reported exceed a certain threshold. We do this in §4 by computing the conditional ASEs of the alternative delay estimators: for a given delay estimator, we compute its ASE as in (3.1) but we restrict our sample to the pairs (a_i, p_i) where a_i exceeds our threshold. That is, in §4, we will study the performances of our delay estimators conditional ASE where we include *all* the pairs (a_i, p_i) in (3.1). We generate point and confidence interval estimates for the ASEs. Let \widehat{ASE} be the point estimate of the ASE that we calculate.

In addition to the ASE, we compute point estimates of the *relative ASE*(RASE). We let \widehat{RASE} be the point estimate of the RASE that we calculate. Then:

$$\widehat{RASE} \equiv \widehat{ASE} / (E[W_{\infty}|W_{\infty} > 0])^2$$
(3.2) {RASE}

where $[W_{\infty}|W_{\infty} > 0]$ is a random variable denoting the steady-state waiting time given that the wait is positive and $E[W_{\infty}|W_{\infty} > 0]$ is our computed point estimate of the mean of $[W_{\infty}|W_{\infty} > 0]$. That is, to compute point estimates for the RASE, we divide our point estimate for the ASE, \widehat{ASE} , by the square of our point estimate for the mean, $(E[W_{\infty}|W_{\infty} > 0])^2$. For large samples, the RASE should agree with the *relative mean squared error* (RMSE) in steady state. The RMSE, approximated by the RASE, is appealing because of its simple form in some cases. In particular, the RMSE of the Full-Information Queue Length Delay Estimator (QL) has a linear form in the M/M/s model and the RMSE of the No-Information Steady-State estimator (NI) is equal to 1 in the GI/M/s model. The simple form of the RMSE makes it appealing to use. That is why we choose to adopt it. Plotting the RMSE as a function of the traffic intensity ρ produces nice linear plots which are easy to analyze. For example, see Figures 12 and 22 below.

An alternative approach, which we do not consider here, would be to compute point esti-

mates for the *relative root average squared error* (RRASE). If \widehat{RRASE} is a point estimate of the RRASE then:

$$\widehat{RRASE} \equiv \sqrt{\widehat{ASE}/E[W_{\infty}|W_{\infty}} > 0]$$
(3.3) {why not?}

That is, a point estimate for the RRASE is given by dividing the square root of our point estimate for the ASE, \sqrt{ASE} , by our point estimate for the mean, $E[W_{\infty}|W_{\infty} > 0]$. For large samples, the RRASE should agree with the root relative mean squared error (RRMSE) in steady state. Approximating the RRMSE (by estimating the RRASE) would be useful because the MSE is in the same scale as the square of the mean, thus \sqrt{MSE} is in the same scale as the mean.

We can analyze the M/M/s model and get closed form expressions for the ASEs of the QL and NI delay estimators. In the M/M/s model, it is well known that $[W_{\infty}|W_{\infty} > 0]$ is distributed as an exponential random variable with mean $\frac{1}{s\mu(1-\rho)} = \frac{1}{s(1-\rho)}$ when $\mu = 1$ (e.g., see section 5.14 of Cooper (1981)). For the NI estimator, the MSE (thus, the ASE) should coincide with:

$$MSE(NI) = var[W_{\infty}|W_{\infty} > 0] = \frac{1}{s^2(1-\rho)^2}$$
(3.4) {MSEni}

We can also analyze the performance of the QL estimator. Let $[Q_{\infty}|Q_{\infty} > 0]$ be a random variable with the conditional distribution of the steady-state queue length upon arrival given that the customer must wait before beginning service. In the M/M/s model, it is known that $[Q_{\infty}|Q_{\infty} > 0] + 1$ has a geometric distribution with mean $1/(1 - \rho)$. Hence,

$$E[MSE(QL)] = E[Var(W_Q(Q_\infty))] = E[Q_\infty + 1] \times \frac{1}{s^2} = \frac{1}{s^2(1-\rho)}$$
(3.5) {MSEql}

We expect the MSEs (hence, the ASEs) of the other estimators to fall between that of the QL estimator (best possible) and the NI (worst possible). We thus always include these two estimators as reference points for the performances of all the other estimators. In addition to this, we have closed form expressions for the RMSEs of the QL and NI delay estimators. It follows directly from the above that:

$$RMSE(QL) = E[MSE(QL)]/(E[W_{\infty}|W_{\infty} > 0])^{2} = 1 - \rho , \qquad (3.6) \quad \{\text{RMSEq1}\}$$

and,

$$RMSE(NI) = MSE(NI)/(E[W_{\infty}|W_{\infty} > 0])^2 = 1$$
 (3.7) {RMSEni}

Note here that the RMSE of the QL delay estimator is linear in ρ .

We are now ready to present our simulation results. In Tables 1-10, we display our point estimates for the ASEs and RASEs of the alternative delay estimators in the M/M/s model for s = 1, 10, 100, 400 and 900. In Tables 11-20 and 21-30, we report these point estimates for the D/M/s and $H_2/M/s$ models respectively, for s = 1, 10, 100, 400 and 900. We vary the traffic intensity ρ , but we focus more on heavy loads: with those loads, customer delays are longer and giving delay predictions is more meaningful. In addition to the tables, we produce figures plotting the ASE and the RASE of our alternative delay estimators as a function of the traffic intensity in the model. In Figures 1-10, we show plots for the ASE and RASE of our delay estimators as a function of the traffic intensity ρ (for a fixed number of servers s) in the M/M/s model with s = 1, 10, 100, 400 and 900. In Figures 11 - 20 and 21 - 30, we do the same for the D/M/s and $H_2/M/s$ models respectively.

For the M/M/s, D/M/s and $H_2/M/s$ models considered, i.e., for all s and all ρ , we can classify the performance of our estimators as follows:

$$QL > LES \approx HOL > RCS \approx RCS - \sqrt{s} > LCS > NI$$

where ">" is to be read as "performs better than" and " \approx " is to be read as "performs nearly the same but slightly better than". This order of performances holds for all models here, in the range of traffic intensities considered.

We first consider the performances of our reference estimators: QL and NI. As expected, QL always takes the lead among all estimators and NI always falls behind. In the M/M/smodel, we see that ASE(QL), ASE(NI), RASE(QL) and RASE(NI) agree closely with the theoretical values given in (3.5), (3.4), (3.6) and (3.7). Consequently, in the M/M/s model, the ratio ASE(NI)/ASE(QL) agrees closely with $1/(1 - \rho)$, as expected. Note that this ratio increases as ρ increases. This explains the deterioration in the performance of NI as the load increases. Throughout, our point estimate for RASE(NI) \approx 1, except for the $H_2/M/900$ model under heavy loading ($\rho = 0.95$ or $\rho = 0.98$) where we see that RASE(NI) ≈ 0.9 , which means that the run length evidently is not long enough. We will see in §4 that NI's performance is good compared to that of the other delay estimators when we restrict our attention to small delays only. This performance deteriorates as we consider longer delays.

We see that LES and HOL outperform the rest of the estimators (except QL). Their

performances are always very close. Also,

$$ASE(HOL)/ASE(QL) \approx ASE(LES)/ASE(QL) \approx (c_a^2 + 1)/\rho \tag{3.8} \quad \{\texttt{approx1}\}$$

with the approximation becoming more accurate as the observed delay increase, especially for large s. This coincides with theoretical results of section 4 of the main paper, where the approximation holds as $sw \to \infty$ where w is the observed delay. We will discuss this at length in section 7 of this supplement. We note here that this explains why the performances of LES, HOL and QL are so close in the D/M/s model. In this model, $c_a^2 = 0$ and the approximation becomes: $ASE(HOL)/ASE(QL) \approx ASE(LES)/ASE(QL) \approx 1/\rho \downarrow 1$ as $\rho \uparrow 1$.

The analysis in sections 4 and 5 of the main paper leads to the approximation:

$$E[MSE(LES)] \approx E[MSE(HOL)] \approx \frac{1}{s^2} \left[(c_a^2 + 1)^2 + \frac{((2\rho - 1)c_a^2 + 4\rho - 3)(c_a^2 + 1)}{2(1 - \rho)} + K \right]$$
(3.9) {approx2}

where $K = 1.5c_a^4 + 4c_a^2 + 4.5 - (2/3)v_a^3$ and $v_a^3 = E[U^3]/(E[U])^3$.

Our simulation results are roughly consistent with this approximation, which is the most accurate in the M/M/s model. We leave a detailed study of the accuracy of this approximation to section 4 of this supplement.

We now consider the three delay estimators based on the delay information of customers who have completed service, namely: LCS, RCS and $RCS - \sqrt{s}$. We will study the difference in performances between RCS and $RCS - \sqrt{s}$ and the effect of delay information at length in section 8 of this supplement. For s = 1, these three estimators perfectly coincide. This is so because with exactly one server, the order of customer departure from the system is the same as that of customer arrival to the system. Hence, the last customer that has completed service is the most recent arrival among all customers that have completed service. On the other hand, $\sqrt{s} = 1$ implies that $RCS - \sqrt{s}$ and LCS are the same. Even when the number of servers is small enough (e.g., s = 10) we see that the performances of LCS, RCS and $RCS - \sqrt{s}$ are still close, see for example Tables 5, 13 and 23 for the respective results of the M/M/10, D/M/10 and $H_2/M/10$ models. But, as the number of servers increases, LCS clearly falls behind; see for example Tables 9, 19 and 29 for the M/M/900, D/M/900 and $H_2/M/900$ models. We emphasize here that customers need not depart in the order of their arrival. Indeed, with exponential service times, each of the s servers is equally likely to generate the next service completion. Thus, it is possible that the last customer to have completed service has experienced his waiting time long ago. We therefore expect the delay information

of the most recent arrival among those customers that have completed service to be more relevant to our current customer (the system changes less over shorter periods of time). Also, when the number of servers is small we see that the performances of LES and LCS are quite close (see for example Tables 1, 11 and 21). The difference between the two becomes more pronounced as the number of servers increases and is dramatic when s = 400 or s = 900. This is so because, with exponential service times, the times between consecutive departures from service are i.i.d exponential with mean 1/s. Then as the number of servers increases, the times between consecutive departures are significantly smaller than the consecutive service times (recall our assumption of mean service time E[V] = 1). That is, the last customer to have completed service may have experienced his waiting time much before the last customer to enter service. We have thus explained the poor performance of LCS compared to LES, which is more pronounced as s increases.

4. Efficiency of the Estimators Conditional on the Level of Delay Observed

{secCondASE}

In the main paper, we presented analytical results comparing the efficiency of some alternative delay estimators in GI/M/s models. These alternative delay estimators, excepting the No-Information Steady-State estimator (NI), are random variables defined by recent customer delay experience in the model at hand. In section 3, we used simulation to compare the performances of our alternative delay estimators in GI/M/s models. There, we quantified the performance of a delay estimator by computing point estimates and confidence intervals of the *average square error* (ASE). The ASE is defined by:

$$ASE \equiv \frac{1}{n} \sum_{i=1}^{n} (a_i - p_i)^2 ,$$
 (4.1) {ASE}

where a_i is the actual delay of the *i*th customer, p_i is the estimated delay of that customer (the value of the delay estimator at hand) and n is the number of customers in our sample. For large samples, the ASE should agree with the mean squared error (MSE) in steady state. Here, we go beyond measuring the overall performance of a given estimator and report average square errors conditional on the level of actual delay observed (a_i in equation (6.3)). We study the performances of our alternative delay estimators specifically when the actual customer delays reported are in pre-specified intervals. We do this here by computing the conditional ASEs of the alternative delay estimators: for a given delay estimator, we compute its ASE as in (6.3) but we restrict our sample to the pairs (a_i, p_i) where a_i is in a pre-specified interval. That is, we study the performances of our delay estimators conditional on the level of the waiting time. We generate point and confidence interval estimates for these conditional ASEs.

Announcing delay estimates is more relevant when the observed delays in the system are long. It's not clear what one should consider to be a long delay. Here, we compare the actual delays experienced by waiting customers to our point estimate of the mean waiting time in the system, conditional on the wait being positive. Let $E[\widehat{W}|\widehat{W}>0]$ be this point estimate. Here, [W|W>0] is a random variable with the distribution of the steady state waiting time conditional on the wait being positive. We describe an actual delay a_i as long if $a_i > E[\widehat{W}|\widehat{W}>0]$. Furthermore, we study different levels of long actual delays, grouping them into 4 intervals: $(E[\widehat{W}|\widehat{W}>0], 2E[\widehat{W}|\widehat{W}>0]), (2E[\widehat{W}|\widehat{W}>0], 4E[\widehat{W}|\widehat{W}>0]),$ $(4E[\widehat{W}|\widehat{W}>0], 6E[\widehat{W}|\widehat{W}>0])$ and $> 6E[\widehat{W}|\widehat{W}>0]$. We compute conditional ASEs for each of the different intervals. For example, the conditional ASEs of the alternative estimators for the first interval, $(E[\widehat{W}|\widehat{W}>0], 2E[\widehat{W}|\widehat{W}>0])$, are computed as follows: in (6.3), we include only pairs (a_i, p_i) where $a_i \in (E[\widehat{W|W} > 0], 2E[\widehat{W|W} > 0])$ and we do this for all delay estimators returning a conditional ASE for each, in that given interval. We only consider in this section models where the number of servers is large, which we choose to be $s \ge 100$. We do this here because we are more interested in the case of large number of servers, which better captures the real-life settings we are interested in, such as medium to large sized call centers. We vary the traffic intensities considered and compute interval and point estimates of the conditional ASEs in each case. In certain cases, particularly for large s, we found that our sample of actual delays collected was not large enough to infer significant statistical estimates. In that case, we simply did not display the results we obtained (e.g., see the D/M/900 results below).

In section 3, we classified the performances of our delay estimators as follows:

$$QL > LES \approx HOL > RCS \approx RCS - \sqrt{s} > LCS > NI$$

where " > " is to be read as "performs better than" and " \approx " is to be read as "performs nearly the same but slightly better than". This order of performances held for all models considered. in the range of traffic intensities considered. In this section, we see interesting results that were not captured in the overall comparisons of section 3. We report our results for the M/M/smodel in Tables 31-41, those for the D/M/s model in Tables 42-50 and those for the $H_2/M/s$ model in Tables 51-61. Each set of tables just mentioned is followed by the corresponding figures plotting the corresponding conditional ASEs. The first observation we make is about the performance of the No-Information Steady-State estimator (NI) when studying conditional ASEs as opposed to the unconditional ASEs of the previous section. NI performs sometimes better than all other delay estimators (except the Full-Information Queue Length Estimator – QL), when the delays considered fall in the $(E[\widehat{W|W} > 0], 2E[\widehat{W|W} > 0])$ interval and the traffic intensities considered are low (e.g., $\rho = 0.9$ in the M/M/100 queue). This result is interesting in that it shows that it is not always necessary to use more state information in order to give better delay estimates for current waiting customers. As we consider intervals for longer delays, however, the performance of NI deteriorates (see, for example, Figure 1 for the M/M/100 model).

In general, when we consider ρ large enough, and restrict our attention to long actual customer delays observed, the classification of our delay estimators is similar to that of section 3. We once more see that LES and HOL's performances are very similar. We also see that RCS and $RCS - \sqrt{s}$ are very close, with RCS performing slightly better. LCS and NI fall behind constantly, except in the case mentioned above. Finally, QL performs always best among all

of our delay estimators.

In a nutshell, to draw conclusions on sections 3 and 4, we can say that given enough information about the system, the QL delay estimator is the most efficient among the estimators considered. Closely behind fall LES and HOL. In third place, we put RCS and $RCS - \sqrt{s}$. Finally, LCS and NI perform the worse with NI falling behind LCS. But, our aim in this supplement is not to determine which delay estimator to use in practice. In fact, the answer to this question lies beyond the scope of this work as it involves other parameters such as the costs involved and the preferences of the system managers. Also, as we just saw, the performances of our delay estimators depend on the length of the delays observed and a clear-cut answer is not always possible. What we do here is merely quantify the performances of alternative delay estimators that we find to be possible candidates for delay estimations in real-life systems. We study these performances by averaging over all delays observed in section 3 and then look more closely at these performances given the level of delays observed in this section.

5. Simulation Experiments for the Efficiency of the LES Delay Estimator

{secLESEff}

In this section we present simulation experiments that test the accuracy of approximations for the mean squared error of the LES delay estimator. Note that we will use HOL and LES interchangeably since we use one to approximate the other. Simulation has shown that their performances are indeed nearly identical in all cases. When the interarrival times in our model have a non-lattice cumulative distribution function, we use theorem 4.2 of the main paper and test the following approximation:

$$E[MSE(LES)] \approx (1-\rho)^2 w^2 + \frac{(2\rho-1)c_a^2 + 4\rho - 3)w}{s} + \frac{K}{s^2}$$
(5.1) {MSE_LES}

where K is given by:

$$K = \frac{3c_a^4}{2} + 4c_a^2 + \frac{9}{2} - \frac{2v_a^3}{3}$$
(5.2) {Eqn_K}

In (5.1), w denotes the delay estimate given under the LES delay estimation scheme. We replace w in what follows by our simulation point estimate for E[W|W > 0] and w^2 by our simulation point estimate for $E[W^2|W > 0]$ where [W|W > 0] denotes a random variable with the distribution of the steady state waiting time conditional on the wait being positive. The reader interested in the derivation of this approximation is referred to section 4 of the main paper. This approximation is useful because it allows us to roughly quantify the performance of the LES (and hence, HOL) delay estimator, in the GI/M/s queueing model, when the general renewal arrival process has non-lattice interrenewal-time distribution. The approximation should work best when $sw \to \infty$.

For deterministic interarrival times (which have a lattice distribution), we propose testing the following approximation:

$$E[MSE(LES)] \approx (1-\rho)^2 w^2 + \frac{\rho w + 2/s}{s}$$
 (5.3) {MSE_LES_2}

We now explain the derivation of this approximation. Suppose that we have deterministic interarrival times. Let $A = \{A(t) : t \ge 0\}$ be the renewal counting process associated with the deterministic interarrival times. Then, ignoring the error term, we can write: $A(w) \approx \rho s w$. Now, from equation (4.3) in the main paper, we get that:

$$var(W_{HOL}(w)) = (\rho sw + 2) \times \frac{1}{s^2}$$
 (5.4) {oui1}

Further, noting that $E[W_{HOL}(w)] = \frac{E[A(w)+2]}{s} = \frac{\rho s w + 2}{s}$:

$$MSE(LES) = E[(W_{HOL}(w) - w)^2] = E[((W_{HOL}(w) - \frac{\rho sw + 2}{s}) + (\frac{\rho sw + 2}{s} - w))^2] \quad (5.5) \quad \{\text{oui2}\}$$

yielding approximately:

$$E[MSE(LES)] \approx (1-\rho)^2 w^2 + \frac{\rho w + 2/s}{s}$$
 (5.6) {MSE_LES_2}

The LES delay estimator is the delay (before starting service) of the last customer to have entered service, prior to our customer's arrival. This estimator is appealing because it is relatively easy to obtain and interpret. Its implementation does not require knowledge of the queueing model's parameters (number of servers, arrival process and service distributions ... etc).

We propose testing the above approximations while varying two parameters: the number of servers, s, and the traffic intensity in the system, ρ . We are particularly interested in the case of large s since we are motivated by applying this research to call centers, but we will include smaller values of s as well. We expect the approximation to be more accurate when the number of servers is large and/or the delays experienced by waiting customers are long. In particular, we consider the values: s = 1, 10, 100, 400 and 900. We test approximation (5.6) with exponential and H_2 ($c_a^2 = 4$) interarrival time distributions. We test approximation (5.1) with deterministic interarrival time distribution. The simulation results reported throughout are usually based on 10 independent replications of about 5 million events each, where each event is either an arrival or a service completion. We do make the runs longer when the traffic intensity is higher, or when the variability in the arrival process is high such as with H_2 arrivals (7 - 10 million events). Our selection criterion for the simulation run length is based solely on our observations when performing the simulations. When we detect high variability in the simulation estimates across runs for given s and ρ , we increase the simulation run length so as to reduce this variability.

We report our results for the M/M/s model in Tables 62 - 22, those of the D/M/s model in Tables 67 - 71 and those of the $H_2/M/s$ model in Tables 72 - 76. In all tables, we consider s = 1, 10, 100, 400 and 900. We also vary the traffic intensity $\rho = \frac{E[V]}{sE[U]}$ and report results for each value considered. In each case, we construct 95 percent confidence intervals for the estimators and report these intervals. To capture the accuracy of the approximation, we compute the relative percent difference (RPD) between ASE(LES) and the corresponding numerical approximation. We define the RPD as:

$$RPD \equiv \frac{ASE(LES) - approx}{approx} \times 100$$
(5.7) {RelDiff}

where approx denotes the corresponding numerical approximation. We report values for all RPDs in the tables below. Studying the reported RPD values allows us to assess the accuracy of the proposed approximation. Studying the values of the reported RPDs, we immediately see that approximation (5.1) works well in the M/M/s and $H_2/M/s$ queues. In these two models, we see that the absolute values of the RPDs reported are always less than 5 percent (except for the M/M/900 case with $\rho = 0.99$ where the $RPD \approx 7$ percent). On the other hand, approximation (5.6) performs well in the D/M/s queue, except when the loads are light and s is small (e.g., $s = 10, \rho = 0.85$). Otherwise, the RPDs reported in the D/M/s model are consistently close to 5 percent, sometimes reaching a remarkably low value (e.g., see the D/M/400 model when $\rho = 0.99$. In general, both approximations work best when the number of servers is large and the value of ρ is high.

6. Relative Efficiency of HOL Compared to QL

In this section we reproduce our simulation results for the HOL and QL delay estimators, but with a different aim in mind. Here, we compare the efficiency of the QL and HOL delay estimators and test the validity of the approximation (which can be found on p.14 of the main paper):

$$\frac{c_{W_{HOL,s}(w)}^2}{c_{W_{O,s}(n)}^2} \approx \frac{c_a^2 + 1}{\rho}$$
(6.1) {Approx}

which holds when sw and n are large. This approximation compares the efficiency of the QL and *refined* HOL delay estimators. The refined HOL delay estimator (which we denote here by $HOL_{refined}$) is an announcement of ρw where w is the elapsed delay of the customer who is at the head of the line when our customer arrives to the system and joins the line of waiting customers.

We stop first on approximation (6.1). The analysis leading to this approximation can be found on pages 12 - 14 of the main paper. *s* denotes the number of servers in our model. c_a^2 is the squared coefficient of variation (SCV) of the arrival process. ρ is the traffic intensity in the system. $W_{HOL,s}(w)$ is a random variable with the conditional distribution of the waiting time of a new arrival given that the new arrival must wait some positive amount of time, that there already is at least one customer in queue and that the customer at the head of the line has already spent time *w* in queue. $W_{Q,s}(n)$ is a random variable with the conditional distribution of the delay of a new arriving customer, given that the arriving customer must wait before starting service and that the queue length at the time of arrival is *n*. Consider the QL and refined HOL estimators. Then, the mean square errors (MSEs) for these estimators coincide with the variances $var(W_{Q,s}(n))$ and $var(W_{HOL,s}(w))$, respectively. Further, the relative mean square errors (RMSE $\equiv MSE/Mean^2$) for these estimators coincide with the respective SCVs. In this section, we test the validity of approximation (6.1) by noting that:

$$\frac{MSE(HOL_{refined})}{MSE(QL)} \approx \frac{RMSE(HOL_{refined})}{RMSE(QL)} = \frac{c_{W_{HOL,s}(w)}^2}{c_{W_{O,s}(n)}^2} \approx \frac{c_a^2 + 1}{\rho}$$
(6.2) {leapOfFaith

To approximate the MSEs of the two candidate delay estimators, we use simulation to compute the corresponding average square errors (ASEs). For large samples, the ASE should agree with the MSE in steady state. Recall that the ASE of an estimator is given by:

$$ASE \equiv \frac{1}{n} \sum_{i=1}^{n} (a_i - p_i)^2, \tag{6.3} \quad \{\text{ASE}\}$$

where a_i is the actual delay observed and p_i is the predicted (estimated) delay. Under heavy loads, $\rho w \uparrow w$ as $\rho \uparrow 1$. There should thus be little difference between the direct and refined HOL delay estimators, in the heavily loaded systems that we consider. We therefore consider the direct HOL delay estimator in this section (announcement of w instead of ρw) and expect that approximation (6.1) above be valid for this direct HOL estimator as well.

We now present our simulation experiments. In each case, we construct 95 percent confidence intervals for the ASEs of our estimators. For a description of our simulation experiments in this supplement, please refer to section 2 of this supplement. We report our results for the M/M/s model in Tables 77 - 81. We report our results for the D/M/s model in Tables 82 - 86. We report our results for the $H_2/M/s$ model in Tables 87 - 91. We consider s = 1, 10, 100, 400and 900. We also vary the traffic intensity $\rho = \frac{E[V]}{sE[U]}$ and report results for each value considered. The half widths of the confidence intervals reported range from less than 1 percent (high loads, small number of servers) to close to 10 percent (lighter loads, large number of servers as when s = 900 and $\rho = 0.93$). This is so because under lighter loads, especially when the number of servers is large, there is less data because fewer customers have to wait before beginning service. Also included in these tables are the approximation values $\frac{c_a^2+1}{\rho}$. To capture the accuracy of the approximation, we compute the relative percent difference (RPD) between ASE(HOL)/ASE(QL) and the corresponding numerical approximation. We define the RPD as:

$$RPD \equiv \frac{ASE(HOL)/ASE(QL) - approx}{approx} \times 100$$
(6.4) {RelDiff

}

where *approx* denotes the corresponding numerical approximation. We report values for all RPDs in the tables below. Studying the reported RPD values allows us to assess the accuracy of the proposed approximation. We see that approximation (6.1) is most accurate in the M/M/s model where the RPD is consistently lower than 2 percent. It performs worse in the D/M/s model where the reported RPDs range from about 2 percent to about 20 percent. Finally, the approximation performs worst in the $H_2/M/s$ model where the reported RPDs range from 3 percent to over 30 percent. In all cases however, we note that our approximation is indeed a useful one, particularly when the traffic intensity considered in the system is high.

7. The Effect of Delay Information: $\mathbf{RCS} - f(s)$

{secRCSEffect

In this section, we consider a family of delay estimators, RCS-f(s), for functions $f(s) = s, 4\sqrt{s}, 2\sqrt{s}, \sqrt{s}$ and log(s). RCS-f(s) announces the delay of the most recent arrival among the last f(s) customers to have completed service. We use simulation to study the differences between these delay estimators. We wish here to quantify the impact of using past delay information: we expect that increasing the amount of information used would lead to an improvement in the performance of the estimator. This improvement of performance is quantified via a decrease in the value of the average squared error reported. However, using more information implies more data processing which could prove to be costly. We report here simulation results showing that using the delay information of all customers who have completed service is not necessary. Indeed, from heavy-traffic analysis, we deduce that the most recent arrival time of a customer that has completed service is very likely to occur among the last $c\sqrt{s}$ customers when s is large. Here, we use simulation to find that value of c, which turns out to be equal to 4.

Under some circumstances, the LCS (Last Customer to Complete Service) and LES (Last Customer to Enter Service) delay estimators will be similar, but they actually can be very different when the number of servers s is large. To see why, assume that s is large; let X denote an exponential random variable with mean E[X] = 1/s and V be a generic service time random variable, i.e., an exponential random variable with mean E[V] = 1. Further, assume that all servers are busy upon arrival of our customer, at time t, and that she found at least one other customer waiting in line ahead of her when she arrived. These assumptions are not unreasonable in the heavily loaded systems that we consider. The last customer to have completed service prior to t must have experienced her delay before time t - V. The last customer to have entered service prior to t must have experienced her delay at most X time units ago. This is so because the random times between successive service completions in the GI/M/s model are exponential random variables with mean 1/s. When s is large, this means that the last customer to complete service may have experienced her delay much before the last customer to enter service. This explains why LCS and LES can be different, especially when s is large, since more recent information is more relevant to the current state of the system.

We are thus lead to propose other candidate delay estimators based on the delay experience of customers that have already completed service. The first is the delay experienced by the customer that arrived most recently (and thus entered service most recently, under our First In First Out service discipline), among those customers who have already completed service (RCS). A disadvantage of the RCS estimator is that we must analyze a lot of data, going arbitrarily far back in the past. From heavy-traffic analysis, we deduce that the most recent arrival time of a customer that has completed service is very likely to occur among the last $c\sqrt{s}$ customers when s is large. Simulation shows that the corresponding value of c is 4. We find the same value of c for all values of ρ , s and interarrival time distributions considered. The reader interested in this heavy-traffic analysis is referred to Section 7 of the main paper (pages 24-25). So, we introduce another estimator which requires less information processing: RCS- $c\sqrt{s}$ is the delay of the customer to have arrived most recently among the last $c\sqrt{s}$ customers who have already completed service.

In the tables below, we report the average square errors (ASE) for each considered RCS-f(s) delay estimator, along with corresponding 95 percent confidence intervals. These confidence intervals and point estimates are based on 10 independent simulation runs of length 1 million events each. Our simulation run length in this section is shorter than in other sections of the supplement because our aim here is determine the value of c; this value of c is not expected to change when we increase the simulation run length. We only consider values of $s \ge 100$ since we don't expect to see substantial differences between our estimators when the number of servers is small. In our tables, we include in parentheses next to the reported ASE point estimate, the value of relative percent difference (RPD). We define the RPD as:

$$RPD \equiv \frac{ASE(RCS - f(s)) - ASE(RCS)}{ASE(RCS)} \times 100$$
(7.1) {RelDiff}

Studying the values of the reported RPDs allows us to assess the impact of the amount of delay information used. The smaller the RPD, the closer the performance of our estimator is to the best possible estimator here, RCS. The following observations are true throughout this section. We see that the performance of RCS- $2\sqrt{s}$ is very close to that of RCS; the RPDs reported are less than 1 percent for all models considered. RCS- \sqrt{s} falls slightly behind but is still very similar to RCS. The RPDs reported in this case are consistently less than 10 percent. Finally, RCS-log(s) performs the worse, nearly as bad as LCS. That is consistent with our analysis, since $log(s) \leq 3$ for all values of s considered. The amount of delay information processed in this case is actually too little.

We report our results for the M/M/s model in Tables 92 - 94, those for the $H_2/M/s$ model in Tables 95 - 97 and those for the D/M/s model in Tables 98 - 100.

8. Distributions of The Delay Estimators

In this section, we look more carefully at the distribution of actual delays observed given that delay estimates announced are in a small interval about a certain value which we choose. Our first aim is to test approximation (4.13) of the main paper. This approximation states that:

$$W_{HOL}(w) \approx N(\rho w, \rho w (c_a^2 + 1)/s)$$
(8.1) {normapprox}

Recall that $W_{HOL}(w)$ denotes a random variable with the conditional distribution of the waiting time (before starting service) of a new arrival given that the new arrival must join the queue, given that there is already at least one customer in queue, and given that the customer at the head of the line has already spent time w in queue. That is, w is the delay estimate given according to the HOL delay estimation scheme. We expect this approximation to hold as $sw \to \infty$. Thus, in order to test it, we only consider models with a large number of servers (here, we consider s = 100), large w (here, we consider $w \approx 2E[W|W > 0]$) and high ρ (here, $\rho = 0.95$). Recall that [W|W > 0] denotes a random variable with the conditional distribution of the steady state waiting time given that the wait is positive.

We proceed as follows: we simulate the model under consideration and collect the actual delays observed when the delay estimate given according to the HOL scheme, w, is approximately equal to 2E[W|W > 0]. By approximately equal we mean that w should be in a small interval about 2E[W|W > 0] (or $2E[\widehat{W|W} > 0]$, our simulation point estimate for this quantity). We choose this small interval so as to have a large enough sample size for the data thus collected. We then plot a histogram for the actual delays collected. This histogram describes the distribution of $W_{HOL}(w)$ for the chosen w in the given model, thus enabling us to test (8.1) above. In Figures 62, 67 and 72, we display histogram plots for the M/M/100, D/M/100 and $H_2/M/100$ models, respectively. To better assess how well the Normal distribution fits the data distribution, we plot a Normal curve with the same mean and variance on top of our histograms. We see in these figures that the distribution of $W_{HOL}(w)$ is indeed very close to being a Normal distribution. In the M/M/100 model, with $\rho = 0.95$, we can compute E[W|W > 0]analytically and get and exact value of: E[W|W > 0] = 0.2. Our simulation point estimate for $E[W_{HOL}(w)]$ based on our sample of actual delays observed is $E[\widehat{W_{HOL}(w)}] \approx 0.4003$, which is slightly larger than $\rho w \approx \rho \times 2E[W|W > 0] \approx 0.38$. Our simulation point estimate for the variance is $var[W_{HOL}(w)] \approx 0.008$ while the variance $var[W_{HOL}(w)]$ in (8.1) yields 0.0076 in this case. Note that approximations (4.7) and (4.9) of the main paper for the mean and variance of $W_{HOL}(w)$ yield respectively 0.4 and 0.0078 which are closer to our simulation values. For completeness, we restate these approximations:

$$sE[W_{HOL,s}(w)] - \rho sw \to \frac{(c_a^2 + 3)}{2}$$
, (8.2) {a4}

and,

$$s^{2}Var(W_{HOL,s}(w)) - \rho sw(c_{a}^{2} + 1) \rightarrow \left(\frac{5(c_{a}^{2} + 1)^{2}}{4} - \frac{2\nu_{a}^{3}}{3} + 1\right)$$
(8.3) {a4c}

Hence, we see that the simulation estimates reported agree quite closely with the above approximation. In the D/M/100 model with $\rho = 0.95$, our simulation point estimate for E[W|W > 0] is given by: $E[\widehat{W|W} > 0] \approx 0.1$. From our simulations we see that: $E[\widehat{W}_{HOL}(w)] \approx$ 0.1996 whereas $\rho w \approx \rho \times 2E[\widehat{W|W} > 0] \approx 0.19$. On the other hand, $var[\widehat{W}_{HOL}(w)] \approx 0.002$ from simulations, and is given by 0.0019 in (8.1). Once more, we see that the simulation estimates reported agree quite closely with the above approximation. Note that approximations (4.7) and (4.9) of the main paper for the mean and variance of $W_{HOL}(w)$ yield respectively 0.205 and 0.0021 which are closer to our simulation values.

Finally, in the $H_2/M/100$ model, we see from simulations that $E[\widehat{W|W} > 0] \approx 0.48$ when $\rho = 0.95$. This yields from (8.1) that $E[W_{HOL}(w)]$ should be approximately equal to $\rho w \approx 0.912$. From our simulations, we see that $\widehat{E[W_{HOL}(w)]} \approx 0.9625$. On the other hand, (8.1) yields $var[W_{HOL}(w)] \approx 0.0456$ and our simulation point estimate is $var[\widehat{W}_{HOL}(w)] \approx 0.0448$. Note that approximations (4.7) and (4.9) of the main paper for the mean and variance of $W_{HOL}(w)$ yield respectively 0.947 and 0.0448 which are closer to our simulation values (remarkably so for the variance).

Thus, we see that our approximation is indeed a valid one in the three models considered above. Based on our simulation results, we see that using the following alternative approximation yields sightly better results:

$$W_{HOL}(w) \approx N(\rho w + \frac{c_a^2 + 3}{2s}, \rho w \frac{c_a^2 + 1}{s} + \frac{5(c_a^2 + 1)^2}{4s^2} - \frac{2\nu_a^3}{3s^2} + \frac{1}{s^2})$$
(8.4) {normapprox2

In this section, we also go beyond studying the distribution of $W_{HOL}(w)$ and include histograms for $W_{LES}(w)$ and $W_{RCS}(w)$. Recall that $W_{LES}(w)$ denotes a random variable with the conditional distribution of the waiting time (before starting service) of a new arrival given that the new arrival must join the queue, given that the last customer to have entered service prior to the new arrival had delay w. Similarly, $W_{RCS}(w)$ denotes a random variable with the conditional distribution of the waiting time (before starting service) of a new arrival given that the new arrival must join the queue, given that the most recent arrival among all customers that have completed service had delay w. Once more, we restrict our w to being approximately equal to 2E[W|W > 0]. We show the corresponding histograms in Figures 65-66 for the M/M/100 model, 69-70 for the D/M/100 model and 75-76 for the $H_2/M/100$ model. To plot the histogram in the LES case for example, we collect the actual delays observed when the delay estimate given according to the LES delay estimation scheme is approximately equal to 2E[W|W > 0] (or our simulation point estimate for this quantity, $2E[\widehat{W|W} > 0]$). We then plot a histogram for the actual delays thus collected, thus plotting the distribution of $W_{LES}(w)$. We do the same for $W_{RCS}(w)$. We see that our simulation point estimates for the mean and variance of $W_{LES}(w)$ and $W_{RCS}(w)$ are quite close to those of $W_{HOL}(w)$ (see for example Figures 65 and 66), and we once more see an approximately Normal distribution.

Finally, we also plot histograms for the distributions of the LES and RCS delay estimations given when the delay estimate given according to the HOL delay estimation scheme, w, is approximately equal to 2E[W|W > 0]. We see throughout that the variance of the LES and RCS estimations is considerably smaller than that of $W_{HOL}(w)$ (e.g., see Figure 63). In the M/M/100 and $H_2/M/100$ models (e.g., see Figures 64 and 74), we see that the LES and RCS predictions given are normally distributed. In the D/M/100 model (see Figure 68), the distribution of the LES estimations thus collected is not Normal. At present, we did not investigate the shape of the distribution observed in this case.

9. Conclusions

In this supplement to the main paper we presented simulation results investigating the performances of 6 delay estimators: (i) the delay of the last customer to enter service (LES), (ii) the delay experienced so far by the customer at the head of the line (HOL), (iii) the delay experienced by the last customer to complete service (LCS), (iv) the delay experienced by the customer to have arrived most recently among those who have completed service (RCS), (v) the delay of our customer in the GI/M/s model assuming full information at the arrival epoch (QL) and (vi) the steady state mean waiting time, assuming no information beyond the model (NI). We concluded that LES and HOL are very similar, with both being more accurate than others. For large s, RCS is far superior to LCS, because customers need not complete service in the same order they arrive. QL outperforms all other delay estimators but has the disadvantage of assuming full knowledge about the model at the arrival epoch. NI almost always falls behind, except when we restrict our attention to low traffic intensities and relatively short delays. We saw that simulation provided additional support to some theoretical results of the main paper thus further validating them. These results relate mostly to the performance of the LES delay estimator (or equivalently, HOL) in the GI/M/s model. We also used simulation to gain insight into the performances of our delay estimators when we didn't have corresponding theoretical results. We did this, for instance, when studying the performances of the RCS and LCS delay estimators. We went beyond studying the overall performances of our estimators and studied differences in performances when restricting attention to specific delay levels in the system. We then saw that these performances can indeed depend on the level of delays considered.

It is important to note finally that the simulation results of this supplement merely aim at quantifying the differences between the performances of the alternative delay estimators that we proposed. In practice, there are a number of issues that we do not consider here, such as the amount of information available to the system manager and the costs involved in implementing the different delay estimation schemes. The question of which delay estimator to use in a real-life problem cannot be answered without incorporating all of these factors.

10. Tables and Figures

0	ASE(QL)	ASE(LES)	ASE(HOL)	ASE(RCS)	$ASE(RCS - \sqrt{s})$	ASE(LCS)	ASE(NI)
0.95	20.09	42.24	42.35	44.09	44.09	44.09	405.0
	± 0.417	± 0.766	± 0.785	± 0.776	± 0.776	± 0.776	± 23.4
).93	14.36	30.56	30.72	32.36	32.36	32.36	207.5
	± 0.186	± 0.371	± 0.385	± 0.373	± 0.373	± 0.373	± 10.4
.0	9.989	21.76	21.96	23.46	23.46	23.46	100.6
	± 0.084	± 0.186	± 0.206	± 0.192	± 0.192	± 0.192	± 3.44
.85	6.678	15.07	15.38	16.63	16.63	16.63	44.94
	± 0.043	± 0.093	± 0.0951	± 0.010	± 0.010	± 0.010	± 0.883

Table 1: A comparison of the efficiency of different real-time delay estimators for the $M/M/s$ queue with $s = 1$ and $\mu = 1$ as a function of
the traffic intensity ρ . We report point estimates for the average squared error – (ASE). Each estimate is shown with the half width of the
95 percent confidence interval.

	RASE(QL)	RASE(LES)	RASE(HOL)	RASE(RCS)	$RASE(RCS - \sqrt{s})$	RASE(LCS)	RASE(NI)
.95	0.05003	0.1052	0.1054	0.1098	0.1098	0.1098	1.008
.93	0.06990	0.1488	0.1495	0.1575	0.1575	0.1575	1.010
6.	0.1002	0.2183	0.2203	0.2354	0.2353	0.2354	1.009
.85	0.1497	0.3379	0.3448	0.3727	0.3727	0.3727	1.008

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	ASE(QL)	ASE(LES)	ASE(HOL)	ASE(RCS)	$ASE(RCS - \sqrt{s})$	ASE(LCS)	ASE(NI)
.98	0.4954	1.005	1.005	1.075	1.088	1.193	25.72
	± 0.0232	± 0.0416	± 0.0413	± 0.0412	± 0.0423	± 0.0412	土4.81
.95	0.1980	0.4162	0.4171	0.4831	0.4940	0.5869	3.9609
	± 0.00251	± 0.00399	± 0.00416	± 0.00394	± 0.00412	± 0.00413	± 0.228
.93	0.1421	0.3033	0.3046	0.3672	0.3774	0.4621	2.0014
	± 0.00133	± 0.00322	± 0.00374	± 0.00357	± 0.00330	± 0.00360	± 0.0659
6.	0.1003	0.2185	0.2204	0.2792	0.2884	0.3629	1.0099
	± 0.00168	± 0.00329	± 0.00416	± 0.00359	± 0.00346	± 0.00362	± 0.0490
85	0.0661	0.1499	0.1528	0.2037	0.2112	0.2692	0.4409
	± 0.000324	± 0.000757	± 0.00124	± 0.000924	± 0.000854	± 0.000970	± 0.00825

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			RASE in the	M/M/s model	with $s = 10$		
θ	RASE(QL)	RASE(LES)	RASE(HOL)	RASE(RCS)	$RASE(RCS - \sqrt{s})$	RASE(LCS)	RASE(NI)
0.98	0.0205	0.0417	0.0417	0.0446	0.0451	0.0495	1.067
0.95	0.05021	0.1055	0.1057	0.1225	0.1252	0.1488	1.004
0.93	0.07042	0.1503	0.1510	0.1820	0.1870	0.2290	0.9918
0.90	0.09994	0.2176	0.2195	0.2781	0.2873	0.3615	1.006
0.85	0.1495	0.3387	0.3455	0.4604	0.4775	0.6085	0.9966

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100	$RCS - \sqrt{s}$ $ASE(LCS)$ $ASE(NI)$	$\times 10^{-2}$ 2.660 $\times 10^{-2}$ 2.549 $\times 10^{-1}$	$ imes 10^{-4}$ $\pm 6.17 imes 10^{-4}$ $\pm 3.55 imes 10^{-2}$	$\times 10^{-3}$ 1.652 $\times 10^{-2}$ 4.180 $\times 10^{-2}$	10^{-5} $\pm 1.72 \times 10^{-5}$ 2.71×10^{-3}	$\times 10^{-3} \qquad 1.306 \times 10^{-2} 2.083 \times 10^{-2}$	$ imes 10^{-5}$ $\pm 1.27 imes 10^{-4}$ $\pm 1.22 imes 10^{-3}$	$\times 10^{-3}$ 9.379×10^{-3} 9.702×10^{-3}	$\times 10^{-9}$ $\pm 2.74 \times 10^{-4}$ $\pm 7.06 \times 10^{-4}$
1/M/s model with $s = 1$	ASE(RCS) ASE(I)	1.252×10^{-2} $1.287 >$	$\pm 5.11 \times 10^{-4} \pm 5.14$	6.380×10^{-3} $6.674 >$	$\pm 6.53 \times 10^{-5}$ 6.96 ×	5.058×10^{-3} $5.317 >$	$\pm 2.71 \times 10^{-5} \pm 3.29$	3.949×10^{-3} 4.157 >	$\pm 8.38 \times 10^{-3} \pm 9.11$
$ASE \ in \ the \ N$	ASE(HOL)	$1.023 imes 10^{-2}$	$\pm 5.09 imes 10^{-4}$	$4.269 imes10^{-3}$	$\pm 6.028 \times 10^{-5}$	$3.084 imes 10^{-3}$	$\pm 2.63 \times 10^{-5}$	$2.185 imes 10^{-3}$	$\pm 5.96 \times 10^{-3}$
	ASE(LES)	$1.023 imes 10^{-2}$	$\pm 5.04 imes 10^{-4}$	4.258×10^{-3}	$\pm 5.85 imes 10^{-5}$	3.069×10^{-3}	$\pm 2.53 imes 10^{-5}$	2.168×10^{-3}	$\pm 5.87 \times 10^{-9}$
	ASE(QL)	$5.033 imes 10^{-3}$	$\pm 2.46 \times 10^{-4}$	$2.041 imes 10^{-3}$	$\pm 4.22 \times 10^{-5}$	$1.442 imes 10^{-3}$	$\pm 1.66 imes 10^{-5}$	$9.940 imes 10^{-4}$	$\pm 2.97 imes 10^{-3}$
	θ	0.98		0.95		0.93		0.90	

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$^{\prime}/s$ queue with $s = 100$ ar	lach estimate is shown wi	
stimators for the M/M	uared error $-$ (ASE). F	
erent real-time delay e	ates for the average sq	
of the efficiency of diff	We report point estim:	nterval.
Table 5: A comparison	the traffic intensity ρ .	95 percent confidence i

_	β	RASE(QL)	RASE(LES)	RASE(HOL)	RASE(RCS)	$RASE(RCS - \sqrt{s})$	RASE(LCS)	RASE(NI)
	0.98	0.01982	0.04031	0.04033	0.04931	0.05075	0.1053	1.004
_	0.95	0.04961	0.1042	0.1043	0.1551	0.1624	0.4022	1.016
<u> </u>	0.93	0.07001	0.1493	0.1506	0.2465	0.2584	0.6342	1.011
\sim	0.0	0.1013	0.2203	0.2226	0.4002	0.4226	0.9512	0.9846

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		$)^{-2}_{-3}$)-3 -4	-4	$)^{-4}$
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	ASE(NI)	1.548×10^{-1} 2.05×10^{-1}	2.565 imes 10 $3.25 imes 10^{-1}$	$1.39 imes 10^{\circ}$ $1.83 imes 10^{\circ}$	6.358 imes 10 $1.32 imes 10^{-1}$
	ASE(LCS)	4.032×10^{-3} 5.36×10^{-5}	$\begin{array}{c} 2.168 \times 10^{-3} \\ 8.94 \times 10^{-5} \end{array}$	$\begin{array}{c} 1.542 \times 10^{-3} \\ 7.80 \times 10^{-5} \end{array}$	$9.574 imes 10^{-4}$ $9.41 imes 10^{-5}$
with $s = 400$	$ASE(RCS - \sqrt{s})$	$9.706 imes 10^{-4}$ $2.35 imes 10^{-5}$	$5.496 imes 10^{-4}$ $1.49 imes 10^{-5}$	$4.523 imes 10^{-4}$ $1.02 imes 10^{-5}$	$3.704 imes 10^{-4}$ $1.39 imes 10^{-5}$
1/M/s model n	ASE(RCS)	9.273×10^{-4} 2.36×10^{-5}	$5.168 imes 10^{-4}$ $1.43 imes 10^{-5}$	$4.263 imes 10^{-4}$ $9.72 imes 10^{-6}$	$\begin{array}{c} 3.486 \times 10^{-4} \\ 1.21 \times 10^{-5} \end{array}$
ASE in the I	ASE(HOL)	6.427×10^{-4} 2.50×10^{-5}	$\begin{array}{c} 2.651 \times 10^{-4} \\ 1.13 \times 10^{-5} \end{array}$	$\frac{1.960\times10^{-4}}{7.35\times10^{-6}}$	$\frac{1.414\times 10^{-4}}{7.77\times 10^{-6}}$
	ASE(LES)	6.424×10^{-4} 2.46×10^{-5}	$\begin{array}{c} 2.646 \times 10^{-4} \\ 1.11 \times 10^{-5} \end{array}$	$\frac{1.951\times 10^{-4}}{7.18\times 10^{-6}}$	$\frac{1.401\times10^{-4}}{7.31\times10^{-6}}$
	ASE(QL)	$\frac{3.145 \times 10^{-4}}{1.35 \times 10^{-5}}$	$\frac{1.262\times 10^{-4}}{5.51\times 10^{-6}}$	$\begin{array}{c} 9.109 \times 10^{-5} \\ 3.77 \times 10^{-6} \end{array}$	$\begin{array}{c} 6.451 \times 10^{-5} \\ 4.43 \times 10^{-6} \end{array}$
	θ	0.98	0.95	0.93	0.9

0	RASE(QL)	RASE(LES)	RASE(HOL)	RASE(RCS)	$RASE(RCS - \sqrt{s})$	RASE(LCS)	RASE(NI)
0.98	0.01959	0.04002	0.04004	0.05777	0.06046	0.2512	0.9643
0.95	0.04945	0.1037	0.1039	0.2025	0.2154	0.8495	1.005
0.93	0.06724	0.1440	0.1447	0.3135	0.3339	1.1380	1.0290
0.0	0.1007	0.2187	0.2207	0.5440	0.5781	1.494	0.9922

l/s queue with $s = 400$ and $\mu = 1$ as a function	(RASE).
, of the efficiency of different real-time delay estimators for the M/N	9. We report point estimates for the relative average squared error –
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	ASE(NI)	$1.513 imes 10^{-2}$	3.44×10^{-3}	$3.244 imes 10^{-3}$	$3.26 imes 10^{-4}$	4.934×10^{-4}	$1.018 imes 10^{-4}$	2.342×10^{-4}	$6.43 imes 10^{-5}$
	ASE(LCS)	$1.923 imes 10^{-3}$	$2.93 imes 10^{-5}$	1.381×10^{-3}	$2.82 imes 10^{-5}$	6.114×10^{-4}	$6.25 imes 10^{-5}$	$3.701 imes 10^{-4}$	$6.83 imes10^{-5}$
vith $s = 900$	$ASE(RCS - \sqrt{s})$	$3.680 imes 10^{-4}$	$1.87 imes 10^{-5}$	$2.252 imes 10^{-4}$	$4.53 imes 10^{-6}$	$1.340 imes 10^{-4}$	$5.49 imes 10^{-6}$	$1.101 imes 10^{-4}$	9.41×10^{-6}
M/M/s model u	ASE(RCS)	$3.542 imes 10^{-4}$	$1.85 imes 10^{-5}$	2.123×10^{-4}	4.43×10^{-6}	1.246×10^{-4}	$5.11 imes 10^{-6}$	$1.042 imes 10^{-4}$	$8.59 imes 10^{-6}$
ASE in the M	ASE(HOL)	$2.669 imes 10^{-4}$	$1.86 imes 10^{-5}$	$1.287 imes 10^{-4}$	4.25×10^{-6}	$5.229 imes 10^{-5}$	$3.95 imes 10^{-6}$	$3.834 imes 10^{-5}$	4.11×10^{-6}
	ASE(LES)	$2.668 imes 10^{-4}$	$1.85 imes 10^{-5}$	1.286×10^{-4}	4.24×10^{-6}	$5.219 imes10^{-5}$	$3.85 imes 10^{-6}$	$3.821 imes 10^{-5}$	4.09×10^{-6}
	ASE(QL)	$1.320 imes10^{-4}$	$8.86 imes 10^{-6}$	6.334×10^{-5}	$2.15 imes 10^{-6}$	2.484×10^{-5}	1.79×10^{-6}	$1.752 imes 10^{-5}$	1.94×10^{-6}
	θ	0.99		0.98		0.95		0.93	

~	K A S H (C) I Y	RACFILES	R A S E(HOI)	RACERCE	$RASE(RCS - \sqrt{e})$	B A S E(I, C, S)	P A S F(NI)
	(ng) nour				$\frac{(e \Lambda - \alpha \gamma)}{(e \Lambda - \Lambda s)}$	(and) Trunt	(TAT) MANY
0.99	0.009346	0.01871	0.01883	0.02494	0.02594	0.1351	1.063
).98	0.01962	0.03971	0.03984	0.06536	0.06952	0.4253	0.9997
).95	0.04973	0.1045	0.1047	0.2495	0.2684	1.225	0.9882
).93	0.07084	0.1541	0.1547	0.4187	0.4447	1.495	0.9451

h $s = 900$ and $\mu = 1$ as a function	
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ie efficiency of different real-time delay estimators for the M/M	report point estimates for the relative average squared error $-($
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Table 10: A comparison	of the traffic intensity $_{\ell}$

			ASE in the	D/M/s model	with $s = 1$		
β	ASE(QL)	ASE(LES)	ASE(HOL)	ASE(RCS)	$ASE(RCS - \sqrt{s})$	ASE(LCS)	ASE(NI)
0.95	10.13	11.61	11.61	12.56	12.56	12.56	101.1
	± 0.148	± 0.146	± 0.157	± 0.148	± 0.148	± 0.148	± 7.23
0.93	7.322	8.791	8.794	9.731	9.731	9.731	52.73
	± 0.0805	± 0.0780	± 0.0862	± 0.0804	± 0.0804	± 0.0804	± 2.42
0.9	5.192	6.644	6.647	7.556	7.556	7.556	26.76
	± 0.0377	± 0.0372	± 0.0409	± 0.0397	± 0.0397	± 0.0397	± 0.942
0.85	3.528	4.955	4.954	5.819	5.819	5.819	12.43
	± 0.0183	± 0.0178	± 0.0203	± 0.0203	± 0.0206	± 0.0203	± 0.357

ant real-time delay estimators for the $D/M/s$ queue with $s = 1$ and $\mu = 1$ as a function of	for the average squared error $-$ (ASE). Each estimate is shown with the half width of the	
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~	RASE(QL)	RASE(LES)	RASE(HOL)	RASE(RCS)	$RASE(RCS - \sqrt{s})$	RASE(LCS)	RASE(NI
.95	0.09916	0.1136	0.1136	0.1230	0.1230	0.1230	0.9899
.93	0.1375	0.1650	0.1651	0.1827	0.1827	0.1827	0.9898
<u>.</u> 90	0.1936	0.2478	0.2479	0.2818	0.2818	0.2818	0.9979
.85	0.2845	0.3995	0.3995	0.4692	0.4692	0.4692	1.002

e with $s = 1$ and $\mu = 1$ as a function of	
M/s quer	(RASE).
12: A comparison of the efficiency of different real-time delay estimators for the D_{i}	affic intensity ρ . We report point estimates for the relative average squared error –
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	ASE(NI)	5.933	± 1.02	1.011	$\pm 8.27 \times 10^{-2}$	5.240×10^{-1}	$\pm 2.89 \times 10^{-2}$	2.658×10^{-1}	$\pm 1.19 imes 10^{-2}$	$1.236 imes 10^{-1}$	$\pm 5.33 \times 10^{-3}$
	ASE(LCS)	$3.572 imes 10^{-1}$	$\pm 8.55 \times 10^{-3}$	$2.004 imes10^{-1}$.	$\pm 1.91 \times 10^{-3}$	$1.664 imes 10^{-1}$	$\pm 1.22 \times 10^{-3}$	$1.365 imes 10^{-1}$	$\pm 7.81 \times 10^{-4}$	$1.063 imes 10^{-1}$	$\pm 4.71 \times 10^{-4}$
<i>ith</i> $s = 10$	$ASE(RCS - \sqrt{s})$	3.050×10^{-1}	$\pm 8.57 \times 10^{-3}$	$1.552 imes 10^{-1}$.	$\pm 1.88 \times 10^{-3}$	$1.257 imes 10^{-1}$	$\pm 1.14 \times 10^{-3}$	$1.021 imes 10^{-1}$	$\pm 6.55 imes 10^{-4}$	8.137×10^{-2}	$\pm 2.79 imes 10^{-4}$
D/M/s model u	ASE(RCS)	2.990×10^{-1}	$\pm 8.51 \times 10^{-3}$	1.498×10^{-1} .	$\pm 1.92 \times 10^{-3}$	$1.207 imes10^{-1}$	$\pm 1.22 \times 10^{-3}$	$9.769 imes 10^{-2}$	$\pm 7.68 \times 10^{-4}$	$7.789 imes 10^{-2}$	$\pm 4.65 \times 10^{-4}$
ASE in the	ASE(HOL)	$2.633 imes 10^{-1}$	$\pm 8.61 \times 10^{-3}$	$1.157 imes10^{-1}$.	$\pm 2.03 \times 10^{-3}$	$8.773 imes 10^{-2}$	$\pm 1.32 \times 10^{-3}$	$6.632 imes 10^{-2}$	$\pm 9.12 \times 10^{-4}$	4.935×10^{-2}	$\pm 5.73 \times 10^{-4}$
	ASE(LES)	$2.633 imes 10^{-1}$	$\pm 8.34 \times 10^{-3}$	$1.158 imes 10^{-1}$.	$\pm 1.79 \times 10^{-3}$	$8.759 imes 10^{-2}$	$\pm 1.05 \times 10^{-3}$	$6.630 imes 10^{-2}$	$\pm 5.70 \times 10^{-4}$	4.943×10^{-2}	$\pm 2.57 \times 10^{-4}$
	ASE(QL)	2.485×10^{-1}	$\pm 8.36 \times 10^{-3}$	$1.010 imes 10^{-1}$	$\pm 1.76 \times 10^{-3}$	7.301×10^{-2}	$\pm 1.04 \times 10^{-3}$	5.182×10^{-2}	$\pm 5.77 imes 10^{-4}$	$3.519 imes 10^{-2}$	$\pm 2.49 \times 10^{-4}$
	θ	0.98		0.95		0.93		0.9		0.85	

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	ASE(NI)	$6.151 imes 10^{-2}$	$3.82 imes 10^{-3}$	$1.008 imes 10^{-2}$	$\pm 3.96 imes 10^{-4}$	$5.197 imes10^{-3}$	$\pm 3.18 \times 10^{-4}$	2.681×10^{-3}	$\pm 2.30 \times 10^{-4}$
	ASE(LCS)	1.033×10^{-2}	$\pm 1.13 imes 10^{-4}$	6.381×10^{-3}	$\pm 1.21 imes 10^{-4}$	4.897×10^{-3}	$\pm 1.32 imes 10^{-4}$	3.44×10^{-3}	$\pm 1.52 \times 10^{-4}$
$ith \ s = 100$	$ASE(RCS - \sqrt{s})$	$3.945 imes 10^{-3}$	$\pm 5.35 imes 10^{-5}$	$2.338 imes 10^{-3}$	$\pm 2.88 \times 10^{-5}$	$1.960 imes 10^{-3}$	$\pm 2.93 imes 10^{-5}$	1.63×10^{-3}	$\pm 3.71 \times 10^{-9}$
D/M/s model w	ASE(RCS)	$3.773 imes 10^{-3}$	$\pm 5.27 imes 10^{-5}$	$2.203 imes10^{-3}$	$\pm 2.73 \times 10^{-5}$	1.848×10^{-3}	$\pm 2.69 \times 10^{-5}$	$1.54 imes 10^{-3}$	$\pm 3.50 imes 10^{-9}$
ASE in the I	ASE(HOL)	$2.624 imes 10^{-3}$	$\pm 5.14 \times 10^{-5}$	$1.153 imes 10^{-3}$	$\pm 1.84 \times 10^{-5}$	$8.713 imes 10^{-4}$	$\pm 1.70 \times 10^{-5}$	$6.64 imes 10^{-4}$	$\pm 1.67 \times 10^{-9}$
	ASE(LES)	$2.624 imes 10^{-3}$	$\pm 5.02 imes 10^{-5}$	1.154×10^{-3}	$\pm 1.78 \times 10^{-5}$	$8.710 imes10^{-4}$	$\pm 1.58 imes 10^{-5}$	$6.65 imes 10^{-4}$	$\pm 1.56 \times 10^{-9}$
	ASE(QL)	$2.476 imes 10^{-3}$	$\pm 5.04 \times 10^{-5}$	$1.007 imes 10^{-3}$	$\pm 1.70 \times 10^{-5}$	$7.250 imes 10^{-4}$	$\pm 1.58 \times 10^{-5}$	$5.189 imes 10^{-4}$	$\pm 1.52 \times 10^{-9}$
	θ	0.98		0.95		0.93		0.9	

delay estimators for the $D/M/s$ queue with $s = 100$ and $\mu = 1$ as a function	verage squared error $-$ (ASE). Each estimate is shown with the half width of	
Table 15: A comparison of the efficiency of different real-time delay estime	of the traffic intensity ρ . We report point estimates for the average squar.	the 95 percent confidence interval.

•	RASE(QL)	RASE(LES)	RASE(HOL)	RASE(RCS)	$RASE(RCS - \sqrt{s})$	RASE(LCS)	RASE(NI)
.98	0.04043	0.04286	0.04285	0.06162	0.06442	0.1688	1.004
.95	0.1000	0.1146	0.1146	0.2189	0.2323	0.6340	1.001
.93	0.1382	0.1661	0.1662	0.3524	0.3739	0.9338	0.9910
<u> </u>	0.1939	0.2484	0.2482	0.5766	0.6084	1.284	1.002

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	ASE(NI)	1.416×10^{-2}	$\pm 1.77 imes 10^{-3}$	$3.895 imes 10^{-3}$	$\pm 2.64 imes 10^{-4}$	$6.451 imes 10^{-4}$	$\pm 7.27 imes 10^{-5}$	$3.325 imes 10^{-4}$	$\pm 3.84 imes 10^{-5}$	$1.415 imes 10^{-4}$	$\pm 4.31 \times 10^{-5}$
	ASE(LCS)	2.224×10^{-3}	$\pm 3.68 \times 10^{-5}$	$1.620 imes 10^{-3}$	$\pm 3.43 \times 10^{-5}$	7.549×10^{-4}	$\pm 3.31 \times 10^{-5}$	4.975×10^{-4}	$\pm 3.97 imes 10^{-5}$	2.703×10^{-4}	$\pm 6.54 \times 10^{-5}$
$ith \ s = 400$	$ASE(RCS - \sqrt{s})$	4.831×10^{-4}	$\pm 1.18 \times 10^{-5}$	$3.264 imes 10^{-4}$	$\pm 3.40 imes 10^{-6}$	$2.094 imes 10^{-4}$	$\pm 4.56 \times 10^{-6}$	$1.758 imes 10^{-4}$	$\pm 6.74 \times 10^{-6}$	$1.372 imes 10^{-4}$	$\pm 1.91 \times 10^{-5}$
D/M/s model w	ASE(RCS)	4.602×10^{-4}	$\pm 1.17 \times 10^{-5}$	$3.060 imes 10^{-4}$	$\pm 3.51 imes 10^{-6}$	$1.945 imes 10^{-4}$	$\pm 4.09 \times 10^{-6}$	$1.642 imes 10^{-4}$	$\pm 5.54 \times 10^{-6}$	$1.277 imes10^{-4}$	$\pm 1.85 \times 10^{-5}$
ASE in the I	ASE(HOL)	3.129×10^{-4}	$\pm 1.17 \times 10^{-5}$	$1.648 imes 10^{-4}$	$\pm 3.42 \times 10^{-6}$	7.245×10^{-5}	$\pm 2.50 \times 10^{-6}$	$5.511 imes 10^{-5}$	$\pm 2.64 \times 10^{-6}$	$3.931 imes 10^{-5}$	$\pm 5.81 \times 10^{-6}$
	ASE(LES)	$3.130 imes 10^{-4}$	$\pm 1.15 \times 10^{-5}$	$1.649 imes 10^{-4}$	$\pm 3.34 \times 10^{-6}$	7.248×10^{-5}	$\pm 2.41 \times 10^{-6}$	$5.497 imes 10^{-5}$	$\pm 2.55 \times 10^{-6}$	$3.988 imes 10^{-5}$	$\pm 5.45 \times 10^{-6}$
	ASE(QL)	$3.035 imes 10^{-4}$	$\pm 1.16 imes 10^{-5}$	$1.556 imes 10^{-4}$	$\pm 3.37 imes 10^{-6}$	6.329×10^{-5}	$\pm 2.41 \times 10^{-6}$	4.583×10^{-5}	$\pm 2.41 \times 10^{-6}$	$3.063 imes 10^{-5}$	$\pm 4.88 \times 10^{-6}$
	β	0.99		0.98		0.95		0.93		0.9	

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$\begin{array}{c ccc} \rho & RASE \\ \hline \rho & 0.09 & 0.0206 \\ \hline 0.98 & 0.0402 \end{array}$	$\frac{7(QL)}{5}$	RASE(LES)					
0.99 0.0206 0.98 0.0402	ۍ ا		RASE(HOL)	RASE(RCS)	$RASE(RCS - \sqrt{s})$	RASE(LCS)	RASE(NI)
0.98 0.0402		0.02129	0.02129	0.03131	0.03287	0.1508	0.9633
	∞	0.04269	0.04268	0.07923	0.08452	0.4195	1.0087
0.95 0.1015		0.1163	0.1162	0.3121	0.3359	1.2112	1.035
0.93 0.1362		0.1634	0.1638	0.4879	0.5225	1.4786	0.9884
0.90 0.1936		0.2521	0.2485	0.8075	0.8676	1.7088	0.9948

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Table 18: A co	of the traffic in

	ASE(NI)	$2.789 imes 10^{-3}$	$\pm 3.66 imes 10^{-4}$	$7.767 imes 10^{-4}$	$6.04 imes 10^{-5}$	$1.618 imes 10^{-4}$	$\pm 4.97 \times 10^{-5}$	$6.099 imes 10^{-5}$	$\pm 1.895 imes 10^{-5}$
	ASE(LCS)	$7.958 imes 10^{-4}$	$\pm 1.96 \times 10^{-5}$	$5.277 imes 10^{-4}$	$2.16 imes 10^{-5}$	2.148×10^{-4}	$\pm 3.53 imes 10^{-5}$	$1.202 imes 10^{-4}$	$\pm 3.370 imes 10^{-5}$
with $s = 900$	$ASE(RCS - \sqrt{s})$	$1.122 imes 10^{-4}$	$\pm 2.87 imes 10^{-6}$	$7.968 imes 10^{-5}$	$\pm 1.32 \times 10^{-6}$	$5.40 imes 10^{-5}$	$\pm 2.89 imes 10^{-6}$	4.345×10^{-5}	$\pm 6.46 \times 10^{-6}$
D/M/s model	ASE(RCS)	$1.056 imes 10^{-4}$	$\pm 2.81 \times 10^{-6}$	7.385×10^{-5}	$\pm 1.24 \times 10^{-6}$	$5.01 imes10^{-5}$	$\pm 2.57 \times 10^{-6}$	4.175×10^{-5}	$\pm 6.55 \times 10^{-6}$
ASE in the	ASE(HOL)	$6.198 imes 10^{-5}$	$\pm 2.62 imes 10^{-6}$	$3.088 imes 10^{-5}$	$\pm 1.00 \times 10^{-6}$	$1.53 imes 10^{-5}$	$\pm 1.52 imes 10^{-6}$	$1.125 imes 10^{-5}$	$\pm 1.77 imes 10^{-6}$
	ASE(LES)	$6.199 imes 10^{-5}$	$\pm 2.59 imes 10^{-6}$	$3.272 imes 10^{-5}$	$\pm 9.82 imes 10^{-7}$	$1.53 imes 10^{-5}$	$\pm 1.47 \times 10^{-6}$	$1.126 imes 10^{-5}$	$\pm 1.73 imes 10^{-6}$
	ASE(QL)	$6.013 imes 10^{-5}$	$\pm 2.60 \times 10^{-6}$	$3.088 imes 10^{-5}$	$\pm 9.77 imes 10^{-7}$	$1.35 imes 10^{-5}$	$\pm 1.50 \times 10^{-6}$	$9.42 imes 10^{-6}$	$\pm 1.68 \times 10^{-6}$
	θ	0.99		0.98		0.95		0.93	

Table 19: A comparison	of the efficiency of different real-time delay estimators for the $D/M/s$ queue with $s = 900$ and $\mu = 1$ as a function
of the traffic intensity ρ .	We report point estimates for the average squared error $-$ (ASE). Each estimate is shown with the half width of
the 95 percent confidence	interval.

0.03861	0.2738 0.68171	0.9596 1.003
0.10294	0.68171	1.003
0.37672	1.49886	1.0289
0.5941	1.644	0.9841
	0.37672 0.5941	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

$\iota = 1$ as a function	
with $s = 900$ and p	
M/s duene	: - (RASE).
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Table 20: A comparise	of the traffic intensity

			ASE in the .	$H_2/M/s mode_{ m c}$	l with s = 1		
θ	ASE(QL)	ASE(LES)	ASE(HOL)	ASE(RCS)	$ASE(RCS - \sqrt{s})$	ASE(LCS)	ASE(NI)
0.95	48.66	226.4	226.5	231.1	231.1	231.1	2339
	± 1.13	± 5.14	± 5.23	± 5.15	± 5.15	± 5.15	± 425
0.93	34.33	154.4	154.4	158.9	158.9	158.9	1151
	± 0.625	± 2.94	± 2.94	± 2.95	± 2.95	± 2.95	± 181
0.9	23.48	101.3	101.4	105.5	105.5	105.5	552.9
	± 0.366	± 2.32	± 2.36	± 2.35	± 2.35	± 2.35	± 103
0.85	14.95	59.99	60.15	63.89	63.89	63.89	224.4
	± 0.104	± 0.515	± 0.530	± 0.514	± 0.514	± 0.514	± 6.20

	RASE(QL)	RASE(LES)	RASE(HOL)	RASE(RCS)	$RASE(RCS - \sqrt{s})$	RASE(LCS)	RASE(NI
95	0.02057	0.09572	0.09572	0.09768	0.09768	0.09768	0.9889
93	0.02933	0.13191	0.1319	0.1358	0.1358	0.1358	0.9836
06	0.04260	0.1837	0.1839	0.1915	0.1915	0.1915	1.0031
85	0.06610	0.2652	0.2659	0.2824	0.2824	0.2824	0.9921

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0	ASE(QL)	ASE(LES)	ASE(HOL)	ASE(RCS)	$ASE(RCS - \sqrt{s})$	ASE(LCS)	ASE(NI)
0.98	1.284	6.259	6.259	6.435	6.512	6.730	159.4
	± 0.0690	± 0.404	± 0.410	± 0.406	± 0.405	± 0.595	± 25.8
0.95	0.481	2.23	2.23	2.39	2.46	2.65	22.9
	± 0.00811	± 0.0466	± 0.0482	± 0.0466	± 0.0470	± 0.0808	± 0.906
.93	0.3424	1.542	1.543	1.698	1.752	1.940	11.53
	± 0.00691	± 0.0346	± 0.0370	± 0.0351	± 0.0349	± 0.0346	0.679
6.(0.2344	1.012	1.013	1.156	1.1799	1.370	5.436
	± 0.00359	± 0.0181	± 0.0203	± 0.0186	± 0.0184	± 0.0179	± 0.291
.85	0.1498	0.6004	0.6022	0.7252	0.7496	0.8968	2.281
	± 0.00217	± 0.0121	± 0.0133	± 0.0122	± 0.0126	± 0.00757	± 0.137

comparison of the efficiency of different real-time delay estimators for the $H_2/M/s$ queue with $s = 10$ and $\mu = 1$ as a function	: intensity ρ . We report point estimates for the average squared error – (ASE). Each estimate is shown with the half width of	nt confidence interval.	
e 23: A comparison	e traffic intensity ρ .	5 percent confidence	
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			RASE in the	$H_2/M/s model$	with $s = 10$		
σ	RASE(QL)	RASE(LES)	RASE(HOL)	RASE(RCS)	$RASE(RCS - \sqrt{s})$	RASE(LCS)	RASE(NI)
0.98	0.007826	0.03816	0.03816	0.03923	0.03970	0.04103	1.020
0.95	0.02101	0.09735	0.09734	0.1045	0.1077	0.1160	0.9996
0.93	0.02933	0.1321	0.1322	0.1455	0.1501	0.1662	0.9875
0.9	0.04281	0.1849	0.1851	0.2112	0.2155	0.2503	0.9928
0.85	0.06653	0.2666	0.2674	0.3220	0.3328	0.3982	1.0127

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	ASE(NI)	1.505	0.2264	2.433×10^{-1}	2.274×10^{-2}	1.214×10^{-1}	1.02×10^{-2}	5.536×10^{-2}	$2.92 imes 10^{-3}$
	ASE(LCS)	1.034×10^{-1}	3.40×10^{-2}	$5.633 imes 10^{-2}$	$5.797 imes 10^{-4}$	4.454×10^{-2}	$1.02 imes 10^{-3}$	3.305×10^{-2}	$5.28 imes 10^{-4}$
with $s = 100$	$ASE(RCS - \sqrt{s})$	$6.702 imes 10^{-2}$	$3.22 imes 10^{-3}$	$2.842 imes 10^{-2}$	4.489×10^{-4}	$2.108 imes 10^{-2}$	$5.00 imes 10^{-4}$	$1.519 imes 10^{-2}$	2.44×10^{-4}
$I_2/M/s model v$	ASE(RCS)	$6.612 imes 10^{-2}$	$3.22 imes 10^{-3}$	2.765×10^{-2}	4.527×10^{-4}	2.039×10^{-2}	4.91×10^{-4}	1.460×10^{-2}	$2.36 imes 10^{-4}$
ASE in the H	ASE(HOL)	6.041×10^{-2}	$3.22 imes 10^{-3}$	2.248×10^{-2}	4.716×10^{-4}	1.554×10^{-2}	4.44×10^{-4}	$1.021 imes 10^{-2}$	$2.12 imes 10^{-4}$
	ASE(LES)	6.040×10^{-2}	$3.21 imes 10^{-3}$	2.248×10^{-2}	4.628×10^{-4}	$1.553 imes 10^{-2}$	4.38×10^{-4}	$1.020 imes 10^{-2}$	$2.11 imes 10^{-4}$
	ASE(QL)	$1.238 imes 10^{-2}$	$6.95 imes 10^{-4}$	4.815×10^{-2}	9.469×10^{-5}	3.444×10^{-3}	$9.45 imes 10^{-5}$	2.350×10^{-3}	$4.03 imes 10^{-5}$
	θ	0.98		0.95		0.93		0.9	

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φ	RASE(QL)	RASE(LES)	RASE(HOL)	RASE(RCS)	$RASE(RCS - \sqrt{s})$	RASE(LCS)	RASE(NI)
0.98	0.007519	0.03670	0.03670	0.04017	0.04072	0.06281	0.9146
0.95	0.02057	0.09600	0.09603	0.1181	0.1214	0.2406	1.039
0.93	0.02927	0.1320	0.1321	0.1733	0.1792	0.3786	1.031
0.9	0.04213	0.1829	0.1830	0.2618	0.2723	0.5924	0.9906

$s=100$ and $\mu=1$ as a function	
of the efficiency of different real-time delay estimators for the $H_2/M/s$ queue with	We report point estimates for the relative average squared error $-$ (RASE).
Table 26: A comparison of	of the traffic intensity ρ .

β	ASE(QL)	ASE(LES)	ASE(HOL)	ASE(RCS)	$ASE(RCS - \sqrt{s})$	ASE(LCS)	ASE(NI)
0.98	$7.676 imes 10^{-4}$	$3.736 imes 10^{-3}$	$3.736 imes 10^{-3}$	4.449×10^{-3}	4.561×10^{-3}	$1.329 imes 10^{-2}$	9.183×10^{-2}
	$2.34 imes 10^{-5}$	$1.05 imes 10^{-4}$	$1.05 imes 10^{-4}$	$1.04 imes 10^{-4}$	$1.05 imes 10^{-4}$	$1.64 imes10^{-4}$	6.34×10^{-3}
0.95	3.018×10^{-4}	1.382×10^{-3}	1.382×10^{-3}	2.015×10^{-3}	$2.105 imes 10^{-3}$	7.525×10^{-3}	1.423×10^{-2}
	$8.57 imes 10^{-6}$	$4.60 imes 10^{-5}$	$4.67 imes 10^{-5}$	$4.99 imes 10^{-5}$	$5.14 imes 10^{-5}$	1.406×10^{-4}	1.471×10^{-3}
0.93	$2.16 imes 10^{-4}$	$9.74 imes 10^{-4}$	$9.752 imes 10^{-4}$	1.561×10^{-3}	$1.635 imes 10^{-3}$	5.736×10^{-3}	7.613×10^{-3}
	$1.07 imes 10^{-5}$	$3.70 imes10^{-5}$	$3.79 imes 10^{-5}$	$4.12 imes 10^{-5}$	$4.14 imes 10^{-5}$	$1.93 imes 10^{-4}$	$1.157 imes 10^{-3}$
0.9	1.443×10^{-4}	$6.294 imes 10^{-4}$	$6.300 imes 10^{-4}$	1.133×10^{-3}	$1.194 imes 10^{-3}$	3.798×10^{-3}	$3.368 imes 10^{-3}$
	4.09×10^{-6}	$2.56 imes 10^{-5}$	$2.59 imes 10^{-5}$	$3.40 imes 10^{-5}$	$3.75 imes 10^{-5}$	$1.55 imes 10^{-4}$	$2.32 imes 10^{-4}$

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0	RASE(QL)	RASE(LES)	RASE(HOL)	RASE(RCS)	$RASE(RCS - \sqrt{s})$	RASE(LCS)	RASE(NI)
0.98	0.008655	0.04212	0.04212	0.05016	0.05142	0.1498	1.035
0.95	0.02034	0.09315	0.09315	0.1358	0.1419	0.5072	0.9590
0.93	0.02909	0.1314	0.1315	0.2098	0.2205	0.7735	1.027
0.0	0.04224	0.1842	0.1844	0.3316	0.3495	1.112	0.9858

ith $s = 400$ and $\mu = 1$ as a function	
, the efficiency of different real-time delay estimators for the $H_2/M/s$ queue with s -	Ve report point estimates for the relative average squared error $-$ (RASE).
Table 28: A comparison o	of the traffic intensity ρ .

	$-\sqrt{s}$) ASE(LCS) ASE(NI)	$\begin{array}{rrr} 4.395 \times 10^{-3} & 1.698 \times 10^{-2} \\ 8.20 \times 10^{-5} & 2.156 \times 10^{-3} \end{array}$	$\begin{array}{rrr} 2.276 \times 10^{-3} & 2.539 \times 10^{-3} \\ 1.85 \times 10^{-4} & 3.354 \times 10^{-4} \end{array}$	$\begin{array}{rrr} 1.630 \times 10^{-3} & 1.440 \times 10^{-3} \\ 1.32 \times 10^{-4} & 2.37 \times 10^{-4} \end{array}$
with $s = 900$	ASE(RCS -	$9.622 imes 10^{-4}$ $3.38 imes 10^{-5}$	$4.757 imes 10^{-4}$ $2.21 imes 10^{-5}$	$3.832 imes 10^{-4}$ $1.75 imes 10^{-5}$
$H_2/M/s model$	ASE(RCS)	$9.297 imes 10^{-4}$ $3.36 imes 10^{-5}$	$\begin{array}{c} 4.512 \times 10^{-4} \\ 2.15 \times 10^{-5} \end{array}$	$\begin{array}{c} 3.619 \times 10^{-4} \\ 1.68 \times 10^{-5} \end{array}$
ASE in the 1	ASE(HOL)	$\frac{7.205 \times 10^{-4}}{3.45 \times 10^{-5}}$	$\begin{array}{c} 2.713 \times 10^{-4} \\ 1.75 \times 10^{-5} \end{array}$	$\begin{array}{c} 1.936 \times 10^{-4} \\ 1.30 \times 10^{-5} \end{array}$
	ASE(LES)	7.206×10^{-4} 3.42×10^{-5}	$2.713 imes 10^{-4}$ $1.72 imes 10^{-5}$	$\begin{array}{c} 1.935 \times 10^{-4} \\ 1.29 \times 10^{-5} \end{array}$
	ASE(QL)	$\frac{1.487 \times 10^{-4}}{7.56 \times 10^{-6}}$	$5.826 imes 10^{-5}$ $3.70 imes 10^{-6}$	$4.292 imes 10^{-5}$ $2.12 imes 10^{-6}$
	θ	0.98	0.95	0.93

of the traffic intensity ρ . We report point estimates for the average squared error – (ASE). Each estimate is shown with the half width of the 95 percent confidence interval. Table 29: A comparison of the efficiency of different real-time delay estimators for the $H_2/M/s$ queue with s = 900 and $\mu = 1$ as a function

Table 30: A comparison of the efficiency of different real-time delay estimators for the $H_2/M/s$ queue with s = 400 and $\mu = 1$ as a function of the traffic intensity ρ . We report point estimates for the relative average squared error – (RASE).



Figure 1: Point estimates of the ASE of alternative real-time delay estimators for the M/M/s queue with s = 1 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 2: Point estimates of the RASE of alternative real-time delay estimators for the M/M/s queue with s = 1 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 3: Point estimates of the ASE of alternative real-time delay estimators for the M/M/s queue with s = 10 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 4: Point estimates of the RASE of alternative real-time delay estimators for the M/M/s queue with s = 10 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 5: Point estimates of the RASE of alternative real-time delay estimators for the M/M/s queue with s = 100 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 6: Point estimates of the RASE of alternative real-time delay estimators for the M/M/s queue with s = 100 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 7: Point estimates of the ASE of alternative real-time delay estimators for the M/M/s queue with s = 400 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 8: Point estimates of the RASE of alternative real-time delay estimators for the M/M/s queue with s = 400 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 9: Point estimates of the ASE of alternative real-time delay estimators for the M/M/s queue with s = 900 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 10: Point estimates of the RASE of alternative real-time delay estimators for the M/M/s queue with s = 900 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 11: Point estimates of the ASE of alternative real-time delay estimators for the D/M/s queue with s = 1 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 12: Point estimates of the RASE of alternative real-time delay estimators for the D/M/s queue with s = 1 and $\mu = 1$ as a function of the traffic intensity ρ .


Figure 13: Point estimates of the ASE of alternative real-time delay estimators for the D/M/s queue with s = 10 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 14: Point estimates of the RASE of alternative real-time delay estimators for the D/M/s queue with s = 10 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 15: Point estimates of the ASE of alternative real-time delay estimators for the D/M/s queue with s = 100 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 16: Point estimates of the RASE of alternative real-time delay estimators for the D/M/s queue with s = 100 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 17: Point estimates of the ASE of alternative real-time delay estimators for the D/M/s queue with s = 400 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 18: Point estimates of the RASE of alternative real-time delay estimators for the D/M/s queue with s = 400 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 19: Point estimates of the ASE of alternative real-time delay estimators for the D/M/s queue with s = 900 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 20: Point estimates of the RASE of alternative real-time delay estimators for the D/M/s queue with s = 900 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 21: Point estimates of the ASE of alternative real-time delay estimators for the $H_2/M/s$ queue with s = 1 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 22: Point estimates of the RASE of alternative real-time delay estimators for the $H_2/M/s$ queue with s = 1 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 23: Point estimates of the ASE of alternative real-time delay estimators for the $H_2/M/s$ queue with s = 10 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 24: Point estimates of the RASE of alternative real-time delay estimators for the $H_2/M/s$ queue with s = 10 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 25: Point estimates of the ASE of alternative real-time delay estimators for the $H_2/M/s$ queue with s = 100 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 26: Point estimates of the RASE of alternative real-time delay estimators for the $H_2/M/s$ queue with s = 100 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 27: Point estimates of the ASE of alternative real-time delay estimators for the $H_2/M/s$ queue with s = 400 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 28: Point estimates of the RASE of alternative real-time delay estimators for the $H_2/M/s$ queue with s = 400 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 29: Point estimates of the ASE of alternative real-time delay estimators for the $H_2/M/s$ queue with s = 900 and $\mu = 1$ as a function of the traffic intensity ρ .



Figure 30: Point estimates of the RASE of alternative real-time delay estimators for the $H_2/M/s$ queue with s = 900 and $\mu = 1$ as a function of the traffic intensity ρ .

	ASE(NI)	2.504×10^{-2} $\pm 3.89 \times 10^{-2}$	$6.453 imes 10^{-2} \pm 6.08 imes 10^{-3}$	1.040×10^{-2} $\pm 2.58 \times 10^{-4}$	$5.263 imes 10^{-3} \pm 1.20 imes 10^{-4}$	$2.516 imes 10^{-3} \pm 1.27 imes 10^{-4}$	
([0 <	ASE(LCS)	4.884×10^{-2} $\pm 2.18 \times 10^{-3}$	$3.565 imes 10^{-2} \ \pm 9.67 imes 10^{-4}$	$2.339 imes 10^{-2} \\ \pm 4.23 imes 10^{-4}$	$1.776 imes 10^{-2} \ \pm 2.67 imes 10^{-4}$	$1.202 imes 10^{-2} \pm 4.45 imes 10^{-4}$	
$\overline{W}[W > 0], 2E[\overline{W}]W$	$ASE(RCS - \sqrt{s})$	3.090×10^{-2} $\pm 2.09 \times 10^{-3}$	$1.744 imes 10^{-2} \pm 7.62 imes 10^{-4}$	$9.145 imes 10^{-3} \pm 1.44 imes 10^{-4}$	$7.421 imes 10^{-3} \pm 8.77 imes 10^{-5}$	$5.850 imes 10^{-3} \pm 1.55 imes 10^{-4}$	
al delays in $(E[$	ASE(RCS)	3.052×10^{-2} $\pm 2.09 \times 10^{-3}$	$1.702 imes 10^{-2} \pm 7.62 imes 10^{-4}$	$8.711 imes 10^{-3} \ \pm 1.37 imes 10^{-4}$	$7.031 imes 10^{-3} \ \pm 8.37 imes 10^{-5}$	5.564×10^{-3} $\pm 1.44 \times 10^{-4}$	
$s = 100 \ for \ actu$	ASE(HOL)	2.782×10^{-2} $\pm 2.08 \times 10^{-3}$	1.421×10^{-2} $\pm 7.42 \times 10^{-4}$	$5.695 imes 10^{-3} \pm 1.06 imes 10^{-4}$	$3.996 imes 10^{-3} \\ \pm 6.00 imes 10^{-5}$	2.734×10^{-3} $\pm 8.47 \times 10^{-5}$	
1/s model with s	ASE(LES)	2.802×10^{-2} $\pm 2.08 \times 10^{-3}$	1.442×10^{-2} $\pm 7.41 \times 10^{-4}$	$5.924 imes 10^{-3} \pm 1.07 imes 10^{-4}$	$4.231 imes 10^{-3} \ \pm 6.17 imes 10^{-5}$	$2.982 imes 10^{-3} \pm 8.61 imes 10^{-5}$	
SE in the M/M	ASE(QL)	1.380×10^{-2} $\pm 9.71 \times 10^{-4}$	$7.006 \times 10^{-3} \\ \pm 3.47 \times 10^{-4}$	$2.772 imes 10^{-3} \pm 5.46 imes 10^{-5}$	$1.925 imes 10^{-3} \pm 2.72 imes 10^{-5}$	$\frac{1.293 \times 10^{-3}}{\pm 4.18 \times 10^{-5}}$	
Conditional A	E[W W > 0]	9.833×10^{-1} $\pm 6.92 \times 10^{-2}$	$5.039 imes 10^{-1} \pm 2.48 imes 10^{-2}$	$2.028 imes 10^{-1} \pm 3.02 imes 10^{-3}$	$1.435 imes 10^{-1} \\ \pm 1.78 imes 10^{-3}$	$9.929 imes 10^{-2} \pm 2.75 imes 10^{-3}$	
	β	0.99	0.98	0.95	0.93	0.90	

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$\widetilde{V} > 0])$	ASE(NI)	2.882	$\pm 3.81 \times 10^{-1}$	3.058×10^{-2}	$\pm 1.62 \times 10^{-3}$	7.834×10^{-1}	$\pm 9.44 \times 10^{-2}$	$1.300 imes 10^{-1}$	$\pm 4.04 \times 10^{-3}$	6.387×10^{-2}	$\pm 1.73 \times 10^{-3}$
$\widetilde{W} > 0], 4E[\widetilde{W}]$	ASE(LCS)	$7.176 imes 10^{-2}$	$\pm 2.93 \times 10^{-3}$	2.731×10^{-2}	$\pm 6.12 \times 10^{-4}$	4.974×10^{-2}	$\pm 1.04 \times 10^{-3}$	$3.762 imes 10^{-2}$	$\pm 6.06 \times 10^{-4}$	3.345×10^{-2}	$\pm 3.60 \times 10^{-4}$
il delays in $(2E[\widetilde{W}])$	$ASE(RCS - \sqrt{s})$	5.417×10^{-2}	$\pm 2.79 \times 10^{-3}$	$1.000 imes 10^{-2}$	$\pm 1.92 imes 10^{-4}$	$3.028 imes 10^{-2}$	$\pm 1.26 imes 10^{-3}$	1.496×10^{-2}	$\pm 1.79 imes 10^{-4}$	$1.187 imes 10^{-2}$	$1.22 imes 10^{-4}$
$s = 100 \ for \ actual$	ASE(RCS)	$5.373 imes 10^{-2}$	$\pm 2.79 imes 10^{-3}$	9.434×10^{-3}	$\pm 1.89 \times 10^{-4}$	$2.986 imes 10^{-2}$	$\pm 1.27 imes 10^{-3}$	1.445×10^{-2}	$\pm 1.56 imes 10^{-4}$	$1.133 imes 10^{-2}$	$\pm 1.20 \times 10^{-4}$
1/s model with s	ASE(HOL)	5.108×10^{-2}	$\pm 2.77 \times 10^{-3}$	5.556×10^{-3}	$\pm 1.58 \times 10^{-4}$	2.706×10^{-2}	$\pm 1.35 \times 10^{-3}$	$1.117 imes 10^{-2}$	$\pm 1.46 \times 10^{-4}$	$7.867 imes 10^{-3}$	$\pm 8.77 \times 10^{-5}$
SE in the M/N	ASE(LES)	5.128×10^{-2}	$\pm 2.77 \times 10^{-3}$	5.842×10^{-3}	$\pm 1.57 \times 10^{-4}$	$2.727 imes 10^{-2}$	$\pm 1.35 \times 10^{-3}$	1.141×10^{-2}	$\pm 1.47 \times 10^{-4}$	$8.119 imes 10^{-3}$	$\pm 8.98 \times 10^{-5}$
Conditional A	ASE(QL)	$2.589 imes 10^{-2}$	$\pm 1.56 \times 10^{-3}$	2.713×10^{-3}	$\pm 7.57 imes 10^{-5}$	$1.359 imes 10^{-2}$	$\pm 6.75 imes 10^{-4}$	5.536×10^{-3}	$\pm 1.26 imes 10^{-4}$	3.886×10^{-2}	$\pm 5.65 imes 10^{-5}$
	θ	0.99		0.98		0.95		0.93		0.90	

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V > 0]	ASE(NI)	11.586	± 1.25	3.542	$\pm 4.31 \times 10^{-10}$	$5.641 imes 10^{-1}$	$\pm 2.70 \times 10^{-10}$	2.860×10^{-10}	$\pm 8.01 \times 10^{-10}$	1.374×10^{-1}	$\pm 6.74 \times 10^{-10}$	
$\widetilde{W} > 0], 6E[\widetilde{W 1}]$	ASE(LCS)	1.088×10^{-1}	$\pm 1.02 \times 10^{-2}$	6.964×10^{-2}	$\pm 3.67 imes 10^{-3}$	$5.039 imes10^{-2}$	$\pm 3.34 \times 10^{-3}$	$5.204 imes10^{-2}$	$\pm 3.18 \times 10^{-3}$	$5.089 imes 10^{-2}$	$\pm 2.52 \times 10^{-2}$	
al delays in $(4E[W])$	$ASE(RCS - \sqrt{s})$	$9.008 imes 10^{-2}$	$\pm 7.23 imes 10^{-3}$	$5.057 imes 10^{-2}$	$\pm 3.13 imes 10^{-3}$	2.403×10^{-2}	$\pm 8.00 \times 10^{-4}$	$1.930 imes 10^{-2}$	$\pm 4.52 \times 10^{-4}$	$1.609 imes 10^{-2}$	$\pm 6.62 \times 10^{-4}$	
$s = 100 \ for \ actual$	ASE(RCS)	8.943×10^{-2}	$\pm 7.24 \times 10^{-3}$	$5.014 imes 10^{-2}$	$\pm 3.09 imes 10^{-3}$	$2.350 imes10^{-2}$	$\pm 8.16 \times 10^{-4}$	$1.870 imes 10^{-2}$	$\pm 4.28 \times 10^{-4}$	$1.531 imes 10^{-2}$	$\pm 6.07 \times 10^{-4}$	
1/s model with s	ASE(HOL)	$8.632 imes 10^{-2}$	$\pm 6.86 \times 10^{-3}$	4.725×10^{-2}	$\pm 3.04 \times 10^{-3}$	$2.011 imes 10^{-2}$	$\pm 6.16 \times 10^{-4}$	$1.487 imes 10^{-2}$	$\pm 2.90 \times 10^{-4}$	$1.073 imes 10^{-2}$	$\pm 3.75 \times 10^{-4}$	
SE in the M/M	ASE(LES)	$8.655 imes 10^{-2}$	$\pm 6.89 \times 10^{-3}$	4.747×10^{-2}	$\pm 3.06 \times 10^{-3}$	$2.037 imes 10^{-2}$	$\pm 6.30 \times 10^{-4}$	$1.515 imes 10^{-2}$	$\pm 3.08 \times 10^{-4}$	$1.105 imes 10^{-2}$	$\pm 3.82 \times 10^{-4}$	
Conditional A	ASE(QL)	$4.937 imes 10^{-2}$	$\pm 7.03 \times 10^{-3}$	$2.479 imes 10^{-2}$	$\pm 1.75 imes 10^{-3}$	$1.052 imes 10^{-2}$	$\pm 2.30 imes 10^{-4}$	7.544×10^{-3}	$\pm 1.97 \times 10^{-3}$	$5.622 imes10^{-3}$	$\pm 2.07 \times 10^{-4}$	
	θ	0.99		0.98		0.95		0.93		0.90		

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					-	1	-2		-2
	ASE(NI)	7.902	± 1.29	1.496	$\pm 1.65 \times 10^{-1}$	7.362×10^{-1}	$\pm 6.29 \times 10^{-10}$	0.3478	$\pm 3.67 \times 10^{-1}$
6E[W]W > 0])	ASE(LCS)	9.012×10^{-2}	$\pm 2.30 \times 10^{-2}$	$6.823 imes 10^{-2}$	$\pm 8.10 \times 10^{-3}$	$6.603 imes 10^{-2}$	$\pm 1.07 \times 10^{-2}$	7.160×10^{-2}	$\pm 6.04 \times 10^{-3}$
for actual delays >	$ASE(RCS - \sqrt{s})$	$7.776 imes 10^{-2}$	$\pm 2.64 imes 10^{-2}$	$3.820 imes 10^{-2}$	$\pm 4.49 \times 10^{-3}$	$2.803 imes 10^{-2}$	$\pm 3.96 imes 10^{-3}$	2.438×10^{-2}	$\pm 1.34 \times 10^{-3}$
lel with $s = 100$	ASE(RCS)	$7.735 imes 10^{-2}$	$\pm 2.63 imes 10^{-2}$	$3.755 imes 10^{-2}$	$\pm 4.20 \times 10^{-3}$	2.739×10^{-2}	$\pm 3.82 \times 10^{-3}$	2.378×10^{-2}	$\pm 1.41 \times 10^{-3}$
the $M/M/s moo$	ASE(HOL)	7.521×10^{-2}	$\pm 2.60 \times 10^{-2}$	$3.319 imes 10^{-2}$	$\pm 4.13 \times 10^{-3}$	2.274×10^{-2}	$\pm 3.06 \times 10^{-3}$	1.828×10^{-2}	$\pm 8.65 \times 10^{-4}$
tional ASE in 1	ASE(LES)	$7.526 imes 10^{-2}$	$\pm 2.61 \times 10^{-2}$	$3.353 imes 10^{-2}$	$\pm 4.14 \times 10^{-3}$	2.306×10^{-2}	$\pm 3.16 \times 10^{-3}$	1.872×10^{-2}	$\pm 8.63 \times 10^{-4}$
Condi	ASE(QL)	3.361×10^{-2}	$\pm 1.21 \times 10^{-2}$	1.862×10^{-2}	$\pm 2.22 imes 10^{-3}$	$1.306 imes 10^{-2}$	$\pm 1.26 \times 10^{-3}$	$1.042 imes 10^{-2}$	$\pm 9.67 \times 10^{-4}$
	θ	0.98		0.95		0.93		0.90	

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Figure 31: Conditional ASE for the alternative delay estimators in the M/M/100 model for actual delays in $(\widehat{E[W|W>0]}, 2\widehat{E[W|W>0]})$, as a function of the traffic intensity ρ



Figure 32: Conditional ASE for the alternative delay estimators in the M/M/100 model for actual delays in $(2E[\widehat{W|W} > 0], 4E[\widehat{W|W} > 0])$, as a function of the traffic intensity ρ



Figure 33: Conditional ASE for the alternative delay estimators in the M/M/100 model for actual delays in $(4E[\widehat{W|W} > 0], 6E[\widehat{W|W} > 0])$, as a function of the traffic intensity ρ



Figure 34: Conditional ASE for the alternative delay estimators in the M/M/100 model for actual delays larger than $6E[\widehat{W|W} > 0]$, as a function of the traffic intensity ρ

[M]W > 0]	$-\sqrt{s}$) $ASE(LCS)$ $ASE(NI)$	7.164×10^{-3} 1.607×10^{-2}	4 $\pm 2.64 \times 10^{-4}$ $\pm 2.70 \times 10^{-3}$	5.733×10^{-3} 4.104×10^{-3}	5 $\pm 1.87 \times 10^{-4}$ $\pm 3.04 \times 10^{-4}$	2.828×10^{-3} 6.487×10^{-4}	$5 \pm 1.52 \times 10^{-4} \pm 4.91 \times 10^{-5}$	$1.937 imes 10^{-3}$ $3.443 imes 10^{-4}$	$5 \pm 1.27 \times 10^{-4} 2.83 \times 10^{-5}$	$1.117 imes 10^{-3}$ $1.637 imes 10^{-4}$	$5 \pm 9.57 \times 10^{-5} \pm 1.51 \times 10^{-5}$	
$\widetilde{W W} > 0], 2I$	ASE(RCS -	2.177×10^{-3}	$\pm 1.40 \times 10^{-1}$	$1.283 imes 10^{-5}$	$\pm 3.70 \times 10^{-1}$	$7.628 imes 10^{-4}$	$\pm 2.25 \times 10^{-1}$	$6.497 imes 10^{-4}$	$\pm 2.25 \times 10^{-1}$	$5.121 imes 10^{-4}$	$\pm 2.27 \times 10^{-1}$	
tal delays in $(E[$	ASE(RCS)	$2.122 imes 10^{-3}$	$\pm 1.42 \times 10^{-4}$	$1.228 imes 10^{-4}$	$\pm 3.70 imes 10^{-5}$	7.136×10^{-4}	$\pm 2.18 imes 10^{-5}$	$6.078 imes 10^{-4}$	$\pm 2.07 imes 10^{-5}$	4.872×10^{-4}	$\pm 1.92 \times 10^{-5}$	
$s = 400 \ for \ actu$	ASE(HOL)	1.782×10^{-3}	$\pm 1.38 \times 10^{-4}$	$8.905 imes 10^{-4}$	$\pm 3.72 imes 10^{-5}$	$3.518 imes 10^{-4}$	$\pm 1.46 \times 10^{-5}$	$2.562 imes 10^{-4}$	$\pm 1.15 \times 10^{-5}$	$1.764 imes 10^{-4}$	$\pm 9.43 \times 10^{-6}$	
1/s model with s	ASE(LES)	1.796×10^{-3}	$\pm 1.38 \times 10^{-4}$	$9.035 imes 10^{-4}$	$\pm 3.73 \times 10^{-4}$	$3.656 imes 10^{-4}$	$\pm 1.47 \times 10^{-5}$	$2.713 imes 10^{-4}$	$\pm 1.16 \times 10^{-5}$	$1.920 imes 10^{-4}$	$\pm 9.48 \times 10^{-6}$	
SE in the M/N	ASE(QL)	8.808×10^{-4}	$\pm 6.75 imes 10^{-5}$	4.388×10^{-4}	$\pm 1.77 \times 10^{-5}$	$1.710 imes10^{-4}$	$\pm 7.90 \times 10^{-6}$	$1.232 imes 10^{-4}$	$\pm 5.77 \times 10^{-6}$	$8.297 imes 10^{-5}$	$\pm 4.83 \times 10^{-6}$	
Conditional A	E[W W > 0]	$2.507 imes 10^{-1}$	$\pm 1.94 imes 10^{-2}$	$1.267 imes 10^{-1}$	$\pm 4.967 \times 10^{-3}$	$5.052 imes 10^{-2}$	$\pm 2.03 \times 10^{-3}$	3.680×10^{-2}	$\pm 1.50 \times 10^{-3}$	2.531×10^{-2}	$\pm 1.31 \times 10^{-3}$	
	β	0.99		0.98		0.95		0.93		0.90		

queue with s = 400 and $\mu = 1$ as a function of the traffic intensity ρ . We report point estimates for the conditional average squared error (ASE) in the interval (E[W|W > 0], 2E[W|W > 0]). Each estimate is shown with the half width of the 95 percent confidence interval.

V > 0]	ASE(NI)	$2.079 imes 10^{-1}$	4.05×10^{-2}	$5.067 imes 10^{-2}$	$6.02 imes 10^{-3}$	$7.913 imes 10^{-3}$	$6.31 imes 10^{-4}$	4.283×10^{-3}	$3.54 imes 10^{-4}$	$2.01 imes 10^{-3}$	$\pm 2.30 \times 10^{-4}$	
$\overline{W} > 0], 4E[\overline{W}]$	ASE(LCS)	8.890×10^{-3}	$\pm 3.06 \times 10^{-4}$	7.654×10^{-3}	$\pm 2.74 \times 10^{-4}$	$6.117 imes10^{-3}$	$2.43 imes 10^{-4}$	4.757×10^{-3}	$2.07 imes 10^{-4}$	$3.207 imes 10^{-3}$	$\pm 2.53 \times 10^{-4}$	
al delays in $(2E[W])$	$ASE(RCS - \sqrt{s})$	3.814×10^{-3}	$\pm 3.20 imes 10^{-4}$	$2.106 imes 10^{-3}$	$\pm 8.26 imes 10^{-5}$	$1.162 imes 10^{-3}$	$\pm 4.49 \times 10^{-5}$	$1.003 imes 10^{-3}$	$\pm 3.61 imes 10^{-5}$	$9.150 imes 10^{-4}$	$\pm 4.96 \times 10^{-5}$	
$s = 400 \ for \ actual$	ASE(RCS)	$3.757 imes 10^{-3}$	$\pm 3.18 imes 10^{-4}$	2.054×10^{-3}	$\pm 8.04 \times 10^{-5}$	1.094×10^{-3}	$\pm 4.12 \times 10^{-5}$	$9.352 imes 10^{-4}$	$\pm 3.41 \times 10^{-5}$	8.439×10^{-4}	$\pm 4.16 \times 10^{-5}$	
I/s model with s	ASE(HOL)	3.417×10^{-3}	$\pm 3.15 imes 10^{-4}$	$1.707 imes 10^{-3}$	$\pm 8.25 imes 10^{-5}$	$6.928 imes 10^{-4}$	$\pm 3.41 \times 10^{-5}$	$5.060 imes10^{-4}$	$\pm 3.05 imes 10^{-5}$	$3.587 imes 10^{-4}$	$\pm 2.49 \times 10^{-5}$	
SE in the M/M	ASE(LES)	3.431×10^{-3}	$\pm 3.15 imes 10^{-4}$	$1.721 imes 10^{-3}$	$\pm 8.22 \times 10^{-5}$	$7.08 imes 10^{-4}$	$\pm 3.42 imes 10^{-5}$	$5.220 imes10^{-4}$	$\pm 3.09 imes 10^{-5}$	$3.763 imes10^{-4}$	$\pm 2.54 imes 10^{-5}$	
Conditional A	ASE(QL)	$1.736 imes 10^{-3}$	$\pm 1.72 imes 10^{-4}$	8.438×10^{-4}	$\pm 4.25 imes 10^{-5}$	$3.460 imes 10^{-4}$	$\pm 1.66 imes 10^{-5}$	$2.475 imes 10^{-4}$	$\pm 1.54 imes 10^{-5}$	$1.753 imes10^{-4}$	$\pm 1.33 imes 10^{-5}$	
	θ	0.99		0.98		0.95		0.93		0.90		

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Ŭ	pnditional ASE	in the $M/M/s$ r	nodel with $s = 4$	00 for actual de	slays in $(4E[W]W >$	0], 6E[W W > 0	[[
σ	ASE(QL)	ASE(LES)	ASE(HOL)	ASE(RCS)	$ASE(RCS - \sqrt{s})$	ASE(LCS)	ASE(NI)
0.99	$3.026 imes 10^{-3}$	$5.939 imes 10^{-3}$	$5.926 imes 10^{-3}$	6.296×10^{-3}	$6.350 imes 10^{-3}$	1.194×10^{-2}	9.053×10^{-1}
	$\pm 4.09 \times 10^{-4}$	$\pm 6.95 \times 10^{-4}$	$\pm 6.96 \times 10^{-4}$	$\pm 7.16 imes 10^{-4}$	$\pm 7.15 imes 10^{-4}$	$\pm 1.41 \times 10^{-3}$	$2.04 imes 10^{-1}$
0.98	1.465×10^{-3}	$2.974 imes 10^{-3}$	$2.961 imes 10^{-3}$	$3.321 imes 10^{-3}$	$3.366 imes 10^{-3}$	9.149×10^{-3}	$2.156 imes 10^{-1}$
$\pm 1.91 imes 10^{-4}$	$\pm 2.53 imes 10^{-4}$	$2.52 imes 10^{-4}$	$\pm 2.70 imes 10^{-4}$	$2.75 imes 10^{-4}$	$5.70 imes10^{-4}$	$1.97 imes 10^{-2}$	
0.95	$6.489 imes 10^{-4}$	1.345×10^{-3}	$1.325 imes 10^{-3}$	$1.808 imes 10^{-3}$	$1.892 imes 10^{-3}$	$1.010 imes 10^{-2}$	$3.573 imes10^{-2}$
	$\pm 3.34 \times 10^{-5}$	$\pm 9.04 \times 10^{-5}$	$9.07 imes 10^{-5}$	$1.15 imes 10^{-4}$	$1.11 imes 10^{-4}$	$7.16 imes 10^{-4}$	$3.59 imes 10^{-3}$
0.93	4.862×10^{-4}	$9.684 imes 10^{-4}$	$9.512 imes 10^{-4}$	1.394×10^{-3}	1.468×10^{-3}	$9.316 imes 10^{-3}$	$1.909 imes 10^{-2}$
	$\pm 3.87\times 10^{-5}$	$\pm 7.86 imes 10^{-5}$	$\pm 7.45 imes 10^{-5}$	$1.33 imes 10^{-4}$	$1.28 imes 10^{-4}$	$6.20 imes 10^{-4}$	$1.52 imes 10^{-3}$
0.90	$3.87 imes 10^{-4}$	$7.00 imes10^{-4}$	$6.801 imes 10^{-4}$	$1.282 imes 10^{-3}$	$1.393 imes 10^{-3}$	7.896×10^{-3}	$8.682 imes 10^{-3}$
$\pm 5.20\times 10^{-5}$	$\pm 8.78 imes 10^{-5}$	$\pm 8.73 \times 10^{-5}$	$\pm 1.73 imes 10^{-4}$	$\pm 1.65 imes 10^{-4}$	$\pm 6.71 imes 10^{-4}$	$\pm 9.75 imes 10^{-4}$	
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4SE in the $M/M/s$ model with $s = 400$ when $delays > 6E[W W > 0]$	$SE(LES)$ $ASE(HOL)$ $ASE(RCS)$ $ASE(RCS - \sqrt{s})$ $ASE(LCS)$ $ASE(NI)$	$36 \times 10^{-3} 1.916 \times 10^{-3} 2.311 \times 10^{-3} 2.310 \times 10^{-3} 1.470 \times 10^{-2} 8.352 \times 10^{-2} 1.470 \times 10^{-2} 1.470 \times 10^{-2} 1.470 \times 10^{-2} 1.410 \times 10^{-2} \times 1$	$3.64 \times 10^{-4} \pm 3.52 \times 10^{-4} \pm 4.74 \times 10^{-4} \pm 4.77 \times 10^{-4} \pm 2.52 \times 10^{-3} \pm 8.66 \times 10^{-3}$	$559 \times 10^{-3} 1.338 \times 10^{-3} 1.886 \times 10^{-3} 2.003 \times 10^{-3} 1.450 \times 10^{-2} 4.483 \times 10^{-2}$	$3.10 \times 10^{-4} \pm 3.03 \times 10^{-4} \pm 5.09 \times 10^{-4} \pm 5.33 \times 10^{-4} \pm 3.65 \times 10^{-3} \pm 4.43 \times 10^{-3} \pm 4.43 \times 10^{-3} \pm 10^$	8×10^{-3} 1.708 × 10 ⁻³ 2.359 × 10 ⁻³ 2.629 × 10 ⁻³ 1.457 × 10 ⁻² 1.965 × 10 ⁻²	$5.16 \times 10^{-4} \pm 4.79 \times 10^{-4} \pm 6.85 \times 10^{-4} \pm 7.33 \times 10^{-4} \pm 2.27 \times 10^{-3} \pm 3.17 \times 10^{-3} \pm 10^{-3} $	
ASE in the $M/M/s$ mod	ASE(LES) $ASE(HOL$	1.936×10^{-3} 1.916×10^{-3}	$\pm 3.64 \times 10^{-4}$ $\pm 3.52 \times 10^{-1}$	1.359×10^{-3} 1.338×10^{-3}	$\pm 3.10 \times 10^{-4}$ $\pm 3.03 \times 10^{-4}$	1.78×10^{-3} 1.708×10^{-3}	$\pm 5.16 \times 10^{-4}$ $\pm 4.79 \times 10^{-3}$	
	$\rho ASE(QL)$	$0.95 9.815 imes 10^{-4}$	$\pm 1.64 imes 10^{-4}$	$0.93 7.0711 imes 10^{-4}$	$\pm 1.61 imes 10^{-4}$	$0.90 9.73 imes 10^{-4}$	$\pm 2.84 imes 10^{-4}$	

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Figure 35: Conditional ASE for the alternative delay estimators in the M/M/400 model for actual delays in $(\widehat{E[W|W>0]}, 2\widehat{E[W|W>0]})$, as a function of the traffic intensity ρ



Figure 36: Conditional ASE for the alternative delay estimators in the M/M/400 model for actual delays in $(2E[\widehat{W|W} > 0], 4E[\widehat{W|W} > 0])$, as a function of the traffic intensity ρ



Figure 37: Conditional ASE for the alternative delay estimators in the M/M/400 model for actual delays in $(4E[\widehat{W|W} > 0], 6E[\widehat{W|W} > 0])$, as a function of the traffic intensity ρ



Figure 38: Conditional ASE for the alternative delay estimators in the M/M/400 model for actual delays larger than $6E[\widehat{W|W} > 0]$), as a function of the traffic intensity ρ

	ASE(NI)	3.636×10^{-3} $\pm 6.01 \times 10^{-4}$	$8.292 imes 10^{-4} \ \pm 5.17 imes 10^{-5}$	$\begin{array}{c} 1.264 \times 10^{-4} \\ \pm 1.55 \times 10^{-5} \end{array}$	$6.386 imes 10^{-5} \pm 1.28 imes 10^{-5}$	
7 > 0])	ASE(LCS)	$2.677 imes 10^{-3} \ \pm 8.31 imes 10^{-5}$	$\begin{array}{c} 1.943 \times 10^{-3} \\ \pm 5.16 \times 10^{-5} \end{array}$	$\begin{array}{l} 7.396 \times 10^{-4} \\ \pm 7.81 \times 10^{-5} \end{array}$	$\begin{array}{c} 4.367 \times 10^{-4} \\ \pm 7.68 \times 10^{-5} \end{array}$	
[W W > 0], 2E[W W	$ASE(RCS - \sqrt{s})$	$4.880 imes 10^{-4} \pm 2.80 imes 10^{-5}$	$2.954 imes 10^{-4} \ \pm 4.58 imes 10^{-6}$	$1.882 imes 10^{-4} imes 8.49 imes 10^{-6}$	$1.554 imes 10^{-4} \pm 1.40 imes 10^{-5}$	
ual delays in $(E[$	ASE(RCS)	$4.721 imes 10^{-4} imes 12.78 imes 10^{-5}$	2.788×10^{-4} $\pm 4.42 \times 10^{-6}$	$1.744 imes 10^{-4} \\ \pm 7.47 imes 10^{-6}$	$\begin{array}{c} 1.460 \times 10^{-4} \\ \pm 1.20 \times 10^{-5} \end{array}$	
$s = 900 \ for \ acti$	ASE(HOL)	$3.749 imes 10^{-4} \pm 2.67 imes 10^{-5}$	1.787×10^{-4} $\pm 4.79 \times 10^{-6}$	$6.924 imes 10^{-5} \ \pm 4.98 imes 10^{-6}$	$4.97 imes 10^{-5} \ \pm 5.52 imes 10^{-6}$	
1/s model with	ASE(LES)	$3.774 imes 10^{-4} \ \pm 2.67 imes 10^{-5}$	$\begin{array}{c} 1.812 \times 10^{-4} \\ \pm 4.79 \times 10^{-6} \end{array}$	$7.204 imes 10^{-5} \\ \pm 4.99 imes 10^{-6}$	$5.26 imes 10^{-5} \pm 5.61 imes 10^{-6}$	
ISE in the M/N	ASE(QL)	1.855×10^{-4} $\pm 1.40 \times 10^{-5}$	$\begin{array}{c} 8.814 \times 10^{-5} \\ \pm 2.69 \times 10^{-6} \end{array}$	$3.357 imes 10^{-5} \pm 2.47 imes 10^{-6}$	$2.347 imes 10^{-5} \pm 2.80 imes 10^{-6}$	
$Conditional$ \measuredangle	E[W]W > 0]	1.193×10^{-1} $\pm 8.27 \times 10^{-3}$	5.690×10^{-2} $\pm 1.86 \times 10^{-3}$	$2.235 imes 10^{-2} \ \pm 1.52 imes 10^{-3}$	$1.573 imes 10^{-2} \pm 1.66 imes 10^{-3}$	
	β	0.99	0.98	0.95	0.93	

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	$Conditional \ A$	ASE in the M/N	$1/s$ model with $\frac{1}{2}$	$s = 900 \ for \ actu$	al delays in $(2E[\widetilde{W}])$	$\widetilde{W} > 0], 4E[\widetilde{W P}]$	V > 0])
θ	ASE(QL)	ASE(LES)	ASE(HOL)	ASE(RCS)	$ASE(RCS - \sqrt{s})$	ASE(LCS)	ASE(NI)
0.99	$3.533 imes 10^{-4}$	$7.072 imes 10^{-4}$	7.046×10^{-4}	7.981×10^{-4}	$8.137 imes 10^{-4}$	$3.079 imes 10^{-3}$	4.602×10^{-2}
	$\pm 2.85 imes 10^{-5}$	$\pm 6.37 \times 10^{-5}$	$\pm 6.37 imes 10^{-5}$	$\pm 6.48 \times 10^{-5}$	$\pm 6.55 imes 10^{-5}$	$\pm 1.83 imes 10^{-4}$	$\pm 8.93 \times 10^{-3}$
0 08	$1 600 \times 10^{-4}$	$3 \ 138 \ < \ 10^{-4}$	$3 \ 111 \ > \ 10^{-4}$	1.130×10^{-4}	Λ 60.4 \sim 10 ⁻⁴	$3.048 < 10^{-3}$	$1.005 < 10^{-2}$
00	$\pm 8.53 imes 10^{-6}$	$\pm 1.26 imes 10^{-5}$	$\pm 1.26 \times 10^{-5}$	$\pm 1.35 \times 10^{-5}$	$\pm 1.36 imes 10^{-5}$	$\pm 9.60 imes 10^{-5}$	$\pm 7.72 imes 10^{-4}$
0.95	$6.793 imes 10^{-5}$	$1.403 imes 10^{-4}$	$1.374 imes 10^{-4}$	$2.563 imes 10^{-4}$	$2.779 imes 10^{-4}$	$1.905 imes 10^{-3}$	$1.602 imes 10^{-3}$
	$\pm 4.61 imes 10^{-6}$	$\pm 1.12 imes 10^{-5}$	$\pm 1.11 imes 10^{-5}$	$\pm 1.02 imes 10^{-5}$	$\pm 1.08 imes 10^{-5}$	$\pm 1.89 \times 10^{-4}$	$\pm 2.36 \times 10^{-4}$
0.93	$4.85 imes 10^{-5}$	$1.054 imes 10^{-4}$	$1.018 imes 10^{-4}$	$2.390 imes 10^{-4}$	$2.608 imes 10^{-4}$	$1.300 imes 10^{-3}$	$8.177 imes 10^{-4}$
	$\pm 5.45 imes 10^{-6}$	$\pm 1.25 imes 10^{-5}$	$\pm 1.23 imes 10^{-5}$	$\pm 2.02 imes 10^{-5}$	$\pm 2.09 imes 10^{-5}$	$\pm 2.27 imes 10^{-4}$	$\pm 1.75 imes 10^{-4}$

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V > 0])	ASE(NI)	1.949×10^{-1}	$3.86 imes 10^{-2}$	4.528×10^{-2}	3.584×10^{-3}	6.899×10^{-3}	$\pm 1.01 \times 10^{-3}$	$3.365 imes 10^{-3}$	$\pm 6.01 \times 10^{-4}$
$W \ge 0], 6E[W W$	ASE(LCS)	4.097×10^{-3}	$\pm 3.10 imes 10^{-4}$	$3.879 imes 10^{-3}$	$3.061 imes 10^{-4}$	$3.829 imes 10^{-3}$	$\pm 3.50 imes 10^{-4}$	$3.078 imes 10^{-3}$	$\pm 4.76 \times 10^{-4}$
al delays in $(4E[ar{W}])$	$ASE(RCS - \sqrt{s})$	1.410×10^{-3}	$\pm 1.16 imes 10^{-4}$	$7.472 imes 10^{-4}$	$5.726 imes10^{-5}$	$3.812 imes 10^{-4}$	$\pm 5.62 imes 10^{-5}$	3.815×10^{-4}	$\pm 4.82 \times 10^{-9}$
$s = 900 \ for \ actu$	ASE(RCS)	1.394×10^{-3}	$\pm 1.16 \times 10^{-4}$	$7.302 imes10^{-4}$	$5.709 imes 10^{-5}$	$3.600 imes10^{-4}$	$\pm 5.29 \times 10^{-5}$	$3.536 imes 10^{-4}$	$\pm 4.61 \times 10^{-9}$
1/s model with s	ASE(HOL)	1.284×10^{-3}	$\pm 1.16 \times 10^{-4}$	$6.296 imes10^{-4}$	4.776×10^{-5}	2.342×10^{-4}	$\pm 3.38 \times 10^{-5}$	$1.812 imes 10^{-4}$	$\pm 2.12 \times 10^{-9}$
SE in the M/M	ASE(LES)	$1.287 imes 10^{-3}$	$\pm 1.15 \times 10^{-4}$	$6.322 imes 10^{-4}$	4.808×10^{-5}	2.370×10^{-4}	$\pm 3.44 \times 10^{-5}$	$1.854 imes 10^{-4}$	$\pm 2.15 \times 10^{-9}$
Conditional A	ASE(QL)	$6.086 imes 10^{-4}$	$\pm 6.72 \times 10^{-5}$	$3.200 imes 10^{-4}$	$1.845 imes 10^{-5}$	$1.211 imes 10^{-4}$	$\pm 1.48 \times 10^{-5}$	$9.350 imes 10^{-5}$	$\pm 2.01 \times 10^{-9}$
	θ	0.99		0.98		0.95		0.93	

ent real-time delay estimators conditional on the level of delay observed for the $M/M/s$	ie traffic intensity ρ . We report point estimates for the conditional average squared error	0]). Each estimate is shown with the half width of the 95 percent confidence interval.
ifferent real-time delay estimators co	of the traffic intensity ρ . We report 1	$\widetilde{r} > 0$]). Each estimate is shown with
Table 41: A comparison of the efficiency of d	queue with $s = 900$ and $\mu = 1$ as a function of	(ASE) in the interval $(4E[W W > 0], 6E[W W$



Figure 39: Conditional ASE for the alternative delay estimators in the M/M/900 model for actual delays in $(\widehat{E[W|W>0]}, 2\widehat{E[W|W>0]})$, as a function of the traffic intensity ρ



Figure 40: Conditional ASE for the alternative delay estimators in the M/M/900 model for actual delays in $(2E[\widehat{W|W} > 0], 4E[\widehat{W|W} > 0])$, as a function of the traffic intensity ρ



Figure 41: Conditional ASE for the alternative delay estimators in the M/M/400 model for actual delays in $(4E[\widehat{W|W} > 0], 6E[\widehat{W|W} > 0])$, as a function of the traffic intensity ρ

	ASE(NI)	$5.764 imes 10^{-2}$	$\pm 3.90 imes 10^{-3}$	$1.547 imes 10^{-2}$	$\pm 9.72 imes 10^{-4}$	$2.567 imes 10^{-3}$	$\pm 9.04 imes 10^{-5}$	$1.332 imes 10^{-3}$	$\pm 4.11 \times 10^{-5}$	$6.811 imes 10^{-4}$	$\pm 3.08 \times 10^{-5}$	for the $D/M/s$
<pre>> 0])</pre>	ASE(LCS)	$1.759 imes 10^{-2}$	$\pm 4.15 \times 10^{-4}$	1.439×10^{-2}	$\pm 3.26 imes 10^{-4}$	8.661×10^{-3}	$\pm 2.24 \times 10^{-4}$	6.259×10^{-3}	$\pm 1.45 \times 10^{-4}$	4.092×10^{-3}	$\pm 1.60 \times 10^{-4}$	f delay observed
[W W > 0], 2E[W W	$ASE(RCS - \sqrt{s})$	8.415×10^{-3}	$\pm 3.13 imes 10^{-4}$	5.246×10^{-3}	$\pm 1.32 \times 10^{-4}$	$3.314 imes 10^{-3}$	$\pm 6.14 \times 10^{-5}$	$2.810 imes10^{-3}$	$\pm 4.07 \times 10^{-5}$	$2.254 imes 10^{-3}$	$\pm 6.53 imes 10^{-5}$	tional on the level o
tal delays in $(E[$	ASE(RCS)	$8.209 imes 10^{-3}$	$\pm 3.09 imes 10^{-4}$	$5.026 imes10^{-3}$	$\pm 1.28 \times 10^{-4}$	$3.111 imes 10^{-3}$	$\pm 5.62 \times 10^{-5}$	$2.650 imes 10^{-3}$	$\pm 3.96 \times 10^{-5}$	$2.152 imes 10^{-3}$	$\pm 5.99 \times 10^{-5}$	stimators condit
$s = 100 \ for \ actual$	ASE(HOL)	6.814×10^{-3}	$\pm 2.89 \times 10^{-4}$	$3.544 imes 10^{-3}$	$\pm 1.21 imes 10^{-4}$	1.476×10^{-3}	$\pm 2.95 \times 10^{-5}$	$1.080 imes 10^{-3}$	$\pm 1.79 \times 10^{-5}$	$7.757 imes 10^{-4}$	$\pm 2.16 \times 10^{-5}$	al-time delay e
1/s model with .	ASE(LES)	6.918×10^{-3}	$\pm 2.91 imes 10^{-4}$	$3.655 imes 10^{-3}$	$\pm 1.22 imes 10^{-4}$	$1.605 imes 10^{-3}$	$\pm 3.09 imes 10^{-5}$	$1.223 imes 10^{-3}$	$\pm 1.77 imes 10^{-5}$	$9.405 imes 10^{-4}$	$2.29 imes 10^{-5}$	ev of different re
4SE in the D/N	ASE(QL)	6.649×10^{-3}	$\pm 2.84 \times 10^{-4}$	$3.376 imes 10^{-3}$	$\pm 1.17 imes 10^{-4}$	$1.307 imes 10^{-3}$	$\pm 2.71 imes 10^{-5}$	$9.116 imes 10^{-4}$	$\pm 1.59 imes 10^{-5}$	$6.199 imes 10^{-4}$	$\pm 1.74 \times 10^{-5}$	n of the efficienc
Conditional 1	$E[\widetilde{W W} > 0]$	4.812×10^{-1}	$\pm 1.92 \times 10^{-2}$	2.475×10^{-1}	$\pm 8.30 \times 10^{-3}$	$1.003 imes 10^{-1}$	$\pm 2.04 \times 10^{-3}$	7.242×10^{-2}	$\pm 1.29 \times 10^{-3}$	$5.173 imes10^{-2}$	$\pm 1.25 \times 10^{-3}$	2: A comparison
	β	0.99		0.98		0.95		0.93		0.90		Pable 45

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		$^{-1}$	1)-2	-2	$)^{-3}$	-2	$)^{-4}$	-3)-4	
<pre>/>> 0])</pre>	ASE(NI)	7.567×10^{-8} 8 45 \times 10 ⁻¹	1 00 < 10	$\pm 1.30 \times 10$	3.160 imes 10	$\pm 1.48 \times 10$	1.647×10^{-10}	$\pm 5.47 \times 10$	8.367 imes 10	$\pm 3.90 \times 10$	
$\widetilde{W} > 0], 4E[\widetilde{W} \widetilde{H}$	ASE(LCS)	2.436×10^{-2} 1 503 \times 10 ⁻³	1.000×10^{-2}	$\pm 6.63 \times 10^{-4}$	$1.637 imes 10^{-2}$	$\pm 4.02 \times 10^{-4}$	1.403×10^{-2}	$\pm 3.66 \times 10^{-4}$	1.084×10^{-2}	$\pm 4.49 \times 10^{-4}$	
l delays in $(2E[W])$	$ASE(RCS - \sqrt{s})$	1.482×10^{-2} 1 001 $<$ 10 ⁻³	$\frac{1.001 \times 10}{8 \text{sth} \times 10^{-3}}$	$\pm 3.31 \times 10^{-4}$	$5.065 imes 10^{-3}$	$\pm 8.24 imes 10^{-5}$	4.584×10^{-3}	$\pm 9.14 \times 10^{-5}$	4.264×10^{-3}	1.02×10^{-4}	
$s = 100 \ for \ actua$	ASE(RCS)	$1.461 imes 10^{-2}$ 0.05 $ imes$ 10 ⁻⁴	8 980 × 10-3	$\pm 3.29 imes 10^{-4}$	$4.782 imes 10^{-3}$	$\pm 7.82 imes 10^{-5}$	4.281×10^{-3}	$\pm 8.43 \times 10^{-5}$	$3.998 imes 10^{-3}$	1.032×10^{-4}	
M/s model with s	ASE(HOL)	$1.321 imes 10^{-2}$ 0.27 $ imes$ 10 ⁻⁴	6.786×10^{-3}	$\pm 2.86 \times 10^{-4}$	2.887×10^{-3}	$\pm 6.336 \times 10^{-5}$	$2.149 imes 10^{-3}$	$\pm 4.17 \times 10^{-5}$	$1.603 imes 10^{-3}$	$\pm 4.03 \times 10^{-5}$	
ASE in the D/N	ASE(LES)	1.332×10^{-2} +0 33 × 10 ⁻⁴	$\pm 0.00 \times 10^{-3}$ 6 001 $\times 10^{-3}$	$\pm 2.88 \times 10^{-4}$	3.026×10^{-3}	$\pm 6.55 imes 10^{-5}$	$2.307 imes 10^{-3}$	$\pm 4.56 \times 10^{-5}$	1.796×10^{-3}	$\pm 4.57 \times 10^{-5}$	
Conditional .	ASE(QL)	1.304×10^{-2} +0.16 × 10^{-4}	$\frac{-100}{6} \times 10^{-3}$	$\pm 2.89 \times 10^{-4}$	2.729×10^{-3}	$\pm 6.39 imes 10^{-5}$	1.988×10^{-3}	$\pm 4.51 \times 10^{-5}$	1.436×10^{-3}	$\pm 4.63 \times 10^{-5}$	
	θ	0.99	0 08		0.95		0.93		0.9		

rs conditional on the level of delay observed for the $D/M/s$	ort point estimates for the conditional average squared error	with the half width of the 95 percent confidence interval.
3: A comparison of the efficiency of different real-time delay estimators c	with $s = 100$ and $\mu = 1$ as a function of the traffic intensity ρ . We report 1	in the interval $(2E[W W > 0], 4E[W W > 0])$. Each estimate is shown with
Table 43: A con	queue with $s = j$	(ASE) in the inte

> 0])	ASE(NI)	3.182	$\pm 2.28 \times 10^{-1}$	$e e n n \sim 1 n^{-1}$	$\pm 8.20 imes 10^{-2}$	1.384×10^{-1}	$\pm 5.05 imes 10^{-3}$	$7.239 imes 10^{-2}$	$\pm 2.68 \times 10^{-3}$	$3.709 imes 10^{-2}$	$\pm 1.65 imes 10^{-3}$	
V > 0], 6E[W W	ASE(LCS)	3.432×10^{-2}	$\pm 1.63 imes 10^{-3}$	9 511 $\sim 10-2$	$\pm 1.103 \times 10^{-3}$	2.517×10^{-2}	$\pm 1.49 \times 10^{-3}$	$2.630 imes 10^{-2}$	$\pm 1.60 imes 10^{-3}$	2.363×10^{-2}	$\pm 1.34 \times 10^{-3}$	
l delays in $(4E[W V$	$ASE(RCS - \sqrt{s})$	2.462×10^{-2}	$\pm 1.15 \times 10^{-3}$	$1/429 \sim 10-2$	$\pm 6.23 imes 10^{-4}$	$7.821 imes 10^{-3}$	$\pm 5.78 imes 10^{-4}$	$6.980 imes 10^{-3}$	$\pm 4.21 \times 10^{-4}$	$6.653 imes 10^{-3}$	$\pm 4.02 \times 10^{-4}$	
$s = 100 \ for \ actua$	ASE(RCS)	2.442×10^{-2}	$\pm 1.15 imes 10^{-3}$	$1 108 \sim 10^{-2}$	$\pm 6.324 imes 10^{-4}$	$7.528 imes 10^{-3}$	$\pm 5.68 imes 10^{-4}$	$6.576 imes 10^{-3}$	$\pm 4.14 imes 10^{-4}$	$6.214 imes 10^{-3}$	$\pm 3.69 imes 10^{-4}$	
M/s model with	ASE(HOL)	$2.299 imes 10^{-2}$	$\pm 1.09 imes 10^{-3}$	1.952×10^{-2}	$\pm 5.69 \times 10^{-4}$	$5.390 imes10^{-3}$	$\pm 3.06 imes 10^{-4}$	$4.107 imes 10^{-3}$	$\pm 1.51 imes 10^{-4}$	$3.092 imes 10^{-3}$	$\pm 1.38 imes 10^{-4}$	
ASE in the D/i	ASE(LES)	$2.309 imes 10^{-2}$	$\pm 1.20 imes 10^{-3}$	1.964×10^{-2}	$\pm 5.72 \times 10^{-4}$	$5.547 imes10^{-3}$	$\pm 3.16 imes 10^{-4}$	$4.287 imes 10^{-3}$	$\pm 1.63 imes 10^{-4}$	3.3197×10^{-3}	$\pm 1.49 imes 10^{-4}$	
Conditional	ASE(QL)	2.335×10^{-2}	$\pm 1.15 imes 10^{-3}$	1.977×10^{-2}	$\pm 5.86 \times 10^{-4}$	$5.707 imes 10^{-3}$	$\pm 3.50 imes 10^{-4}$	4.426×10^{-3}	$\pm 1.78 imes 10^{-4}$	$3.394 imes 10^{-3}$	$\pm 1.50 imes 10^{-4}$	
	θ	0.99		0 00	00.0	0.95		0.93		0.90		

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Table \cdot	dueue	(ASE)

	11	I							
	ASE(NI)	2.0537	$1.50 imes10^{-1}$	3.550×10^{-1}	$\pm 2.26 \times 10^{-2}$	1.849×10^{-2}	$\pm 1.84 \times 10^{-2}$	9.402×10^{-2}	$\pm 1.52 \times 10^{-2}$
6E[W W > 0]	ASE(LCS)	$3.167 imes 10^{-2}$	$6.83 imes 10^{-3}$	2.970×10^{-2}	$\pm 6.70 \times 10^{-3}$	$3.527 imes 10^{-2}$	$\pm 6.92 \times 10^{-3}$	3.986×10^{-2}	$\pm 5.39 imes 10^{-3}$
for actual delays >	$ASE(RCS - \sqrt{s})$	1.981×10^{-2}	$\pm 3.42 \times 10^{-3}$	$1.032 imes 10^{-2}$	$\pm 1.16 imes 10^{-3}$	$9.711 imes 10^{-3}$	$\pm 1.05 \times 10^{-3}$	$9.994 imes 10^{-3}$	$\pm 1.53 imes 10^{-3}$
<i>del with</i> $s = 100$	ASE(RCS)	1.952×10^{-2}	$\pm 3.41 \times 10^{-3}$	$9.960 imes 10^{-3}$	$\pm 1.05 \times 10^{-3}$	$9.382 imes 10^{-3}$	$\pm 1.00 \times 10^{-3}$	$9.406 imes 10^{-3}$	$\pm 1.40 \times 10^{-3}$
the $D/M/s$ moo	ASE(HOL)	$1.780 imes 10^{-2}$	$\pm 2.87 \times 10^{-3}$	7.804×10^{-3}	$\pm 4.12 \times 10^{-4}$	6.439×10^{-3}	$\pm 5.21 imes 10^{-4}$	$5.103 imes10^{-3}$	$\pm 5.79 imes 10^{-4}$
itional ASE in	ASE(LES)	1.791×10^{-2}	$\pm 2.87 \times 10^{-3}$	7.954×10^{-3}	$\pm 4.46 \times 10^{-4}$	$6.65 imes 10^{-3}$	$\pm 5.67 \times 10^{-4}$	$5.353 imes 10^{-3}$	$\pm 6.46 \times 10^{-4}$
Cond	ASE(QL)	1.965×10^{-2}	$\pm 3.03 imes 10^{-3}$	$9.421 imes 10^{-3}$	$\pm 4.31 \times 10^{-4}$	$8.018 imes 10^{-3}$	$\pm 5.23 imes 10^{-4}$	$6.789 imes 10^{-3}$	$\pm 5.85 imes 10^{-4}$
	θ	0.98		0.95		0.93		0.90	

mal on the level of delay observed for the $D/M/s$	stimates for the conditional average squared error	1 of the 95 percent confidence interval.
: A comparison of the efficiency of different real-time delay estimators conditional on	th $s = 100$ and $\mu = 1$ as a function of the traffic intensity ρ . We report point estimat	ir delays larger than $6E[W W > 0]$). Each estimate is shown with the half width of the
Table 4	queue v	(ASE)



Figure 42: Conditional ASE for the alternative delay estimators in the D/M/100 model for actual delays in $(\widehat{E[W|W>0]}, 2\widehat{E[W|W>0]})$, as a function of the traffic intensity ρ



Figure 43: Conditional ASE for the alternative delay estimators in the D/M/100 model for actual delays in $(2E[\widehat{W|W} > 0], 4E[\widehat{W|W} > 0])$, as a function of the traffic intensity ρ



Figure 44: Conditional ASE for the alternative delay estimators in the D/M/100 model for actual delays in $(4E[\widehat{W|W} > 0], 6E[\widehat{W|W} > 0])$, as a function of the traffic intensity ρ



Figure 45: Conditional ASE for the alternative delay estimators in the D/M/100 model for actual delays larger than $6E[\widehat{W|W} > 0]$, as a function of the traffic intensity ρ

[> 0])	ASE(LCS) ASE(NI)	$3.040 imes10^{-3}$	$\pm 8.88 imes 10^{-5}$	6 0 1 0 0 0 0	$2.286 imes 10^{-3}$	$\pm 1.00 imes 10^{-4}$	$9.087 imes 10^{-4}$	$\pm 6.55 imes 10^{-5}$	$5.798 imes 10^{-4}$	$\pm4.57 imes10^{-5}$	$3.035 imes 10^{-4}$	$\pm 6.97 imes 10^{-5}$	
V W > 0], 2E[W W	$ASE(RCS - \sqrt{s})$	6.263×10^{-4}	$\pm 2.02 imes 10^{-5}$		4.339×10^{-4}	$\pm 1.22 imes 10^{-5}$	$2.982 imes 10^{-4}$	$\pm 9.30 \times 10^{-6}$	$2.501 imes 10^{-4}$	$\pm 1.16 imes 10^{-5}$	$1.818 imes 10^{-4}$	$\pm 2.80 imes 10^{-5}$	
d delays in $(E[W]$	ASE(RCS)	5.988×10^{-4}	$\pm 2.01 imes 10^{-5}$		4.049×10^{-4}	$\pm 1.21 imes 10^{-5}$	$2.778 imes 10^{-4}$	$\pm 8.98 \times 10^{-6}$	$2.367 imes 10^{-4}$	$\pm 1.08 imes 10^{-5}$	$1.738 imes 10^{-4}$	$\pm 2.41 \times 10^{-5}$	
$= 400 \ for \ actua$	ASE(HOL)	4.301×10^{-4}	$\pm 1.85 imes 10^{-5}$		2.235×10^{-4}	$\pm 9.54 \times 10^{-6}$	$9.215 imes 10^{-5}$	$\pm 4.35 \times 10^{-6}$	$6.897 imes 10^{-5}$	$\pm 3.60 imes 10^{-6}$	$4.621 imes 10^{-5}$	$\pm 6.21 \times 10^{-6}$	
/s model with s	ASE(LES)	4.367×10^{-4}	$\pm 1.85 imes 10^{-5}$		2.304×10^{-4}	$\pm 9.61 imes 10^{-6}$	$1.003 imes 10^{-4}$	$\pm 4.29 imes 10^{-6}$	$7.804 imes 10^{-5}$	$\pm 3.57 imes 10^{-6}$	$5.615 imes10^{-5}$	$\pm 6.28 imes 10^{-6}$	
NSE in the D/M	ASE(QL)	4.197×10^{-4}	$\pm 1.82 imes 10^{-5}$		$2.130 imes 10^{-4}$	$\pm 9.31 imes 10^{-6}$	8.154×10^{-5}	$\pm 4.07 \times 10^{-6}$	$5.830 imes10^{-5}$	$\pm 3.25 imes 10^{-6}$	$3.700 imes 10^{-5}$	$\pm 5.029 \times 10^{-6}$	
Conditional A	E[W]W > 0]	1.212×10^{-1}	$\pm 5.38 imes 10^{-3}$		6.214×10^{-4}	$\pm 2.56 imes 10^{-3}$	2.497×10^{-2}	$\pm 1.18 imes 10^{-3}$	1.834×10^{-2}	$\pm 8.68 \times 10^{-4}$	$1.258 imes 10^{-2}$	$\pm 1.478 \times 10^{-3}$	
	σ	0.99		(0.98		0.95		0.93		0.9		

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V > 0])	ASE(NI)	4.858×10^{-2}	$\pm 6.58 \times 10^{-3}$	1.204×10^{-2}	$\pm 1.016 \times 10^{-3}$	$1.960 imes 10^{-3}$	$\pm 1.85 imes 10^{-4}$	$1.069 imes 10^{-3}$	$\pm 1.022 \times 10^{-4}$	$5.099 imes 10^{-4}$	$\pm 1.15 imes 10^{-4}$	
W > 0], 4E[W V	ASE(LCS)	$3.770 imes 10^{-3}$	$\pm 1.85 imes 10^{-4}$	$3.614 imes 10^{-3}$	$\pm 1.53 imes 10^{-4}$	$2.322 imes 10^{-3}$	$\pm 1.51 imes 10^{-4}$	$1.638 imes 10^{-3}$	$\pm 1.067 \times 10^{-4}$	$1.008 imes 10^{-3}$	$\pm 2.04 imes 10^{-4}$	
ual delays in $(2E[ec{W}]$	$ASE(RCS - \sqrt{s})$	$1.050 imes 10^{-3}$	$\pm 5.59 imes 10^{-5}$	6.49×10^{-4}	$\pm 2.20 imes 10^{-5}$	$4.532 imes 10^{-4}$	$\pm 1.11 imes 10^{-5}$	$4.309 imes 10^{-4}$	$\pm 2.07 imes 10^{-5}$	$3.655 imes 10^{-4}$	$\pm 5.73 imes 10^{-5}$	
$s = 400 \ for \ acta$	ASE(RCS)	$1.021 imes 10^{-3}$	$\pm 5.41 \times 10^{-5}$	$6.172 imes 10^{-4}$	$\pm 2.25 imes 10^{-5}$	4.143×10^{-4}	$\pm 8.48 \times 10^{-6}$	$3.921 imes 10^{-4}$	$\pm 1.85 imes 10^{-5}$	$3.382 imes 10^{-4}$	$\pm 5.19 \times 10^{-5}$	
M/s model with	ASE(HOL)	8.479×10^{-4}	$\pm 5.23 imes 10^{-5}$	4.311×10^{-4}	$\pm 1.99 \times 10^{-5}$	$1.810 imes 10^{-4}$	$\pm 8.49 \times 10^{-6}$	$1.362 imes 10^{-4}$	$\pm 4.70 \times 10^{-6}$	$9.450 imes 10^{-5}$	$\pm 1.35 \times 10^{-5}$	
ASE in the D/i	ASE(LES)	8.547×10^{-4}	$\pm 5.24 \times 10^{-5}$	4.382×10^{-4}	$\pm 2.00 \times 10^{-5}$	$1.895 imes 10^{-4}$	$\pm 8.67 \times 10^{-6}$	1.465×10^{-4}	$\pm 4.76 \times 10^{-6}$	$1.060 imes 10^{-4}$	$\pm 1.44 \times 10^{-5}$	
Conditional .	ASE(QL)	8.373×10^{-4}	$\pm 5.22 imes 10^{-5}$	4.214×10^{-4}	$\pm 2.02 \times 10^{-5}$	$1.710 imes 10^{-4}$	$\pm9.11\times10^{-6}$	1.264×10^{-4}	$\pm 5.59 \times 10^{-6}$	$8.507 imes10^{-5}$	$\pm 1.43 \times 10^{-5}$	
	σ	0.99		0.98		0.95		0.93		0.9		.

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7 > 0])	ASE(NI)	1.981×10^{-1}	$\pm 1.72 \times 10^{-2}$	E 160 10-2	1.400×10 $\pm 5.92 \times 10^{-4}$	10.20×10	8.892×10^{-3}	$\pm 1.00 \times 10^{-3}$	c	4.525×10^{-3}	$\pm 4.48 \times 10^{-4}$	$2.101 imes 10^{-3}$	$\pm 6.53 imes 10^{-4}$		
V > 0], 6E[W]W	ASE(LCS)	4.402×10^{-3}	$\pm 3.88 \times 10^{-4}$	1 210 110-3	$\pm 3.012 \times 10$ $\pm 5.90 \times 10^{-4}$	$T 0.73 \times 10$	4.796×10^{-3}	$\pm 1.88 \times 10^{-4}$	c	4.051×10^{-3}	$\pm 3.87 imes 10^{-4}$	$2.797 imes 10^{-3}$	$\pm 7.33 imes 10^{-4}$		
l delays in $(4E[W])$	$ASE(RCS - \sqrt{s})$	1.635×10^{-3}	$\pm 1.16 imes 10^{-4}$	0,000,0010-4	3.303×10 $\pm 1.18 \times 10^{-5}$	Т4.10 × 1U	$6.308 imes 10^{-4}$	$\pm 3.71 imes 10^{-5}$		$6.412 imes 10^{-4}$	$\pm 1.17 imes 10^{-4}$	$9.401 imes 10^{-4}$	$\pm 3.72 imes 10^{-4}$		
$= 400 \ for \ actua$	ASE(RCS)	1.608×10^{-3}	$\pm 1.17 imes 10^{-4}$	4001 10-4	9.034×10 $\pm 4.75 \sim 10^{-5}$	\pm 4.10 \wedge 10	$5.831 imes 10^{-4}$	$\pm 3.67 imes 10^{-5}$	-	$5.750 imes 10^{-4}$	$\pm 8.49 \times 10^{-5}$	$8.031 imes 10^{-4}$	$\pm 3.37 imes 10^{-4}$		
'/s model with s	ASE(HOL)	1.432×10^{-3}	$\pm 1.06 \times 10^{-4}$	7 667 \(10-4	1.001×10 $\pm 113 \times 10^{-5}$	Т. 1 .1.0 × 10	$3.238 imes 10^{-4}$	$\pm 1.87 imes 10^{-5}$	-	2.340×10^{-4}	$\pm 1.50 imes 10^{-5}$	$2.275 imes 10^{-4}$	$\pm 8.06 \times 10^{-5}$		
SE in the D/M	ASE(LES)	1.440×10^{-3}	$\pm 1.062 imes 10^{-4}$		1.142×10^{-5}		$3.346 imes 10^{-4}$	$\pm 1.87 imes 10^{-5}$		2.450×10^{-4}	$\pm 1.59 imes 10^{-5}$	$2.549 imes 10^{-4}$	$\pm 9.52 imes 10^{-5}$		
Conditional A	ASE(QL)	1.459×10^{-3}	$\pm 1.07 imes 10^{-4}$		1.041×10 $\pm 1.00 \times 10^{-5}$	T4.20 × 10	$3.415 imes 10^{-4}$	$\pm 2.05 imes 10^{-5}$		2.609×10^{-4}	$\pm 1.84 \times 10^{-5}$	$2.475 imes 10^{-4}$	$\pm 8.86 \times 10^{-5}$		
	β	0.99		000	0.30		0.95			0.93		0.9			

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Figure 46: Conditional ASE for the alternative delay estimators in the D/M/400 model for actual delays in $(\widehat{E[W|W>0]}, 2\widehat{E[W|W>0]})$, as a function of the traffic intensity ρ



Figure 47: Conditional ASE for the alternative delay estimators in the D/M/400 model for actual delays in $(2E[\widehat{W|W} > 0], 4E[\widehat{W|W} > 0])$, as a function of the traffic intensity ρ



Figure 48: Conditional ASE for the alternative delay estimators in the D/M/400 model for actual delays in $(4E[\widehat{W|W} > 0], 6E[\widehat{W|W} > 0])$, as a function of the traffic intensity ρ

	ASE(NI)	$7.251 imes10^{-4}$	$\pm 8.05 \times 10^{-5}$	$1.954 imes 10^{-4}$	$\pm 1.58 \times 10^{-5}$	$3.582 imes 10^{-5}$	$7.230 imes 10^{-6}$	$2.002 imes 10^{-5}$	6.15×10^{-6}	
ilde W[W] > 0], 2E[ilde W[W] > 0])	ASE(LCS)	$1.117 imes 10^{-3}$	$\pm 5.12 \times 10^{-5}$	$7.203 imes10^{-4}$	$\pm 4.00 \times 10^{-5}$	2.458×10^{-4}	4.465×10^{-5}	$1.416 imes 10^{-4}$	$\pm 3.86 \times 10^{-5}$	
	$ASE(RCS - \sqrt{s})$	1.445×10^{-4}	$\pm 5.44 \times 10^{-6}$	$1.072 imes 10^{-4}$	$\pm 2.63 imes 10^{-6}$	$7.550 imes10^{-5}$	$6.201 imes 10^{-6}$	$6.352 imes 10^{-5}$	$\pm 8.47 \times 10^{-6}$	
tal delays in $(E[$	ASE(RCS)	$1.363 imes 10^{-4}$	$\pm 5.27 imes 10^{-6}$	$9.869 imes 10^{-5}$	$\pm 2.57 \times 10^{-6}$	$7.079 imes 10^{-5}$	5.254×10^{-6}	$6.063 imes 10^{-5}$	$\pm 7.30 \times 10^{-6}$	
$s = 900 \ for \ actual \ det{actual}$	ASE(HOL)	$8.547 imes 10^{-5}$	$\pm 5.09 \times 10^{-6}$	4.465×10^{-5}	$\pm 2.15 \times 10^{-6}$	1.948×10^{-5}	$1.979 imes 10^{-6}$	1.419×10^{-5}	2.35×10^{-6}	
1/s model with .	ASE(LES)	$8.680 imes 10^{-5}$	$\pm 5.10 \times 10^{-6}$	4.604×10^{-5}	$\pm 2.17 \times 10^{-6}$	$2.104 imes 10^{-5}$	1.983×10^{-6}	$1.595 imes 10^{-5}$	$2.27 imes 10^{-6}$	
ISE in the D/N	ASE(QL)	$8.339 imes 10^{-5}$	$\pm 5.01 \times 10^{-6}$	4.254×10^{-5}	$\pm 2.09 \times 10^{-6}$	$1.732 imes 10^{-5}$	$1.853 imes 10^{-6}$	$1.214 imes 10^{-5}$	$\pm 2.08 \times 10^{-6}$	
Conditional A	E[W W > 0]	5.391×10^{-2}	$\pm 3.09 \times 10^{-3}$	$2.782 imes 10^{-2}$	$\pm 1.23 \times 10^{-3}$	1.1972×10^{-2}	$1.275 imes 10^{-3}$	$8.552 imes 10^{-3}$	$\pm 1.39 \times 10^{-3}$	
	β	0.99		0.98		0.95		0.93		

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Table 4	dueue v	(ASE)

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V > 0])	ASE(NI)	9.410×10^{-3}	$\pm 1.214 \times 10^{-1}$	2.486×10^{-3}	$\pm 2.56 \times 10^{-4}$	$4.66 imes 10^{-4}$	$\pm 1.06 \times 10^{-4}$	2.152×10^{-4}	$6.52 imes 10^{-5}$
$r > 0], 4E[\tilde{W} W$	ASE(LCS)	$1.522 imes 10^{-3}$	$\pm 9.62 imes 10^{-5}$	1.404×10^{-3}	$\pm 8.40 \times 10^{-5}$	$7.120 imes 10^{-4}$	$\pm 1.09 imes 10^{-4}$	4.251×10^{-4}	$1.06 imes 10^{-4}$
al delays in $(2E[ilde W]V$	$ASE(RCS - \sqrt{s})$	2.248×10^{-4}	$\pm 1.25 imes 10^{-5}$	$1.514 imes 10^{-4}$	$\pm 4.69 \times 10^{-6}$	$1.23493 imes 10^{-4}$	$\pm 6.39 \times 10^{-6}$	$9.681 imes 10^{-5}$	$\pm 9.56 \times 10^{-6}$
s = 900 for actual	ASE(RCS)	$2.163 imes 10^{-4}$	$\pm 1.25 imes 10^{-5}$	$1.417 imes 10^{-4}$	$\pm 4.26 \times 10^{-6}$	$1.11378 imes 10^{-4}$	$\pm 5.902 \times 10^{-6}$	$9.176 imes 10^{-5}$	$\pm 1.04 \times 10^{-5}$
'M/s model with	ASE(HOL)	$1.660 imes 10^{-4}$	$\pm 1.16 \times 10^{-5}$	8.641×10^{-5}	$\pm 4.19 \times 10^{-6}$	$3.862 imes 10^{-5}$	$\pm 4.75 \times 10^{-6}$	2.734×10^{-5}	$\pm 4.87 \times 10^{-6}$
ASE in the $D/$	ASE(LES)	$1.674 imes 10^{-4}$	$\pm 1.16 \times 10^{-5}$	$8.783 imes 10^{-5}$	$\pm 4.19 \times 10^{-6}$	4.034×10^{-5}	$\pm 4.75 \times 10^{-6}$	2.918×10^{-5}	$\pm 4.88 \times 10^{-6}$
Conditional	ASE(QL)	1.640×10^{-4}	$\pm 1.17 \times 10^{-5}$	8.442×10^{-5}	$\pm 4.18 \times 10^{-6}$	$3.684 imes 10^{-5}$	$\pm 4.98 \times 10^{-6}$	$2.605 imes 10^{-5}$	$\pm 5.32 \times 10^{-6}$
	β	0.99		0.98		0.95		0.93	

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he efficiency	1 as a functi	$ \overline{W} > 0], 4E[V$
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lable 50: A co	ueue with $s =$	ASE) in the in



x 10⁻⁴ DM900 – Conditional ASE for delays in (E[W|W>0], 2E[W|W>0])

Figure 49: Conditional ASE for the alternative delay estimators in the D/M/900 model for actual delays in $(\widehat{E[W|W>0]}, 2\widehat{E[W|W>0]})$, as a function of the traffic intensity ρ



Figure 50: Conditional ASE for the alternative delay estimators in the D/M/900 model for actual delays in $(2E[\widehat{W|W} > 0], 4E[\widehat{W|W} > 0])$, as a function of the traffic intensity ρ

	ASE(NI)	4.222×10^{-1}	$\pm 7.18 \times 10^{-2}$	$5.927 imes 10^{-2}$	$\pm 3.74 \times 10^{-3}$	$2.982 imes 10^{-2}$	$\pm 1.43 \times 10^{-3}$	$1.411 imes 10^{-2}$	$\pm 4.22 \times 10^{-4}$	
$\widetilde{V}[W>0], 2E[\widetilde{W} W>0])$	ASE(LCS)	$1.381 imes 10^{-1}$	$\pm 8.43 \times 10^{-3}$	7.849×10^{-2}	$\pm 1.31 \times 10^{-3}$	$6.222 imes 10^{-2}$	$\pm 1.34 \times 10^{-3}$	4.563×10^{-2}	$\pm 8.59 \times 10^{-4}$	
	$ASE(RCS - \sqrt{s})$	$9.481 imes 10^{-2}$	$7.78 imes 10^{-3}$	$3.819 imes 10^{-2}$	$\pm 7.69 imes 10^{-4}$	2.826×10^{-2}	$\pm 6.61 imes 10^{-4}$	$2.075 imes 10^{-2}$	$\pm 3.88 \times 10^{-4}$	
ual delays in $(E[$	ASE(RCS)	$9.381 imes 10^{-2}$	$\pm 7.80 \times 10^{-3}$	$3.720 imes 10^{-2}$	$\pm 7.70 imes 10^{-4}$	$2.729 imes 10^{-2}$	$\pm 6.39 \times 10^{-4}$	1.987×10^{-2}	$\pm 3.77 \times 10^{-4}$	
$s = 100 \ for \ actual$	ASE(HOL)	8.740×10^{-2}	$\pm 7.780 \times 10^{-3}$	3.0739×10^{-2}	$\pm 8.15 \times 10^{-4}$	2.089×10^{-2}	$\pm 5.72 imes 10^{-4}$	1.351×10^{-2}	$\pm 2.89 \times 10^{-4}$	
M/s model with	ASE(LES)	$8.789 imes 10^{-2}$	$\pm 7.78 \times 10^{-3}$	$3.123 imes 10^{-1}$	$\pm 8.15 \times 10^{-4}$	$2.137 imes 10^{-2}$	$\pm 5.72 imes 10^{-4}$	$1.399 imes 10^{-2}$	$\pm 2.96 \times 10^{-4}$	
$4SE$ in the $H_2/$	ASE(QL)	$1.804 imes 10^{-2}$	$\pm 1.54 \times 10^{-3}$	$6.728 imes 10^{-3}$	$\pm 2.05 imes 10^{-4}$	4.750×10^{-3}	$\pm 1.32 imes 10^{-4}$	3.226×10^{-3}	$\pm 6.53 \times 10^{-5}$	
Conditional ,	E[W]W > 0]	1.283	$\pm 1.13 \times 10^{-1}$	4.838×10^{-1}	$\pm 1.54 \times 10^{-2}$	$3.430 imes 10^{-1}$	$\pm 8.45 \times 10^{-3}$	$2.3620 imes 10^{-1}$	$\pm 3.83 \times 10^{-3}$	
	β	0.98		0.95		0.93		0.9		

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· _> 0])	ASE(NI)	5.513	1.02	7.268×10^{-1}	$\pm 4.55 \times 10^{-2}$	$3.640 imes10^{-1}$	$\pm 1.66 imes 10^{-2}$	1.744×10^{-1}	$\pm 0.20 \times 10^{\circ}$
$V > 0], 4E[\overline{W} W$	ASE(LCS)	2.270×10^{-1}	$\pm 1.80 \times 10^{-2}$	1.170×10^{-1}	$\pm 2.44 \times 10^{-3}$	$10.00 imes 10^{-2}$	$\pm 1.98 \times 10^{-3}$	$8.357 imes 10^{-2}$	_ 01 × 10.1Ξ
A delays in $(2E[\widetilde{W}])$	$ASE(RCS - \sqrt{s})$	1.818×10^{-1}	$\pm 1.73 \times 10^{-2}$	$6.800 imes 10^{-2}$	$\pm 1.57 imes 10^{-3}$	4.963×10^{-2}	$\pm 1.13 imes 10^{-3}$	$3.565 imes 10^{-2}$, 10^{-4}	$_{-}$ 01 × 60.1 \pm
$= 100 \ for \ actua$	ASE(RCS)	$1.807 imes 10^{-1}$	$\pm 1.73 \times 10^{-2}$	$6.692 imes 10^{-2}$	$\pm 1.53 \times 10^{-3}$	4.854×10^{-2}	$\pm 1.12 imes 10^{-3}$	$3.449 imes 10^{-2}$	±1.31 × 10 -
I/s model with s =	ASE(HOL)	1.739×10^{-1}	$\pm 1.719 \times 10^{-2}$	$6.000 imes 10^{-2}$	$\pm 1.65 \times 10^{-3}$	4.156×10^{-2}	$\pm 1.08 \times 10^{-3}$	2.734×10^{-2}	- 01 × 60.6Ξ
ASE in the H_2/l	ASE(LES)	1.744×10^{-1}	$\pm 1.72 imes 10^{-2}$	$6.052 imes 10^{-2}$	$\pm 1.642 \times 10^{-3}$	4.210×10^{-2}	$\pm 1.08 \times 10^{-3}$	2.788×10^{-2}	$\pm 0.92 \times 10^{-2}$
Conditional 1	ASE(QL)	$3.596 imes 10^{-2}$	$\pm 3.88 \times 10^{-3}$	1.296×10^{-2}	$\pm 4.66 \times 10^{-4}$	9.234×10^{-3}	$\pm 2.52 imes 10^{-4}$	$6.391 imes 10^{-3}$	王 177 × 10
	θ	0.98		0.95		0.93		0.9	

it real-time delay estimators conditional on the level of delay observed for the $H_2/M/s$	traffic intensity ρ . We report point estimates for the conditional average squared error). Each estimate is shown with the half width of the 95 percent confidence interval.
sney of different real-time delay estimators conc	unction of the traffic intensity ρ . We report po	4E[W W > 0]). Each estimate is shown with t
Table 52: A comparison of the efficie	queue with $s = 100$ and $\mu = 1$ as a fi	(ASE) in the interval $(2E[W W > 0])$,

V > 0])	ASE(NI)	21.34	± 4.21	3.305	$2.26 imes 10^{-1}$	1.654	$\pm 7.37 imes 10^{-2}$	$7.745 imes 10^{-1} \pm 2.59 imes 10^{-2}$	
$W \ge 0], 6E[\tilde{W} W$	ASE(LCS)	3.348×10^{-1}	$\pm 5.16 \times 10^{-2}$	$1.706 imes 10^{-1}$	$\pm 9.69 imes 10^{-3}$	1.466×10^{-1}	$\pm 7.65 \times 10^{-3}$	1.291×10^{-1} $\pm 5.48 \times 10^{-3}$	
al delays in $(4E[\tilde{W}])$	$ASE(RCS - \sqrt{s})$	$2.904 imes 10^{-1}$	$\pm 4.83 \times 10^{-2}$	$1.201 imes 10^{-1}$	$\pm 7.31 imes 10^{-3}$	$8.661 imes 10^{-2}$	$\pm 4.66 \times 10^{-3}$	$6.276 imes 10^{-2}$ $3.082 imes 10^{-3}$	
$s = 100 \ for \ actu$	ASE(RCS)	$2.894 imes 10^{-1}$	$\pm 4.81 \times 10^{-2}$	$1.190 imes 10^{-1}$	$\pm 7.30 \times 10^{-3}$	8.548×10^{-2}	$\pm 4.66 \times 10^{-3}$	$6.140 imes 10^{-2} \pm 3.00 imes 10^{-3}$	
M/s model with :	ASE(HOL)	2.831×10^{-1}	$\pm 4.71 \times 10^{-2}$	$1.116 imes 10^{-1}$	$\pm 6.96 \times 10^{-3}$	7.7659×10^{-2}	$\pm 4.26 \times 10^{-3}$	$5.307 imes 10^{-2} \pm 2.36 imes 10^{-3}$	
SE in the H_2/N	ASE(LES)	$2.836 imes 10^{-1}$	$\pm 4.72 \times 10^{-2}$	$1.122 imes 10^{-1}$	$\pm 7.04 \times 10^{-3}$	7.824×10^{-2}	$\pm 4.30 \times 10^{-3}$	$5.373 imes 10^{-2} \pm 2.37 imes 10^{-3}$	
Conditional A	ASE(QL)	$5.359 imes 10^{-2}$	$\pm 6.15 \times 10^{-3}$	$2.376 imes 10^{-2}$	$\pm 1.64 \times 10^{-3}$	$1.696 imes 10^{-2}$	$\pm 8.96 \times 10^{-4}$	$\begin{array}{c} 1.184 \times 10^{-2} \\ \pm 3.24 \times 10^{-4} \end{array}$	
	θ	0.98		0.95		0.93		0.90	

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compar	s = 100	e interva
e 53: A	e with s	\overline{c}) in the
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					0^{-1}		0^{-1}	
	ASE(NI)	8.023	± 1.11	4.280	$\pm 4.31 \times 10$	1.977	$\pm 1.11 \times 10$	
$6E[\widetilde{W W}>0]$	ASE(LCS)	2.454×10^{-1}	$\pm 5.08 imes 10^{-2}$	$1.962 imes 10^{-1}$	$\pm 2.54 \times 10^{-2}$	1.849×10^{-1}	$\pm 1.30 \times 10^{-2}$	
for actual delays >	$ASE(RCS - \sqrt{s})$	1.832×10^{-1}	$\pm 3.97 imes 10^{-2}$	$1.353 imes 10^{-1}$	$\pm 1.59 imes 10^{-2}$	$1.037 imes 10^{-1}$	$\pm 7.64 imes 10^{-3}$	
del with $s = 100$	ASE(RCS)	1.821×10^{-1}	$\pm 3.97 imes 10^{-2}$	$1.340 imes 10^{-1}$	$\pm 1.56 imes 10^{-2}$	$1.019 imes 10^{-1}$	$\pm 7.60 imes 10^{-3}$	
the $H_2/M/s$ mo	ASE(HOL)	1.734×10^{-1}	$\pm 3.83 imes 10^{-2}$	$1.263 imes 10^{-1}$	$\pm 1.43 \times 10^{-2}$	9.158×10^{-2}	$\pm 7.55 imes 10^{-3}$	
$itional \ ASE \ in$	ASE(LES)	1.741×10^{-1}	$\pm 3.85 imes 10^{-2}$	$1.270 imes10^{-1}$	$\pm 1.44 \times 10^{-2}$	$9.237 imes 10^{-2}$	$\pm 7.55 imes 10^{-3}$	
Cond	ASE(QL)	$3.210 imes 10^{-2}$	$\pm 7.23 imes 10^{-3}$	2.661×10^{-2}	$\pm 1.97 \times 10^{-3}$	1.884×10^{-2}	$\pm 2.73 \times 10^{-3}$	
	θ	0.95		0.93		0.9		

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Figure 51: Conditional ASE for the alternative delay estimators in the $H_2/M/100$ model for actual delays in $(E[\widehat{W|W} > 0], 2E[\widehat{W|W} > 0])$, as a function of the traffic intensity ρ



Figure 52: Conditional ASE for the alternative delay estimators in the $H_2/M/900$ model for actual delays in $(2E[\widehat{W|W} > 0], 4E[\widehat{W|W} > 0])$, as a function of the traffic intensity ρ



Figure 53: Conditional ASE for the alternative delay estimators in the $H_2/M/100$ model for actual delays in $(4E[\widehat{W|W} > 0], 6E[\widehat{W|W} > 0])$, as a function of the traffic intensity ρ



Figure 54: Conditional ASE for the alternative delay estimators in the $H_2/M/100$ model for actual delays larger than $6E[\widehat{W|W} > 0]$, as a function of the traffic intensity ρ

	ASE(NI)	2.273×10^{-2}	$\pm 2.87 \times 10^{-3}$	$3.715 imes 10^{-3}$	$\pm 2.65 \times 10^{-4}$	$1.882 imes 10^{-3}$	$\pm 1.14 \times 10^{-4}$	$8.672 imes 10^{-4}$	$\pm 3.99 \times 10^{-5}$
<pre>/ > 0])</pre>	ASE(LCS)	$1.759 imes 10^{-2}$	$\pm 7.86 \times 10^{-4}$	$1.068 imes 10^{-2}$	$\pm 4.10 \times 10^{-4}$	$7.568 imes 10^{-3}$	$\pm 3.18 \times 10^{-4}$	4.736×10^{-3}	$\pm 2.15 \times 10^{-4}$
W W > 0], 2E W W	$ASE(RCS - \sqrt{s})$	6.01×10^{-3}	$\pm 3.26 imes 10^{-4}$	2.868×10^{-3}	$\pm 8.57 imes 10^{-5}$	$2.205 imes 10^{-3}$	$\pm 6.36 \times 10^{-5}$	$1.657 imes 10^{-3}$	$\pm 4.76 \times 10^{-5}$
tal delays in $(E]$	ASE(RCS)	$5.882 imes 10^{-3}$	$\pm 3.28 \times 10^{-4}$	2.738×10^{-3}	$\pm 8.65 \times 10^{-5}$	$2.083 imes 10^{-3}$	$\pm 5.82 \times 10^{-5}$	$1.562 imes 10^{-3}$	$\pm 4.12 \times 10^{-5}$
$s = 400 \ for \ actu$	ASE(HOL)	$5.091 imes 10^{-3}$	$\pm 3.21 \times 10^{-4}$	$1.939 imes 10^{-3}$	$\pm 7.67 \times 10^{-5}$	$1.309 imes 10^{-3}$	$\pm 5.06\times 10^{-5}$	8.283×10^{-4}	$\pm 3.07 \times 10^{-5}$
1/s model with s	ASE(LES)	5.122×10^{-3}	$\pm 3.22 \times 10^{-4}$	$1.969 imes 10^{-3}$	$\pm 7.66 imes 10^{-5}$	$1.339 imes 10^{-3}$	$\pm 5.05 imes 10^{-5}$	$8.575 imes 10^{-4}$	$\pm 3.11 \times 10^{-5}$
SE in the H_2/N	ASE(QL)	$1.056 imes 10^{-3}$	$\pm 7.32 \times 10^{-5}$	$4.293 imes 10^{-4}$	$\pm 1.78 \times 10^{-5}$	$2.977 imes 10^{-4}$	$\pm 9.76 \times 10^{-6}$	$1.981 imes 10^{-4}$	$\pm 5.65 \times 10^{-6}$
Conditional A	$E[\widetilde{W W} > 0]$	2.978×10^{-1}	$\pm 1.906 \times 10^{-2}$	$1.218 imes 10^{-1}$	$\pm 4.93 \times 10^{-3}$	8.611×10^{-2}	$\pm 2.55 \times 10^{-3}$	5.845×10^{-2}	$\pm 1.58 \times 10^{-3}$
	β	0.98		0.95		0.93		0.9	

of delay observed for the $H_2/M/s$	e conditional average squared error	5 percent confidence interval.
omparison of the efficiency of different real-time delay estimators conditional on the level of de	= 100 and $\mu = 1$ as a function of the traffic intensity ρ . We report point estimates for the con-	nterval $(E[W W > 0], 2E[W W > 0])$. Each estimate is shown with the half width of the 95 per
Table 55: A	queue with s	(ASE) in the

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7 > 0	ASE(NI)	$2.787 imes 10^{-1}$	$\pm 3.99 imes 10^{-2}$	4.616×10^{-2}	$\pm 3.516\times 10^{-3}$	$2.271 imes 10^{-3}$	$\pm 1.45 \times 10^{-3}$	$1.07 imes 10^{-2}$	$\pm 5.48 \times 10^{-4}$
W > 0], 4E[W W	ASE(LCS)	2.333×10^{-2}	$\pm 1.37 \times 10^{-3}$	$1.812 imes 10^{-2}$	$\pm 5.503 imes 10^{-4}$	$1.553 imes 10^{-2}$	$\pm 4.58 \times 10^{-4}$	$1.159 imes 10^{-2}$	$\pm 4.76 \times 10^{-4}$
tal delays in $(2E[W$	$ASE(RCS - \sqrt{s})$	1.054×10^{-2}	$\pm 8.53 \times 10^{-4}$	$4.752 imes 10^{-3}$	$\pm 2.089 \times 10^{-4}$	$3.595 imes 10^{-3}$	$\pm 1.24 \times 10^{-4}$	$2.74 imes 10^{-3}$	$\pm 1.025 imes 10^{-4}$
$i \ s = 400 \ for \ actu$	ASE(RCS)	1.041×10^{-2}	$\pm 8.47 \times 10^{-4}$	$4.616 imes 10^{-3}$	$\pm 2.105 imes 10^{-4}$	$3.450 imes10^{-3}$	$\pm 1.17 imes 10^{-4}$	$2.596 imes 10^{-3}$	$\pm 1.022 imes 10^{-4}$
/M/s model with	ASE(HOL)	$9.605 imes 10^{-3}$	$\pm 8.15 \times 10^{-4}$	$3.778 imes 10^{-3}$	$\pm 1.997 imes 10^{-4}$	$2.594 imes 10^{-3}$	$\pm 1.09 imes 10^{-4}$	$1.699 imes 10^{-3}$	$\pm 7.61 imes 10^{-5}$
ASE in the H_2	ASE(LES)	$9.638 imes 10^{-3}$	$\pm 8.16 \times 10^{-4}$	3.811×10^{-3}	$\pm 1.99 \times 10^{-4}$	$2.626 imes 10^{-3}$	$\pm 1.09 \times 10^{-4}$	$1.732 imes 10^{-3}$	$\pm 7.59 imes 10^{-5}$
Conditional	ASE(QL)	$2.012 imes 10^{-3}$	$\pm 1.35 imes 10^{-4}$	8.336×10^{-4}	$\pm 2.97 imes 10^{-5}$	$5.766 imes 10^{-4}$	$\pm 1.97 imes 10^{-5}$	$3.927 imes 10^{-4}$	$\pm 1.51 imes 10^{-5}$
	θ	0.98		0.95		0.93		0.9	

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-	Conditional ASE	\vec{s} in the $H_2/M/s$	model with $s = -$	400 for actual a	elays in $(4E[W W >$	> 0], 6E[W W > 0]	([[
θ	ASE(QL)	ASE(LES)	ASE(HOL)	ASE(RCS)	$ASE(RCS - \sqrt{s})$	ASE(LCS)	ASE(NI)
0.98	3.238×10^{-3}	1.661×10^{-2}	$1.659 imes 10^{-2}$	1.738×10^{-2}	$1.751 imes 10^{-2}$	$3.017 imes 10^{-2}$	1.193
$\pm 3.68 \times 10^{-4}$	$\pm 1.71 \times 10^{-3}$	$\pm 1.71 imes 10^{-3}$	$\pm 1.76 \times 10^{-3}$	$\pm 1.76 \times 10^{-3}$	$\pm 2.80 imes 10^{-3}$	$\pm 1.73 \times 10^{-1}$	
0.95	1.540×10^{-3}	$6.717 imes 10^{-3}$	$6.683 imes 10^{-3}$	$7.603 imes 10^{-3}$	$7.773 imes 10^{-3}$	$2.514 imes 10^{-2}$	2.064×10^{-1}
	$\pm 8.31 \times 10^{-5}$	$\pm 7.02 imes 10^{-4}$	$\pm 7.00 \times 10^{-4}$	$\pm 8.77 \times 10^{-4}$	$\pm 8.95 imes 10^{-4}$	$\pm 2.64 \times 10^{-3}$	$\pm 1.854 \times 10^{-2}$
0.93	$1.073 imes 10^{-3}$	$5.139 imes10^{-3}$	$5.099 imes 10^{-3}$	$6.217 imes 10^{-3}$	$6.421 imes 10^{-3}$	$2.722 imes 10^{-2}$	$1.014 imes 10^{-1}$
	$\pm 1.21 \times 10^{-4}$	$\pm 3.462 \times 10^{-4}$	$\pm 3.45 \times 10^{-4}$	$\pm 3.54 \times 10^{-4}$	$\pm 3.75 imes 10^{-4}$	$1.650 imes 10^{-3}$	$7.38 imes 10^{-3}$
0.9	$7.412 imes 10^{-4}$	$3.253 imes 10^{-3}$	$3.218 imes 10^{-3}$	4.174×10^{-3}	4.330×10^{-3}	2.287×10^{-2}	4.737×10^{-2}
	$\pm 6.18 \times 10^{-5}$	$\pm 2.93 imes 10^{-4}$	$\pm 2.91 \times 10^{-4}$	$\pm 3.63 \times 10^{-4}$	$\pm 4.09 \times 10^{-4}$	$\pm 1.788 \times 10^{-3}$	$\pm 3.61 \times 10^{-3}$

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	ASE(NI)	4.950×10^{-1}	$\pm 1.03 \times 10^{-1}$	2.461×10^{-1}	$5.43 imes 10^{-2}$	$1.123 imes 10^{-1}$	$\pm 1.18 \times 10^{-2}$
$6E[\widehat{W W}>0]$	ASE(LCS)	3.872×10^{-2}	$\pm 1.88 \times 10^{-2}$	$3.952 imes 10^{-2}$	$\pm 1.01 \times 10^{-2}$	$3.704 imes 10^{-2}$	$\pm 7.61 \times 10^{-3}$
for actual delays >	$ASE(RCS - \sqrt{s})$	1.355×10^{-2}	$\pm 4.48 \times 10^{-3}$	$8.022 imes 10^{-3}$	$\pm 1.56 imes 10^{-3}$	$7.381 imes 10^{-3}$	$\pm 1.51 imes 10^{-3}$
del with $s = 100$	ASE(RCS)	1.345×10^{-2}	$\pm 4.42 \times 10^{-3}$	$7.791 imes 10^{-3}$	$\pm 1.47 \times 10^{-3}$	$7.135 imes 10^{-3}$	$\pm 1.41 \times 10^{-3}$
the $H_2/M/s$ mo	ASE(HOL)	1.256×10^{-2}	$\pm 3.99 \times 10^{-3}$	$7.00 imes10^{-3}$	$\pm 1.22 imes 10^{-3}$	$5.745 imes 10^{-3}$	$\pm 1.62 \times 10^{-3}$
itional ASE in	ASE(LES)	1.259×10^{-2}	$\pm 4.00 \times 10^{-3}$	$7.045 imes 10^{-3}$	$\pm 1.22 imes 10^{-3}$	$5.797 imes 10^{-3}$	1.63×10^{-3}
Cond	ASE(QL)	2.162×10^{-3}	$\pm 4.97 \times 10^{-4}$	$1.516 imes 10^{-3}$	$\pm 4.30 \times 10^{-4}$	$1.413 imes 10^{-3}$	$\pm 2.90 \times 10^{-4}$
	θ	0.95		0.93		0.9	

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Figure 55: Conditional ASE for the alternative delay estimators in the $H_2/M/400$ model for actual delays in $(E[\widehat{W|W} > 0], 2E[\widehat{W|W} > 0])$, as a function of the traffic intensity ρ



Figure 56: Conditional ASE for the alternative delay estimators in the $H_2/M/400$ model for actual delays in $(2E[\widehat{W|W} > 0], 4E[\widehat{W|W} > 0])$, as a function of the traffic intensity ρ


Figure 57: Conditional ASE for the alternative delay estimators in the $H_2/M/400$ model for actual delays in $(4E[\widehat{W|W} > 0], 6E[\widehat{W|W} > 0])$, as a function of the traffic intensity ρ



Figure 58: Conditional ASE for the alternative delay estimators in the $H_2/M/400$ model for actual delays larger than $6E[\widehat{W|W} > 0]$, as a function of the traffic intensity ρ

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θ	E[W W > 0]	ASE(QL)	ASE(LES)	ASE(HOL)	ASE(RCS)	$ASE(RCS - \sqrt{s})$	ASE(LCS)	ASE(NI)
0.98	1.3919×10^{-1}	$2.174 imes 10^{-4}$	$1.059 imes 10^{-3}$	$1.053 imes 10^{-3}$	$1.287 imes 10^{-3}$	$1.325 imes 10^{-3}$	$6.215 imes 10^{-3}$	4.946×10^{-3}
	$\pm 1.45 \times 10^{-2}$	$\pm 2.42 imes 10^{-5}$	$\pm 1.144 imes 10^{-4}$	$1.14 imes 10^{-4}$	$\pm 1.14 imes 10^{-4}$	$\pm 1.14 imes 10^{-4}$	$\pm 3.57 imes 10^{-4}$	$\pm 1.04 \times 10^{-3}$
	10-0 10-0						0 000 10-3	
0.95	5.322×10^{-2}	8.219 × 10 °	3.820×10^{-4}	3.760×10^{-4}	0.045×10^{-5}	0.409×10^{-5}	3.003×10^{-5}	7.254×10^{-4}
	- 01 × 07.0工	- 01 X 07.0T	$\pm 4.90 \times 10^{-5}$	- 01 × 00.7T	- 01 × 60.7T	- 01 X 00.7T	$\pm 0.1 \times 0.07$	二 OT × 17.7工
0.93	3.811×10^{-2}	$5.850 imes10^{-5}$	$2.620 imes 10^{-4}$	2.561×10^{-4}	$4.827 imes 10^{-4}$	$5.149 imes 10^{-4}$	$2.003 imes 10^{-3}$	$3.676 imes 10^{-4}$
	$\pm 1.92 imes 10^{-3}$	$\pm 3.12 \times 10^{-6}$	$\pm 1.42 imes 10^{-5}$	$\pm 1.41 imes 10^{-5}$	$\pm 1.69 \times 10^{-5}$	$\pm 1.87 imes 10^{-5}$	$\pm 1.58 imes 10^{-4}$	$\pm 3.75 imes 10^{-5}$
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Table	59: A comparise	on of the emcien	icy of different re	al-time delay esi	timators conditi	onal on the level of	delay observed	IOT THE $\pi_2/M/s$

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T20.54 × 10 ° TZ.79 × 10 ° TZ.60 × 10 ° T2.50 × 10 ° T2.42 × 10 ° T2.20 × 10 ° T4.03 × 10 °	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c c} ASE(HOL) \\ \hline 3 & 1.976 \times 10^{-3} \\ -4 & \pm 2.13 \times 10^{-4} \\ \hline 4 & 7.299 \times 10^{-4} \\ -5 & \pm 5.20 \times 10^{-5} \\ \hline 4 & 5.121 \times 10^{-4} \\ -6 & -9.76 \times 10^{-5} \end{array}$	$\begin{array}{c} ASE(RCS) \\ 2.208 \times 10^{-3} \\ \pm 2.15 \times 10^{-4} \\ 9.746 \times 10^{-4} \\ \pm 5.19 \times 10^{-5} \\ 7.738 \times 10^{-4} \end{array}$	$\begin{array}{c} ASE(RCS - \sqrt{s}) \\ 2.246132 \times 10^{-3} \\ \pm 2.15 \times 10^{-4} \\ 1.015 \times 10^{-3} \\ \pm 5.43 \times 10^{-5} \\ 8.171 \times 10^{-4} \\ 8.171 \times 10^{-4} \end{array}$	$\begin{array}{c} ASE(LCS) \\ 8.250 \times 10^{-3} \\ \pm 3.31 \times 10^{-4} \\ 6.091 \times 10^{-3} \\ \pm 5.55 \times 10^{-5} \\ 4.944 \times 10^{-3} \\ \end{array}$	$\frac{ASE(NI)}{6.160 \times 10^{-2}}$ $\pm 1.27 \times 10^{-2}$ 8.771×10^{-3} $\pm 3.77 \times 10^{-4}$ 4.500×10^{-3}
	0.04 × 10 - ±2.19 × 10	TZ.10 X 10	INT X OP.CT	T01 X 74.0T	T01 × 67.6T	$\pm 4.03 \times 10^{-1}$

of the efficiency of different real-time delay estimators conditional on the level of delay observed for the $H_2/M/s$	$\mu = 1$ as a function of the traffic intensity ρ . We report point estimates for the conditional average squared error	$E[\tilde{W} W > 0]$, $4E[\tilde{W} W > 0]$). Each estimate is shown with the half width of the 95 percent confidence interval.
Table 60: A comparison of the efficiency of	queue with $s = 900$ and $\mu = 1$ as a function	(ASE) in the interval $(2E[\tilde{W} W > 0], 4E[\tilde{W}])$



Figure 59: Conditional ASE for the alternative delay estimators in the $H_2/M/900$ model for actual delays in $(E[\widehat{W|W} > 0], 2E[\widehat{W|W} > 0])$, as a function of the traffic intensity ρ



Figure 60: Conditional ASE for the alternative delay estimators in the $H_2/M/900$ model for actual delays in $(2E[\widehat{W|W} > 0], 4E[\widehat{W|W} > 0])$, as a function of the traffic intensity ρ



Figure 61: Conditional ASE for the alternative delay estimators in the $H_2/M/900$ model for actual delays in $(4E[\widehat{W|W}>0], 6E[\widehat{W|W}>0])$, as a function of the traffic intensity ρ

s = 1	RPD (percent)	0.571	-0.0327	-1.09	-1.70
vodel with	approx	42	30.57	22	15.33
ASE in the $M/M/s$ m	ASE(LES) (simulation)	42.24 ± 0.766	30.56 ± 0.371	$\begin{array}{c} 21.76 \\ \pm 0.186 \end{array}$	$\frac{15.07}{\pm 0.093}$
	β	0.95	0.93	0.9	0.85

Table 62: Testing the accuracy of the numerical approximation for the average square error (ASE) of the LES delay estimator in the M/M/squeue with s = 1 and $\mu = 1$ as a function of the traffic intensity ρ . Each simulation estimate is shown with the half width of the 95 percent confidence interval.

i = 10	RPD (percent) -1.44	-0.905	-0.785	-0.682	-2.22	e error (ASE) of t estimate is shov
odel with s	approx 1.02	0.42	0.3057	0.22	0.1533	arage squar simulation
ASE in the $M/M/s$ m	$\frac{ASE(LES) \text{ (simulation)}}{1.005} \pm 0.0416$	0.4162 ± 0.00399	0.3033 ± 0.00322	0.2185 ± 0.00329	0.1499 ± 0.000757	d approximation for the ave he traffic intensity ρ . Each
	ρ 0.98	0.95	0.93	0.9	0.85	umerica ion of tj

100	RPD (percent)	0.294	1.38	0.393	-1.45
model with s =	approx	$1.02 imes 10^{-2}$	4.2×10^{-3}	$3.057 imes 10^{-3}$	$2.2 imes 10^{-3}$
$ASE \ in \ the \ M/M/s$	ASE(LES) (simulation)	$1.023 imes 10^{-2} \pm 5.04 imes 10^{-4}$	$4.258 imes 10^{-3} \pm 5.85 imes 10^{-5}$	$3.069 imes 10^{-3} \pm 2.53 imes 10^{-5}$	$2.168 imes10^{-3} \pm 5.87 imes10^{-5}$
	θ	0.98	0.95	0.93	0.90

Table 64: Testing the accuracy of the numerical approximation for the average square error (ASE) of the LES delay estimator in the M/M/s queue with s = 100 and $\mu = 1$ as a function of the traffic intensity ρ . Each simulation estimate is shown with the half width of the 95 percent confidence interval.

400	RPD (percent)	0.769	0.8	2.09	1.89
model with s =	approx	6.375×10^{-4}	$2.625 imes 10^{-4}$	$1.911 imes 10^4$	$1.375 imes 10^{-4}$
ASE in the $M/M/s$	ASE(LES) (simulation)	$6.424 imes 10^{-4} \pm 2.46 imes 10^{-5}$	$2.646 imes 10^{-4} \pm 1.11 imes 10^{-5}$	$1.951 imes 10^{-4} \pm 7.18 imes 10^{-6}$	$1.401 imes 10^{-4} \ \pm 7.31 imes 10^{-6}$
	β	0.98	0.95	0.93	0.9

Table 65: Testing the accuracy of the numerical approximation for the average square error (ASE) of the LES delay estimator in the M/M/s queue with s = 400 and $\mu = 1$ as a function of the traffic intensity ρ . Each simulation estimate is shown with the half width of the 95 percent confidence interval.

006	RPD (percent)	6.98	2.14	0.656	2.06
model with s =	approx	2.494×10^{-4}	$1.259 imes 10^{-4}$	5.185×10^{-5}	$3.774 imes 10^{-5}$
ASE in the $M/M/s$	ASE(LES) (simulation)	$2.668 imes 10^{-4} imes 1.85 imes 10^{-5}$	$1.286 imes 10^{-4} \ \pm 4.24 imes 10^{-6}$	$5.219 imes 10^{-5} \pm 3.85 imes 10^{-6}$	$3.821 imes 10^{-5} \ \pm 4.09 imes 10^{-6}$
	β	0.99	0.98	0.95	0.93

Table 66: Testing the accuracy of the numerical approximation for the average square error (ASE) of the LES delay estimator in the M/M/s queue with s = 900 and $\mu = 1$ as a function of the traffic intensity ρ . Each simulation estimate is shown with the half width of the 95 percent confidence interval.

s = 1	RPD (percent)	-4.14	-5.57	-7.68	-10.7
odel with	approx	12.11	9.310	7.197	5.551
ASE in the $D/M/s$ m	ASE(LES) (simulation)	11.61 ± 0.146	8.791 ± 0.0780	6.644 ± 0.0372	4.955 ± 0.0178
	θ	0.95	0.93	0.9	0.85

Table 67: Testing the accuracy of the numerical approximation for the average square error (ASE) of the LES delay estimator in the D/M/s queue with s = 1 and $\mu = 1$ as a function of the traffic intensity ρ . Each simulation estimate is shown with the half width of the 95 percent confidence interval.

10	RPD (percent)	-1.92	-4.32	-5.89	-7.88	-11.0
model with $s =$	approx	2.684×10^{-1}	$1.210 imes 10^{-1}$	$9.305 imes 10^{-2}$	$7.197 imes 10^{-2}$	$5.552 imes 10^{-2}$
ASE in the $D/M/s$	ASE(LES) (simulation)	$2.633 imes 10^{-1} imes 18.34 imes 10^{-3}$	$1.158 imes 10^{-1} \pm 1.79 imes 10^{-3}$	$8.759 imes 10^{-2} \pm 1.05 imes 10^{-3}$	$6.630 imes 10^{-2} \pm 5.70 imes 10^{-4}$	$4.943 imes 10^{-2} \pm 2.57 imes 10^{-4}$
	β	0.98	0.95	0.93	0.9	0.85

Table 68: Testing the accuracy of the numerical approximation for the average square error (ASE) of the LES delay estimator in the D/M/squeue with s = 10 and $\mu = 1$ as a function of the traffic intensity ρ . Each simulation estimate is shown with the half width of the 95 percent confidence interval.

100	RPD (percent)	-1.88	-4.12	-5.82	-7.53
model with $s =$	approx	2.674×10^{-3}	$1.203 imes 10^{-3}$	$9.249 imes 10^{-4}$	7.191×10^{-4}
ASE in the $D/M/s$	ASE(LES) (simulation)	$2.624 imes 10^{-3} imes 5.02 imes 10^{-5}$	$egin{array}{llllllllllllllllllllllllllllllllllll$	$8.710 imes 10^{-4} \pm 1.58 imes 10^{-5}$	$6.650 imes 10^{-4} \ \pm 1.56 imes 10^{-5}$
	θ	0.98	0.95	0.93	0.9

Table 69: Testing the accuracy of the numerical approximation for the average square error (ASE) of the LES delay estimator in the D/M/s queue with s = 100 and $\mu = 1$ as a function of the traffic intensity ρ . Each simulation estimate is shown with the half width of the 95 percent confidence interval.

400	RPD (percent)	-0.791	-1.75	-3.24	-5.95	-9.29
model with $s =$	approx	3.155×10^{-4}	$1.678 imes 10^{-4}$	7.491×10^{-5}	$5.844 imes 10^{-5}$	4.396×10^{-5}
ASE in the $D/M/s$	ASE(LES) (simulation)	$3.130 imes 10^{-4} \pm 1.15 imes 10^{-5}$	$1.649 imes 10^{-4} \pm 3.34 imes 10^{-6}$	$7.248 imes 10^{-5} \pm 2.41 imes 10^{-6}$	$5.497 imes 10^{-5} \pm 2.55 imes 10^{-6}$	$3.988 imes 10^{-5} \pm 5.45 imes 10^{-6}$
	θ	0.99	0.98	0.95	0.93	0.9

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900	RPD (percent)	-0.586	-1.99	-3.24	-6.34
model with s =	approx	6.235×10^{-5}	3.338×10^{-5}	$1.582 imes 10^{-5}$	$1.202 imes 10^{-5}$
ASE in the $D/M/s$	ASE(LES) (simulation)	$6.199 imes 10^{-5} \pm 2.59 imes 10^{-6}$	$3.272 imes 10^{-5} \pm 9.82 imes 10^{-7}$	$egin{array}{llllllllllllllllllllllllllllllllllll$	$1.126 imes 10^{-5} \pm 1.73 imes 10^{-6}$
	β	0.99	0.98	0.95	0.93

Table 71: Testing the accuracy of the numerical approximation for the average square error (ASE) of the LES delay estimator in the D/M/s queue with s = 900 and $\mu = 1$ as a function of the traffic intensity ρ . Each simulation estimate is shown with the half width of the 95 percent confidence interval.

s = 1	RPD (percent)	-1.69	-2.46	-3.25	-4.49
100 notel with	approx	230.3	158.3	104.7	62.81
ASE in the $H_2/M/s$ n	ASE(LES) (simulation)	226.4 ±5.14	154.4 ± 2.94	101.3 ± 2.32	59.99 ± 0.515
	θ	0.95	0.93	0.9	0.85

Table 72: Testing the accuracy of the numerical approximation for the average square error (ASE) of the LES delay estimator in the $H_2/M/s$ queue with s = 1 and $\mu = 1$ as a function of the traffic intensity ρ . Each simulation estimate is shown with the half width of the 95 percent confidence interval.

s = 10	RPD (percent)	-0.223	-1.50	-2.47	-3.07	-4.18
odel with	approx	6.273	2.264	1.581	1.044	0.6266
ASE in the $H_2/M/s$ m	ASE(LES) (simulation)	6.259 ± 0.404	$\begin{array}{c} 2.23 \\ \pm 0.0466 \end{array}$	$\begin{array}{c} 1.542 \\ \pm 0.0346 \end{array}$	1.012 ± 0.0181	0.6004 ±0.0121
	β	0.98	0.95	0.93	0.9	0.85

e 73: Testing the accuracy of the numerical approximation for the average square error (ASE) of the LES delay estimator in the $H_2/M/s$	ie with $s = 10$ and $\mu = 1$ as a function of the traffic intensity ρ . Each simulation estimate is shown with the half width of the 95 percent	dence interval.
Table 73:	queue witl	confidence

100	RPD (percent)	-3.87		-1.87		-2.14		-3.22	
model with s =	approx	6.283×10^{-2}		$2.291 imes 10^{-2}$		$1.587 imes 10^{-2}$		1.054×10^{-2}	
ASE in the $H_2/M/s$	ASE(LES) (simulation)	6.040×10^{-2}	±3.21 × 10 °	$2.248 imes 10^{-2}$	$\pm 4.628 imes 10^{-4}$	$1.553 imes 10^{-2}$	$\pm 4.38 \times 10^{-4}$	$1.020 imes 10^{-2}$	$\pm 2.11 imes 10^{-4}$
	θ	0.98		0.95		0.93		0.9	

Table 74: Testing the accuracy of the numerical approximation for the average square error (ASE) of the LES delay estimator in the $H_2/M/s$ queue with s = 100 and $\mu = 1$ as a function of the traffic intensity ρ . Each simulation estimate is shown with the half width of the 95 percent confidence interval.

: 400	RPD (percent)	2.55	-4.16	-2.25	-3.42
model with s =	approx	3.643×10^{-3}	1.442×10^{-3}	9.964×10^{-4}	$6.517 imes 10^{-4}$
ASE in the $H_2/M/s$	ASE(LES) (simulation)	$3.736 imes 10^{-3} \pm 1.05 imes 10^{-4}$	$1.382 imes 10^{-3} \pm 4.60 imes 10^{-5}$	$9.74 imes 10^{-4} \pm 3.70 imes 10^{-5}$	$6.294 imes 10^{-4} \pm 2.56 imes 10^{-5}$
	β	0.98	0.95	0.93	0.9

Table 75: Testing the accuracy of the numerical approximation for the average square error (ASE) of the LES delay estimator in the $H_2/M/s$ queue with s = 400 and $\mu = 1$ as a function of the traffic intensity ρ . Each simulation estimate is shown with the half width of the 95 percent confidence interval.

= 900	RPD (percent)	-4.83	-3.03	-1.23
model with s =	approx	7.572×10^{-4}	2.798×10^{-4}	1.959×10^{-4}
ASE in the $H_2/M/s$	ASE(LES) (simulation)	$7.206 imes 10^{-4} \pm 3.42 imes 10^{-5}$	$2.713 imes 10^{-4} \pm 1.72 imes 10^{-5}$	$1.935 imes 10^{-4} \pm 1.29 imes 10^{-5}$
	β	0.98	0.95	0.93

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Table 77: A comparison of the efficiency of the HOL and QL delay estimators for the $M/M/s$ queue with $s = 1$ and $\mu = 1$ as a function of
the traffic intensity ρ . We report point estimates for the average squared error – (ASE). Each estimate is shown with the half width of the
95 percent confidence interval. Also included are corresponding values of the approximation ASE(HOL)/ASE(QL) and the relative percent
lifference RPD.

	Compari	son of $ASE(QL)$) and $ASE(HOL)$ in the $M/M/$	s model with $s = 10$	
θ	ASE(QL)	ASE(HOL)	ASE(HOL)/ASE(QL) (sim)	ASE(HOL)/ASE(QL) (approx)	RPD
0.98	0.4954	1.005	2.03	2.04	-0.556
	± 0.0232	± 0.0413			
0.95	0.1980	0.4171	2.10	2.11	-0.163
	± 0.00251	± 0.00416			
0.93	0.1421	0.3046	2.14	2.15	-0.299
	± 0.00133	± 0.00374			
0.9	0.1003	0.2204	2.20	2.22	-1.02
	± 0.00168	± 0.00416			
0.85	$6.613 imes 10^{-2}$	$1.528 imes 10^{-1}$	2.31	2.35	-1.63
	$\pm 3.24 \times 10^{-4}$	$\pm 1.24 \times 10^{-4}$			

: A comparison of the efficiency of the HOL and QL delay estimators for the $M/M/s$ queue with $s = 10$ and $\mu = 1$ as a function of	ic intensity ρ . We report point estimates for the average squared error – (ASE). Each estimate is shown with the half width of the	int confidence interval. Also included are corresponding values of the approximation ASE(HOL)/ASE(QL) and the relative percent	e RPD.
Lable 78: A c	he traffic inte	5 percent co.	lifference RP

FC				
ASE(HUL)	Ē	ASE(HOL)/ASE(QL) (sim) ASI	E(HOL)/ASE(QL) (approx)	RPD
$023 imes 10^{-2}$ $5.09 imes 10^{-4}$	$)^{-2}$	2.03	2.04	-0.363
$269 imes 10^{-3}$ $6.03 imes 10^{-5}$	$)^{-3}$	2.09	2.11	-0.871
084×10^{-3} 2.63×10^{-5}	$)^{-3}$	2.14	2.15	-0.526
$185 imes 10^{-3}$ $5.96 imes 10^{-5}$	$)^{-3}$	2.20	2.22	-0.982

	RPD	0.175	-0.444	0.0799	-1.27
s model with $s = 400$	ASE(HOL)/ASE(QL) (approx)	2.04	2.11	2.15	2.22
and $ASE(HOL)$ in the $M/M/s$	ASE(HOL)/ASE(QL) (sim)	2.04	2.10	2.15	2.19
on of $ASE(QL)$	ASE(HOL)	$6.427 imes 10^{-4} \pm 2.50 imes 10^{-5}$	$2.651 imes 10^{-4} \pm 1.13 imes 10^{-5}$	$1.960 \times 10^{-4} \pm 7.35 \times 10^{-6}$	1.414×10^{-4} $\pm 7.77 \times 10^{-6}$
Comparis	ASE(QL)	3.145×10^{-4} $\pm 1.35 \times 10^{-5}$	$1.262 \times 10^{-4} \pm 5.51 \times 10^{-6}$	$9.109 imes 10^{-5} \pm 3.77 imes 10^{-6}$	6.451×10^{-5} $\pm 4.43 \times 10^{-6}$
	θ	0.98	0.95	0.93	0.0

and QL delay estimators for the $M/M/s$ queue with $s = 400$ and $\mu = 1$ as a function of	the average squared error $-$ (ASE). Each estimate is shown with the half width of the	esponding values of the approximation ASE(HOL)/ASE(QL) and the relative percent	
Table 80: A comparison of the efficiency of the HOL and QL delay estimators f	the traffic intensity ρ . We report point estimates for the average squared error	95 percent confidence interval. Also included are corresponding values of the a	difference RPD.

חממ	NLD	0.197	-0.397	-0.234	1.78
$\frac{1}{A \operatorname{CF}(\operatorname{HOI}) / \operatorname{ACF}(\operatorname{OI}) / \frac{1}{A \operatorname{CF}(\operatorname{OI})}}$	ADE(DUD)/ADE(UUD)/ADE(UUD)/ADE(UD)/A	2.02	2.04	2.11	2.15
ASF(HOL) / ASF(OL) (""")	ADE(DUD)/ADE(UUD)/ADE(UD)	2.02	2.03	2.11	2.19
رسام ADE(ACE(HOL)	(UUU)JGA	$2.669 imes 10^{-4} \pm 1.86 imes 10^{-5}$	$1.287 imes 10^{-4} \\ \pm 4.25 imes 10^{-6}$	$5.229 imes 10^{-5} \pm 3.95 imes 10^{-6}$	3.834×10^{-5} $\pm 4.11 \times 10^{-6}$
VOTT V	ADE(UL)	$1.320 imes 10^{-4} \pm 8.86 imes 10^{-6}$	$\begin{array}{c} 6.334 \times 10^{-5} \\ \pm 2.15 \times 10^{-6} \end{array}$	$\begin{array}{c} 2.484 \times 10^{-5} \\ \pm 1.79 \times 10^{-6} \end{array}$	$\frac{1.752\times 10^{-5}}{\pm 1.94\times 10^{-6}}$
	β	0.99	0.98	0.95	0.93

	RPD	9.15		11.2		15.3		19.0	
M/s model with s = 1	ASE(HOL)/ASE(QL) (approx)	1.05		1.08		1.11		1.18	
QL) and $ASE(HOL)$ in the D/J	ASE(HOL)/ASE(QL) (sim)	1.15		1.20		1.28		1.40	
ison of ASE(0	ASE(HOL)	11.61	± 0.157	8.794	± 0.0862	6.647	± 0.0409	4.954	± 0.0203
$Compa_1$	ASE(QL)	10.13	± 0.148	7.322	± 0.0805	5.192	± 0.0377	3.528	± 0.0183
	β	0.95		0.93		0.9		0.85	

Table 82: A comparison of the efficiency of the HOL and QL delay estimators for the $D/M/s$ queue with $s = 1$ and $\mu = 1$ as a function o	the traffic intensity ρ . We report point estimates for the average squared error – (ASE). Each estimate is shown with the half width of the	95 percent confidence interval. Also included are corresponding values of the approximation ASE(HOL)/ASE(QL) and the relative percent	difference RPD.
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	Compari	son of $ASE(QL)$) and $ASE(HOL)$ in the $D/M/$	s model with $s = 10$	
θ	ASE(QL)	ASE(HOL)	ASE(HOL)/ASE(QL) (sim)	ASE(HOL)/ASE(QL) (approx)	RPD
0.98	2.485×10^{-1} $\pm 8.36 \times 10^{-3}$	2.633×10^{-1} $\pm 8.61 \times 10^{-3}$	1.06	1.02	3.88
0.95	$1.010 imes 10^{-1} \pm 1.76 imes 10^{-3}$	$\begin{array}{c} 1.157 \times 10^{-1} \\ \pm 2.03 \times 10^{-3} \end{array}$	1.15	1.05	9.10
0.93	$\begin{array}{c} 7.301 \times 10^{-2} \\ \pm 1.04 \times 10^{-3} \end{array}$	$8.773 imes 10^{-2} \pm 1.32 imes 10^{-3}$	1.20	1.08	10.8
0.9	$5.182 imes 10^{-2} \pm 5.77 imes 10^{-4}$	$6.632 \times 10^{-2} \pm 9.12 \times 10^{-4}$	1.28	1.11	15.3
0.85	$3.519 imes 10^{-2} \pm 2.49 imes 10^{-4}$	$\begin{array}{l} 4.935 \times 10^{-2} \\ \pm 5.73 \times 10^{-4} \end{array}$	1.40	1.18	18.8

and $\mu = 1$ as a function of	with the half width of the	L) and the relative percent	
timators for the $D/M/s$ queue with $s = 1$	red error $-$ (ASE). Each estimate is show	s of the approximation ASE(HOL)/ASE(
efficiency of the HOL and QL delay est	rt point estimates for the average squa	Also included are corresponding values	
Table 83: A comparison of the ϵ	the traffic intensity ρ . We report	95 percent confidence interval.	difference RPD.

	RPD	3.90	9.05	11.3	15.3
s model with $s = 100$	ASE(HOL)/ASE(QL) (approx)	1.02	1.05	1.08	1.11
and $ASE(HUL)$ in the $D/M/s$	ASE(HOL)/ASE(QL) (sim)	1.06	1.14	1.20	1.28
son of ASE(UL)	ASE(HOL)	2.624×10^{-3} $\pm 5.14 \times 10^{-5}$	1.153×10^{-3} $\pm 1.84 \times 10^{-5}$	8.713×10^{-4} $\pm 1.70 \times 10^{-5}$	$6.641 imes 10^{-4} \pm 1.67 imes 10^{-5}$
Compari	ASE(QL)	2.476×10^{-3} $\pm 5.04 \times 10^{-5}$	$1.007 imes 10^{-3} \pm 1.70 imes 10^{-5}$	$7.250 imes 10^{-4} \pm 1.58 imes 10^{-5}$	$5.189 imes 10^{-4} \pm 1.52 imes 10^{-5}$
	θ	0.98	0.95	0.93	0.0

for the $D/M/s$ queue with $s=100$ and $\mu=1$ as a function of	r - (ASE). Each estimate is shown with the half width of the	approximation ASE(HOL)/ASE(QL) and the relative percent	
Table 84: A comparison of the efficiency of the HOL and	the traffic intensity ρ . We report point estimates for the	95 percent confidence interval. Also included are corresp	difference RPD.

	Compari	son of $ASE(QL)$	and $ASE(HOL)$ in the $D/M/s$	s model with $s = 400$	
θ	ASE(QL)	ASE(HOL)	ASE(HOL)/ASE(QL) (sim)	ASE(HOL)/ASE(QL) (approx)	RPD
0.99	3.035×10^{-4} $\pm 1.16 \times 10^{-5}$	3.129×10^{-4} $\pm 1.17 \times 10^{-5}$	1.03	1.01	2.08
0.98	$1.556 imes 10^{-4} \\ \pm 3.37 imes 10^{-6}$	1.648×10^{-4} $\pm 3.42 \times 10^{-6}$	1.06	1.02	3.84
0.95	$6.329 imes 10^{-5} \pm 2.41 imes 10^{-6}$	$7.245 \times 10^{-5} \pm 2.50 \times 10^{-6}$	1.14	1.05	9.02
0.93	$4.583 \times 10^{-5} \pm 2.41 \times 10^{-6}$	$5.511 imes 10^{-5} \pm 2.64 imes 10^{-6}$	1.20	1.08	11.3
0.9	3.063×10^{-5} $\pm 4.88 \times 10^{-6}$	$3.931 \times 10^{-5} \pm 5.81 \times 10^{-6}$	1.28	1.11	15.6

omparison of the efficiency of the HOL and QL delay estimators for the $D/M/s$ queue with $s = 400$ and $\mu = 1$ as a function of	ensity ρ . We report point estimates for the average squared error – (ASE). Each estimate is shown with the half width of the	nfidence interval. Also included are corresponding values of the approximation ASE(HOL)/ASE(QL) and the relative percent	Ū.
able 85: A comparison of	e traffic intensity $\rho.$ We	percent confidence inter	ference RPD.

	RPD	2.06	-1.96	8.08	10.5
s model with $s = 900$	ASE(HOL)/ASE(QL) (approx)	1.01	1.02	1.05	1.08
and $ASE(HOL)$ in the $D/M/s$	ASE(HOL)/ASE(QL) (sim)	1.03	1.00	1.13	1.19
ion of ASE(QL)	ASE(HOL)	6.198×10^{-5} $\pm 2.62 \times 10^{-6}$	$3.088 \times 10^{-5} \pm 1.00 \times 10^{-7}$	$1.532 imes 10^{-5} \pm 1.52 imes 10^{-6}$	$1.125 imes 10^{-5} \pm 1.77 imes 10^{-6}$
Comparis	ASE(QL)	6.013×10^{-5} $\pm 2.60 \times 10^{-6}$	$3.088 \times 10^{-5} \pm 9.77 \times 10^{-7}$	$1.351 imes 10^{-5} \pm 1.50 imes 10^{-6}$	$9.424 imes 10^{-6} \pm 1.68 imes 10^{-6}$
	β	0.99	0.98	0.95	0.93

Table 86: A comparison of the efficiency of the HOL and QL delay estimators for the $D/M/s$ queue with $s = 900$ and $\mu = 1$ as a function of the traffic intensity ρ . We report point estimates for the average squared error – (ASE). Each estimate is shown with the half width of the 55 percent confidence interval. Also included are corresponding values of the approximation ASE(HOL)/ASE(QL) and the relative percent
lifference RPD.

	RPD	-11.5		-16.4		-22.3		-31.6	
M/s model with s = 1	ASE(HOL)/ASE(QL) (approx)	5.26		5.38		5.56		5.88	
$\mathcal{P}L$) and $ASE(HOL)$ in the $H_2/$	ASE(HOL)/ASE(QL) (sim)	4.65		4.50		4.32		4.02	
ison of $ASE(Q)$	ASE(HOL)	226.5 - E 93	07.0H	154.4	± 2.94	101.4	± 2.36	60.15	± 0.530
Compar	ASE(QL)	48.66	01.1 T	34.33	± 0.625	23.48	± 0.366	14.95	± 0.104
	θ	0.95		0.93		0.9		0.85	

β	ASE(QL)	ASE(HOL)	ASE(HOL)/ASE(QL) (sim)	ASE(HOL)/ASE(QL) (approx)	RPD
0.98	1.284 ± 0.0690	6.259 ± 0.410	4.87	4.59	6.20
0.95	0.4812 ± 0.00811	2.231 ± 0.0482	4.64	5.26	-11.9
0.93	0.3424 ± 0.00691	1.5436 ± 0.0370	4.51	5.38	-16.20
0.9	0.2344 ± 0.00359	1.013 ± 0.0203	4.32	5.56	-22.3
0.85	0.1498 ± 0.00217	0.6022 ± 0.0133	4.02	5.88	-31.6

10 and $\mu = 1$ as a function of	'n with the half width of the	QL) and the relative percent	
delay estimators for the $H_2/M/s$ queue with $s =$	rage squared error $-$ (ASE). Each estimate is sho	ing values of the approximation ASE(HOL)/ASE	
able 88: A comparison of the efficiency of the HOL and QL	ie traffic intensity ρ . We report point estimates for the aver	ò percent confidence interval. Also included are correspondi	fference RPD.

	RPD	-4.32	-11.2	-16.1	-21.9
s model with s = 100	ASE(HOL)/ASE(QL) (approx)	5.10	5.26	5.38	5.56
and $ASE(HOL)$ in the $H_2/M/$	ASE(HOL)/ASE(QL) (sim)	4.88	4.67	4.51	4.34
on of $ASE(QL)$	ASE(HOL)	$6.041 imes 10^{-2} ext{ \pm 3.22 imes 10^{-3}}$	2.248×10^{-2} $\pm 4.72 \times 10^{-4}$	$\frac{1.554\times 10^{-2}}{\pm 4.44\times 10^{-4}}$	$1.021 imes 10^{-2}$ $2.12 imes 10^{-4}$
Comparis	ASE(QL)	1.238×10^{-2} $\pm 6.95 \times 10^{-4}$	$\begin{array}{l} 4.815 \times 10^{-3} \\ \pm 9.47 \times 10^{-5} \end{array}$	$\begin{array}{c} 3.444 \times 10^{-3} \\ \pm 9.45 \times 10^{-5} \end{array}$	$\begin{array}{c} 2.350 \times 10^{-3} \\ 4.03 \times 10^{-5} \end{array}$
	θ	0.98	0.95	0.93	0.9

Table 89: A comparison of the efficiency of the HOL and QL delay estimators for the $H_2/M/s$ queue with $s = 100$ and $\mu = 1$ as a function of the metric of the metric for the metric of the metric
or the traine mention p . We report point estimates for the average squared effor $-(ADE)$. Each estimate is shown with the half with or the
95 percent confidence interval. Also included are corresponding values of the approximation ASE(LES)/ASE(QL) and the relative percent
difference RPD.
RPD

ASE(LES)/ASE(QL) (approx)
ASE(LES)/ASE(QL) (sim)
ASE(LES)
ASE(QL)
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Table 90: A comparison of the efficiency of the HOL and QL delay estimators for the $H_2/M/s$ queue with $s = 400$ and $\mu = 1$ as a function
of the traffic intensity ρ . We report point estimates for the average squared error – (ASE). Each estimate is shown with the half width of the
95 percent confidence interval. Also included are corresponding values of the approximation ASE(HOL)/ASE(QL) and the relative percent
difference RPD.

	RPD	-4.99	-11.5	-16.2
s model with $s = 900$	ASE(HOL)/ASE(QL) (approx)	5.10	5.26	5.38
and $ASE(HOL)$ in the $H_2/M/$	ASE(HOL)/ASE(QL) (sim)	4.85	4.66	4.51
on of $ASE(QL)$	ASE(HOL)	7.205×10^{-4} $\pm 3.45 \times 10^{-5}$	$2.713 imes 10^{-4} \pm 1.75 imes 10^{-5}$	$\begin{array}{c} 1.936 \times 10^{-4} \\ \pm 1.30 \times 10^{-5} \end{array}$
Comparise	ASE(QL)	1.487×10^{-4} $\pm 7.56 \times 10^{-6}$	$5.826 imes 10^{-5} \pm 3.70 imes 10^{-6}$	$4.292 imes 10^{-5} \pm 2.12 imes 10^{-6}$
	θ	0.98	0.95	0.93

nsity ρ . We report point estimates for the average squared ϵ
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	ASE(RCS - log(s))	$1.548 imes 10^{-2} (64.0)$	$\pm 4.50 \times 10^{-4}$	$1.388 imes 10^{-2} ({f 72.0})$	$\pm 3.34 imes 10^{-4}$	$1.135 imes 10^{-2} (80.7)$	$\pm 3.01 imes 10^{-4}$	$9.017 imes 10^{-3} (83.7)$	$\pm 1.91 imes 10^{-4}$	$6.892 imes 10^{-3}$ (76.9)	$\pm 1.92 \times 10^{-4}$	
	$ASE(RCS - \sqrt{(s)})$	$9.779 imes 10^{-3} (3.60)$	$\pm 3.18 imes 10^{-4}$	$8.395 imes 10^{-3} (4.03)$	$\pm 1.80 imes 10^{-4}$	$6.571 imes 10^{-3} (4.63)$	$\pm 1.82 imes 10^{-4}$	$5.161 imes 10^{-3} ({f 5.15})$	$\pm 1.23 imes 10^{-4}$	$4.108 imes 10^{-3} ({f 5.41})$	$\pm 1.01 imes 10^{-4}$	y e y e - E -
/s model with $s = 100$	$ASE(RCS - 2\sqrt{(s)})$	$9.452 imes 10^{-3} (0.138)$	$\pm 3.13 imes 10^{-4}$	$8.083 imes 10^{-3} (0.161)$	$\pm 1.72 imes 10^{-4}$	$6.295 imes 10^{-3} (0.239)$	$\pm 1.79 imes 10^{-4}$	$4.918 \times 10^{-3} (0.204)$	$\pm 1.23 imes 10^{-4}$	$3.906 imes 10^{-3} (0.696)$	$\pm 9.62 imes 10^{-5}$	
$ASE in the M/M_{\prime}$	$ASE(RCS - 4\sqrt{(s)})$	$9.439 imes 10^{-3}$	$\pm 3.13 imes 10^{-4}$	$8.070 imes10^{-3}$	$\pm 1.71 imes 10^{-4}$	$6.280 imes10^{-3}$	$\pm 1.78 imes 10^{-4}$	4.908×10^{-3}	$\pm 1.22 imes 10^{-4}$	$3.897 imes 10^{-3}$	$\pm 9.62 imes 10^{-5}$	
	ASE(RCS-s)	9.439×10^{-3}	$\pm 3.13 imes 10^{-4}$	$8.070 imes 10^{-3}$	$\pm 1.71 imes 10^{-4}$	$6.280 imes10^{-3}$	$\pm 1.78 imes 10^{-4}$	4.908×10^{-3}	$\pm 1.22 imes 10^{-4}$	$3.897 imes 10^{-3}$	$\pm 9.62 imes 10^{-5}$	8
	ASE(RCS)	9.439×10^{-3}	$\pm 3.13 imes 10^{-4}$	$8.070 imes 10^{-3}$	$\pm 1.71 imes 10^{-4}$	$6.280 imes 10^{-3}$	$\pm 1.78 imes 10^{-4}$	4.908×10^{-3}	$\pm 1.22 imes 10^{-4}$	$3.897 imes 10^{-3}$	$\pm 9.62 \times 10^{-5}$	
	β	0.98		0.97		0.95		0.93		0.9		

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	ASE(RCS - log(s))	$1.663 imes 10^{-3} (128)$	$\pm 7.14 \times 10^{-5}$	$1.512 imes 10^{-3} ({f 134})$	$\pm 3.69 imes 10^{-5}$	$1.184 imes 10^{-3} ({f 133})$	$\pm 7.21 imes 10^{-5}$	$9.242 imes 10^{-4} (124)$	$\pm 6.68 imes 10^{-5}$
ASE in the $M/M/s$ model with $s = 400$	$ASE(RCS - \sqrt{(s)})$	$7.696 imes 10^{-4} ({f 5.45})$	$\pm 3.59 imes 10^{-5}$	$6.849 imes 10^{-4} (6.07)$	$\pm 1.37 imes 10^{-5}$	$5.416 imes 10^{-4} ({f 6.47})$	$\pm 2.72 imes 10^{-5}$	$4.351 \times 10^{-4} ({\bf 5.66})$	$\pm 1.98 \times 10^{-5}$
	$ASE(RCS - 2\sqrt{(s)})$	$7.318 imes 10^{-4} (0.274)$	$\pm 3.45 imes 10^{-5}$	$6.475 imes 10^{-4} (0.279)$	$\pm 1.38 imes 10^{-5}$	$5.102 imes 10^{-4} (0.295)$	$\pm 2.60 imes 10^{-5}$	$4.124 imes 10^{-4} (0.146)$	$\pm 2.08 imes 10^{-5}$
	$ASE(RCS - 4\sqrt{(s)})$	$7.298 imes 10^{-4}$	$\pm 3.45 imes 10^{-5}$	$6.457 imes 10^{-4}$	$\pm 1.37 imes 10^{-5}$	$5.087 imes 10^{-4}$	$\pm 2.61 imes 10^{-5}$	$4.118 imes 10^{-4}$	$\pm 2.07 imes 10^{-5}$
	ASE(RCS-s)	$7.298 imes 10^{-4}$	$\pm 3.45 imes 10^{-5}$	$6.457 imes 10^{-4}$	$\pm 1.37 imes 10^{-5}$	$5.087 imes 10^{-4}$	$\pm 2.61 imes 10^{-5}$	$4.118 imes 10^{-4}$	$\pm 2.07 imes 10^{-5}$
	ASE(RCS)	$7.298 imes 10^{-4}$	$\pm 3.45 \times 10^{-5}$	$6.457 imes 10^{-4}$	$\pm 1.37 imes 10^{-5}$	$5.087 imes 10^{-4}$	$\pm 2.61 \times 10^{-5}$	4.118×10^{-4}	$\pm 2.07 imes 10^{-5}$
	θ	0.98		0.97		0.95		0.93	

n $s = 400$ and $\mu = 1$ as a	ate is shown with the half	ference $-$ (RPD).
estimators for the $M/M/s$ queue wi	ge squared $\operatorname{error} - (ASE)$. Each estin	re the values of the relative percent d
sucy of the candidate $RCS-f(s)$ delay	e report point estimates for the avera	terval. Also included in parentheses a
Table 93: A comparison of the efficie	function of the traffic intensity ρ . W	width of the 95 percent confidence in

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	ASE(RCS - log(s))	$5.262 imes 10^{-4} ({f 221})$	$\pm 1.24 \times 10^{-5}$	$4.717 imes 10^{-4} (220)$	$\pm 3.46 imes 10^{-5}$	$3.477 imes 10^{-4} (193)$	$\pm 4.11 \times 10^{-5}$	$2.847 imes 10^{-4} ({f 164})$	$\pm 8.02 \times 10^{-5}$
ASE in the $M/M/s$ model with $s = 900$	$ASE(RCS - \sqrt{(s)})$	$1.753 imes 10^{-4}$ (7.02)	$\pm 5.11 imes 10^{-6}$	$1.586 imes 10^{-4}$ (7.60)	$\pm 6.45 imes 10^{-6}$	$1.272 imes 10^{-4}$ (7.34)	$\pm 8.71 imes 10^{-6}$	$1.149 imes 10^{-4} (6.49)$	$\pm 2.11 \times 10^{-5}$
	$ASE(RCS - 2\sqrt{(s)})$	$1.642 imes 10^{-4} (0.244)$	$\pm 4.73 \times 10^{-6}$	$1.479 imes 10^{-4} (0.339)$	$\pm 6.19 imes 10^{-6}$	$1.190 imes 10^{-4} (0.421)$	$\pm 8.45 imes 10^{-6}$	$1.084 imes 10^{-4} (0.463)$	$\pm 1.92 imes 10^{-5}$
	$ASE(RCS - 4\sqrt{(s)})$	$1.638 imes 10^{-4}$	$\pm 4.66 \times 10^{-6}$	$1.474 imes 10^{-4}$	$\pm 6.21 imes 10^{-6}$	$1.185 imes 10^{-4}$	$\pm 8.44 \times 10^{-6}$	$1.079 imes 10^{-4}$	$\pm 1.91 imes 10^{-5}$
	ASE(RCS-s)	$1.638 imes 10^{-4}$	$\pm 4.66 \times 10^{-6}$	$1.474 imes 10^{-4}$	$\pm 6.21 \times 10^{-6}$	$1.185 imes 10^{-4}$	$\pm 8.44 \times 10^{-6}$	$1.079 imes 10^{-4}$	$\pm 1.91 \times 10^{-5}$
	ASE(RCS)	$1.638 imes 10^{-4}$	$\pm 4.66 \times 10^{-6}$	$1.474 imes 10^{-4}$	$\pm 6.21 \times 10^{-6}$	$1.185 imes 10^{-4}$	$\pm 8.44 \times 10^{-6}$	$1.079 imes 10^{-4}$	$\pm 1.91 imes 10^{-5}$
	θ	0.98		0.97		0.95		0.93	

delay estimators for the $M/M/s$ queue with $s = 900$ and $\mu = 1$ as a	werage squared error $-$ (ASE). Each estimate is shown with the half	ses are the values of the relative percent difference $-$ (RPD).
1: A comparison of the efficiency of the candidate $RCS-f(s)$ delay estimate	of the traffic intensity ρ . We report point estimates for the average square	f the 95 percent confidence interval. Also included in parentheses are the val
Table	funct.	width

		I									1	,
	ASE(RCS - log(s))	$3.724 imes 10^{-2} ({f 52.7})$	$\pm 6.81 \times 10^{-4}$	$3.566 imes 10^{-2} (55.6)$	$\pm 5.80 imes 10^{-4}$	$3.175 imes 10^{-2} (59.6)$	$\pm 5.45 imes 10^{-4}$	$2.800 imes 10^{-2} (63.2)$	$\pm 5.89 imes 10^{-4}$	$2.233 imes 10^{-2} (66.3)$	$\pm 8.61 \times 10^{-4}$	-
	$ASE(RCS - \sqrt{(s)})$	$2.511 imes 10^{-2} ({f 2.95})$	$\pm 4.92 imes 10^{-4}$	$2.367 imes 10^{-2} ({f 3.28})$	$\pm 4.73 imes 10^{-4}$	$2.058 imes 10^{-2} (3.48)$	$\pm 3.64 \times 10^{-4}$	$1.780 imes 10^{-2} (3.78)$	$\pm 3.60 imes 10^{-4}$	$1.399 imes 10^{-2} (4.06)$	$\pm 4.99 \times 10^{-4}$	
/s model with $s = 100$	$ASE(RCS-2\sqrt{(s)})$	$2.442 imes 10^{-2} (0.123)$	$\pm 4.88 \times 10^{-4}$	$2.229 imes 10^{-2} (0.141)$	$\pm 4.73 \times 10^{-4}$	$1.992 imes 10^{-2} (0.136)$	$\pm 3.67 imes 10^{-4}$	$1.718 imes 10^{-2} (0.150)$	$\pm 3.54 imes 10^{-4}$	$1.347 imes 10^{-2} (0.182)$	$\pm 4.89 \times 10^{-4}$	-
ASE in the H_2/M	$ASE(RCS - 4\sqrt{(s)})$	$2.439 imes 10^{-2}$	$\pm 4.84 \times 10^{-4}$	$2.229 imes 10^{-2}$	$\pm 4.70 imes 10^{-4}$	$1.989 imes 10^{-2}$	$\pm 3.67 imes 10^{-4}$	$1.715 imes 10^{-2}$	$\pm 3.56 imes 10^{-4}$	$1.344 imes 10^{-2}$	$\pm 4.90 \times 10^{-4}$	
	ASE(RCS-s)	2.439×10^{-2}	$\pm 4.84 \times 10^{-4}$	$2.229 imes 10^{-2}$	$\pm 4.70 imes 10^{-4}$	$1.989 imes 10^{-2}$	$\pm 3.67 imes 10^{-4}$	$1.715 imes 10^{-2}$	$\pm 3.56 imes 10^{-4}$	1.344×10^{-2}	$\pm 4.90 \times 10^{-4}$	
	ASE(RCS)	$2.439 imes 10^{-2}$	$\pm 4.84 \times 10^{-4}$	$2.229 imes 10^{-2}$	$\pm 4.70 \times 10^{-4}$	$1.989 imes 10^{-2}$	$\pm 3.67 imes 10^{-4}$	$1.715 imes 10^{-2}$	$\pm 3.56 \times 10^{-4}$	1.344×10^{-2}	$\pm 4.90 \times 10^{-4}$	-
	θ	0.98		0.97		0.95		0.93		0.90		

l/s queue with $s=100$ and $\mu=1$ as a). Each estimate is shown with the half	ve percent difference $-$ (RPD).
, efficiency of the candidate RCS- $f(s)$ delay estimators for the H_{2}	ρ . We report point estimates for the average squared error – (AS	nce interval. Also included in parentheses are the values of the rel
Table 95: A comparison of the	function of the traffic intensity	width of the 95 percent confider

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ASE in the $H_2/M/s$ model with $s = 400$	ASE(RCS - log(s))	$3.634 imes 10^{-3} (102)$	$\pm 7.32 imes 10^{-5}$	$3.561 imes 10^{-3} (106)$	$\pm 9.86 \times 10^{-5}$	$3.133 imes 10^{-3} (108)$	$\pm 1.10 imes 10^{-4}$	$2.848 imes 10^{-3} (109)$	$\pm 6.63 \times 10^{-5}$
	$ASE(RCS - \sqrt{(s)})$	$1.883 imes 10^{-3} (4.90)$	$\pm4.13 imes10^{-5}$	$1.819 imes 10^{-3} ({f 5.02})$	$\pm 4.50 imes 10^{-5}$	$1.579 imes 10^{-3} (4.86)$	$\pm 4.70 imes 10^{-5}$	$1.434 imes 10^{-3} (5.32)$	$\pm 2.94 \times 10^{-5}$
	$ASE(RCS - 2\sqrt{(s)})$	$1.799 imes 10^{-3} (0.249)$	$\pm 3.96 imes 10^{-5}$	$1.737 imes 10^{-3} (0.242)$	$\pm 4.11 \times 10^{-5}$	$1.509 imes 10^{-3} (0.194)$	$\pm 4.57 imes 10^{-5}$	$1.365 imes 10^{-3} (0.248)$	$\pm 2.73 \times 10^{-5}$
	$ASE(RCS - 4\sqrt{(s)})$	$1.795 imes 10^{-3}$	$\pm 3.97 imes 10^{-5}$	$1.733 imes 10^{-3}$	$\pm 4.06 \times 10^{-5}$	$1.506 imes 10^{-3}$	$\pm 4.57 imes 10^{-5}$	$1.361 imes 10^{-3}$	$\pm 2.70 imes 10^{-5}$
	ASE(RCS-s)	$1.795 imes 10^{-3}$	$\pm 3.97 imes 10^{-5}$	$1.733 imes10^{-3}$	$\pm 4.06 \times 10^{-5}$	$1.506 imes 10^{-3}$	$\pm 4.57 \times 10^{-5}$	$1.361 imes 10^{-3}$	$\pm 2.70 \times 10^{-5}$
	ASE(RCS)	$1.795 imes 10^{-3}$	$\pm 3.97 imes 10^{-5}$	$1.733 imes 10^{-3}$	$\pm 4.06 \times 10^{-5}$	$1.506 imes 10^{-3}$	$\pm 4.57 \times 10^{-5}$	1.361×10^{-3}	$\pm 2.70 \times 10^{-5}$
	θ	0.98		0.97		0.95		0.93	

of the candidate RCS- $f(s)$ delay estimators for the $H_2/M/s$ queue with $s=400$ and $\mu=1$ as a	port point estimates for the average squared error $-$ (ASE). Each estimate is shown with the half	al. Also included in parentheses are the values of the relative percent difference $-$ (RPD).
cy of the candidate RCS	report point estimates fo	rval. Also included in pa
96: A comparison of the efficier	on of the traffic intensity ρ . We	of the 95 percent confidence into
Table	functi	width

ASE in the $H_2/M/s$ model with $s = 900$	ASE(RCS - log(s))	$1.075 imes 10^{-3} (161)$	$\pm 2.82 imes 10^{-5}$	$1.019 imes 10^{-3} (160)$	$\pm 1.94 \times 10^{-5}$	$9.449 imes 10^{-4} (167)$	$\pm 3.48 \times 10^{-5}$	$8.371 imes 10^{-4} (161)$	$\pm 5.42 \times 10^{-5}$
	$ASE(RCS - \sqrt{(s)})$	$4.356 imes 10^{-4} ({f 5.56})$	$\pm 1.03 imes 10^{-5}$	$4.124 imes 10^{-4} ({f 5.37})$	$\pm 6.96 imes 10^{-6}$	$3.768 imes 10^{-4} (6.37)$	$\pm 1.23 imes 10^{-5}$	$3.412 imes 10^{-4} (6.16)$	$\pm 1.68 \times 10^{-5}$
	$ASE(RCS - 2\sqrt{(s)})$	$4.139 imes 10^{-4} (0.307)$	$\pm 9.92 imes 10^{-6}$	$3.925 imes 10^{-4} (0.300)$	$\pm 6.81 imes 10^{-6}$	$3.553 imes 10^{-4} (0.304)$	$\pm 1.19 imes 10^{-5}$	$3.221 imes 10^{-4} (0.243)$	$\pm 1.53 imes 10^{-5}$
	$ASE(RCS - 4\sqrt{(s)})$	$4.216 imes 10^{-4}$	$\pm 9.97 imes 10^{-6}$	$3.914 imes 10^{-4}$	$\pm 6.87 imes 10^{-6}$	$3.543 imes 10^{-4}$	$\pm 1.18 imes 10^{-5}$	$3.213 imes 10^{-4}$	$\pm 1.53 imes 10^{-5}$
	ASE(RCS-s)	$4.216 imes 10^{-4}$	$\pm 9.97 \times 10^{-6}$	$3.914 imes 10^{-4}$	$\pm 6.87 \times 10^{-6}$	$3.543 imes 10^{-4}$	$\pm 1.18 \times 10^{-5}$	$3.213 imes 10^{-4}$	$\pm 1.53 imes 10^{-5}$
	ASE(RCS)	$4.216 imes 10^{-4}$	$\pm 9.97 \times 10^{-6}$	$3.914 imes 10^{-4}$	$\pm 6.87 \times 10^{-6}$	$3.543 imes 10^{-4}$	$\pm 1.18 \times 10^{-5}$	$3.213 imes 10^{-4}$	$\pm 1.53 imes 10^{-5}$
	θ	0.98		0.97		0.95		0.93	

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ators for the $D/M/s$ queue with $s = 100$ and $\mu = 1$ as a	ared error $-$ (ASE). Each estimate is shown with the half	values of the relative percent difference $-$ (RPD).
Table 98: A comparison of the efficiency of the candidate RCS-	function of the traffic intensity ρ . We report point estimates for	width of the 95 percent confidence interval. Also included in par

ASE in the $D/M/s$ model with $s = 400$	ASE(RCS - log(s))	$7.908 imes 10^{-4}$ (161)	$\pm 3.10 imes 10^{-5}$	$6.604 imes 10^{-4} (165)$	$\pm 2.99 \times 10^{-5}$	$4.954 imes 10^{-4} (149)$	$\pm 4.20 \times 10^{-5}$	$3.692 imes 10^{-4} (122)$	$\pm 6.34 \times 10^{-5}$
	$ASE(RCS - \sqrt{(s)})$	$3.241 imes 10^{-4} (6.93)$	$\pm 1.37 imes 10^{-5}$	$2.687 imes 10^{-4}$ (7.70)	$\pm 1.03 imes 10^{-5}$	$2.144 imes 10^{-4}$ (7.79)	$\pm 1.38 imes 10^{-5}$	$1.789 imes 10^{-4}$ (7.64)	$\pm 2.14 \times 10^{-5}$
	$ASE(RCS - 2\sqrt{(s)})$	$3.041 imes 10^{-4} (0.330)$	$\pm 1.38 imes 10^{-5}$	$2.504 imes 10^{-4} (0.361)$	$\pm 9.75 imes 10^{-6}$	$1.996 imes 10^{-4} (0.352)$	$\pm 1.33 imes 10^{-5}$	$1.667 imes 10^{-4} (0.300)$	$\pm 2.02 imes 10^{-5}$
	$ASE(RCS - 4\sqrt{(s)})$	$3.031 imes 10^{-4}$	$\pm 1.32 imes 10^{-5}$	$2.495 imes 10^{-4}$	$\pm 9.33 imes 10^{-6}$	$1.989 imes 10^{-4}$	$\pm 1.27 imes 10^{-5}$	$1.662 imes 10^{-4}$	$\pm 1.88 \times 10^{-5}$
	ASE(RCS-s)	$3.031 imes 10^{-4}$	$\pm 1.32 imes 10^{-5}$	$2.495 imes 10^{-4}$	$\pm 9.33 \times 10^{-6}$	$1.989 imes 10^{-4}$	$\pm 1.27 imes 10^{-5}$	$1.662 imes 10^{-4}$	$\pm 1.88 \times 10^{-5}$
	ASE(RCS)	$3.031 imes 10^{-4}$	$\pm 1.32 imes 10^{-5}$	2.495×10^{-4}	$\pm 9.33 \times 10^{-6}$	$1.989 imes 10^{-4}$	$\pm 1.27 imes 10^{-5}$	$1.662 imes 10^{-4}$	$\pm 1.88 \times 10^{-5}$
	θ	0.98		0.97		0.95		0.93	

= 900	$2\sqrt{(s)}$ $ASE(RCS - \sqrt{(s)})$ $ASE(RCS - log(s))$	0.461) 7.987×10^{-5} (8.27) 2.744×10^{-4} (272)	$)^{-6}$ $\pm 4.13 \times 10^{-6}$ $\pm 1.85 \times 10^{-5}$	0.507) 6.878×10^{-5} (9.05) 2.309×10^{-4} (266)	$)^{-6}$ $\pm 4.22 \times 10^{-6}$ $\pm 2.66 \times 10^{-5}$	$\textbf{0.566} \qquad 5.955 \times 10^{-5} \textbf{(8.69)} \qquad 1.792 \times 10^{-4} \textbf{(227)}$	$)^{-6}$ $\pm 5.36 \times 10^{-6}$ $\pm 4.18 \times 10^{-5}$	0.815) 4.117×10^{-5} (8.29) 1.008×10^{-4} (165)	$)^{-5}$ $\pm 1.15 \times 10^{-5}$ $\pm 5.01 \times 10^{-5}$
ASE in the $D/M/s$ model with $s = 900$	$\rho \qquad ASE(RCS) ASE(RCS-s) ASE(RCS-4\sqrt{(s)}) ASE(RCS-2\sqrt{(s)}) ASE(RCS-2\sqrt{(s)}) ASE(RCS) A$	$0.98 7.377 \times 10^{-5} 7.377 \times 10^{-5} 7.377 \times 10^{-5} 7.411 \times 10^{-5} \\ \textbf{(0.461)} 7.987 \times 10^{-5} 7.411 \times 10^{-5} \\ \textbf{(0.461)} 7.987 \times 10^{$	$\pm 4.01 \times 10^{-6}$ $\pm 4.01 \times 10^{-6}$ $\pm 4.01 \times 10^{-6}$ $\pm 4.01 \times 10^{-6}$ $\pm 3.87 \times 10^{-6}$ $\pm 4.13 \times 10^{-6}$	$0.97 6.307 \times 10^{-5} 6.307 \times 10^{-5} 6.307 \times 10^{-5} 6.339 \times 10^{-5} (0.507) 6.878 \times$	$\pm 3.83 \times 10^{-6}$ $\pm 3.83 \times 10^{-6}$ $\pm 3.83 \times 10^{-6}$ $\pm 3.83 \times 10^{-6}$ $\pm 4.22 \times 10^{-6}$	$0.95 5.479 \times 10^{-5} 5.479 \times 10^{-5} 5.479 \times 10^{-5} 5.479 \times 10^{-5} 5.510 \times 10^{-5} (0.566) 5.955 \times 10^{-5} 0.566 5.955 \times 10^{-5} \times 10^{-5} 0.566 5.955 \times 10^{-5} \times 10^{-5} \times 10^{-5} $	$\pm 4.83 \times 10^{-6}$ $\pm 4.83 \times 10^{-6}$ $\pm 4.83 \times 10^{-6}$ $\pm 4.83 \times 10^{-6}$ $\pm 5.36 \times 10^{-6}$ $\pm 5.36 \times 10^{-6}$	$0.93 3.802 \times 10^{-5} 3.802 \times 10^{-5} 3.802 \times 10^{-5} 3.833 \times 10^{-5} \\ \textbf{(0.815)} 4.117 \times 10^{-5} \\ \textbf{(0.815)} $	$\pm 1.03 \times 10^{-5}$ $\pm 1.03 \times 10^{-5}$ $\pm 1.03 \times 10^{-5}$ $\pm 1.03 \times 10^{-5}$ $\pm 1.15 \times 10^{-5}$

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$00 \text{ and } \mu$	hown with	- (RPD).
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$\operatorname{RCS-}f(s)$	es for the	n parenthe
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Table 10	function	width o



Figure 62: Distribution of $W_{HOL}(w)$ when $w \approx 2E[W|W > 0]$ in the M/M/100 model when $\rho = 0.95$

{MM100_actual



Figure 63: Distribution of the LES delay estimations given when the HOL estimations, w, are such that w $\approx 2E[W|W > 0]$ in the M/M/100 model when $\rho = 0.95$



Figure 64: Distribution of the RCS delay estimations given when the HOL estimations, w, are such that w $\approx 2E[W|W > 0]$ in the M/M/100 model when $\rho = 0.95$



Figure 65: Distribution of the actual delays observed when the LES delay estimation, w, is such that: w $\approx 2E[W|W > 0]$ in the M/M/100 model when $\rho = 0.95$



Figure 66: Distribution of the actual delays observed when the RCS delay estimation, w, is such that: w $\approx 2E[W|W > 0]$ in the M/M/100 model when $\rho = 0.95$



Figure 67: Distribution of $W_{HOL}(w)$ when w $\approx 2E[W|W>0]$ in the D/M/100 model when $\rho = 0.95$

{DM100_actual



Figure 68: Distribution of the LES delay estimations given when the HOL estimations, w, are such that w $\approx 2E[W|W > 0]$ in the D/M/100 model when $\rho = 0.95$



Figure 69: Distribution of the RCS delay estimations given when the HOL estimations, w, are such that w $\approx 2E[W|W > 0]$ in the D/M/100 model when $\rho = 0.95$



Figure 70: Distribution of the actual delays observed when the LES delay estimation, w, is such that: w $\approx 2E[W|W > 0]$ in the D/M/100 model when $\rho = 0.95$



Figure 71: Distribution of the actual delays observed when the RCS delay estimation, w, is such that: w $\approx 2E[W|W > 0]$ in the D/M/100 model when $\rho = 0.95$



Figure 72: Distribution of $W_{HOL}(w)$ when $w \approx 2E[W|W > 0]$ in the $H_2/M/100$ model when $\rho = 0.95$

{HM100_actual



Figure 73: Distribution of the LES delay estimations given when the HOL estimations, w, are such that w $\approx 2E[W|W > 0]$ in the $H_2/M/100$ model when $\rho = 0.95$



Figure 74: Distribution of the RCS delay estimations given when the HOL estimations, w, are such that w $\approx 2E[W|W > 0]$ in the $H_2/M/100$ model when $\rho = 0.95$



Figure 75: Distribution of the actual delays observed when the LES delay estimation, w, is such that: w $\approx 2E[W|W > 0]$ in the $H_2/M/100$ model when $\rho = 0.95$



Figure 76: Distribution of the actual delays observed when the RCS delay estimation, w, is such that: w $\approx 2E[W|W > 0]$ in the $H_2/M/100$ model when $\rho = 0.95$

References

- Abate, J. and W. Whitt. 1994. A heavy-traffic expansion for asymptotic decay rates of tail probabilities in multichannel queues. Operations Res. Letters 15, 223–230.
- Abramowitz, M. and I. A. Stegun. 1972. Handbook of Mathematical Functions, National Bureau of Standards, U. S. Dept. of Commerce, Washington, D.C.
- Armony, M., N. Shimkin and W. Whitt. 2006. The impact of delay announcements in manyserver queues with abandonments. Submitted for publication. Available at http://columbia.edu/~ww2040.
- Asmussen, S. 2003. Applied Probability and Queues, second edition, Springer, New York.
- Avramidis, A. N., A. Deslauriers and P. L'Ecuyer. 2004. Modeling daily arrivals to a telephone call center. *Management Sci.* 50, 896–908.
- Brown, L., N. Gans, A. Mandelbaum, A. Sakov, H. Shen, S. Zeltyn and L. Zhao. 2005. Statistical analysis of a telephone call center: a queueing-science perspective. J. Amer. Statist. Assoc. 100, 36–50.
- Choudhury, G. L. and W. Whitt. 1994. Heavy-traffic asymptotic expansions for the asymptotic decay rates in the BMAP/G/1 Queue. *Stochastic Models* 10, 453–498.
- Coates, M., A. O. Hero, III, R. Nowak and B. Yu. 2002. Internet tomography. IEEE Signal Processing Magazine 19, 47–65.
- Cooper, R. B. 1981. Introduction to Queueing Theory, second edition, North-Holland, New York.
- Doytchinov, B., J. Lehoczky and S. Shreve. 2001. Real-time queues in heavy traffic with earliest-deadline-first queue discipline. *Ann. Appl. Probab.* 11, 332-378.
- Gans, N., G. Koole and A. Mandelbaum. 2003. Telephone call centers: tutorial, review and research prospects. *Manufacturing and Service Opns. Mgmt.* 5, 79–141.
- Glynn, P. W. and W. Whitt. 1989. Indirect estimation via $L = \lambda W$. Operations Research 37, 82–103.

- Glynn, P. W. and W. Whitt. 1994. Logarithmic Asymptotics for steady-state tail probabilities in a single-server queue. J. Appl. Prob. 31A, 131–156 (also called Studies in Applied Probability, Papers in Honour of Lajos Takes, J. Galambos and J. Gani (eds.), Applied Probability Trust, Sheffield, England).
- Glynn, P. W. and A. J. Zeevi. 2000. Estimating tail probabilities in queues via extremal statistics. In Analysis of Communication Networks: Call Centres, Traffic and Performance, D. R. McDonald and S. R. E. Turner (eds.), Fields Institute Communications 28, American Math. Soc., Providence, RI, 135–158.
- Guo, P. and P. Zipkin 2004. Analysis and comparison of queues with different levels of delay information. Duke University.
- Halfin, S. and W. Whitt. 1981. Heavy-traffic limits for queues with many exponential servers. Operations Research 29, 567–588.
- Hassin, R. and M. Haviv. 2003. To Queue or Not to Queue: Equilibrium Behavior in Queueing Systems, Kluwer.
- Ibrahim, R. E. and W. Whitt. 2006. Real time delay estimation based on delay history in the GI/M/s queue. IEOR Department, Columbia University, New York, NY.
- Iglehart, D. L. and W. Whitt. 1970. Multiple channel queues in heavy traffic II: sequences, networks, and batches. *Advances in Applied Probability* 2, 355–369.
- Jelenkovic P., A. Mandelbaum A. and P. Momcilovic. 2004. Heavy traffic limits for queues with many deterministic servers. *Queueing Systems* 47, 53–69.
- Mandelbaum A., A. Sakov and S. Zeltyn. 2000. Empirical analysis of a call center. Technical Report, Faculty of Industiral Engineering and Management, The Technion, Israel.
- Nakibly, E. 2002. *Predicting Waiting Times in Telephone Service Systems*, MS thesis, the Technion, Haifa, Israel.
- Neuts, M. F. 1986. The caudal characteristic curve of queues. *Adv. Appl. Probab.* 18, 221–254.
- Puhalskii, A. A. and M. I. Reiman. 2000. The multiclass GI/PH/N queue in the Halfin-Whitt regime. Adv. Appl. Prob. 32, 564–595.

- Reiman, M. I. 1982. The heavy traffic diffusion approximation for sojourn times in Jackson networks. In Applied Probability – Computer Science, the Interface, II, R. L. Disney and T. J. Ott (eds.), Birhauser, Boston, 409–422.
- Ross, S. M. 1996. Stochastic Processes, second edition, Wiley, New York.
- Shimkin, N. and A. Mandelbaum. 2004. Rational abandonment from tele-queues: nonlinear waiting cost with heterogeneous preferences. *Queueing Systems* 47, 117–146.
- Smith, W. L. 1953. On the distribution of queueing times. Proc. Camb. Phil. Soc. 49, 449–461.
- Ward, A. W. and W. Whitt. 2000. Predicting response times in processor-sharing queues. In Analysis of Communication Networks: Call Centres, Traffic and Performance, D. R. McDonald and S. R. E. Turner (eds.), Fields Institute Communications 28, American Math. Society, Providence, RI, 1-29.
- Whitt, W. 1982. Approximating a point process by a renewal process: two basic methods. Operations Research 30, 125–147.
- Whitt, W. 1984. On approximations for queues, I: extremal distributions. *AT&T Bell Lab. Tech. J.* 63, 115–138.
- Whitt, W. 1989. Planning queueing simulations. Management Sc. 35, 1341–1366.
- Whitt, W. 1999. Predicting queueing delays. Management Sci. 45, 870–888.
- Whitt, W. 2002. Stochastic-Process Limits, Springer, New York.
- Whitt, W. 2004. A diffusion approximation for the G/GI/n/m queue. Operations Research 52, 922–941.
- Whitt, W. 2005. Heavy-traffic limits for the G/H2*/n/m queue. Math. Oper. Res. 30, 1–27.