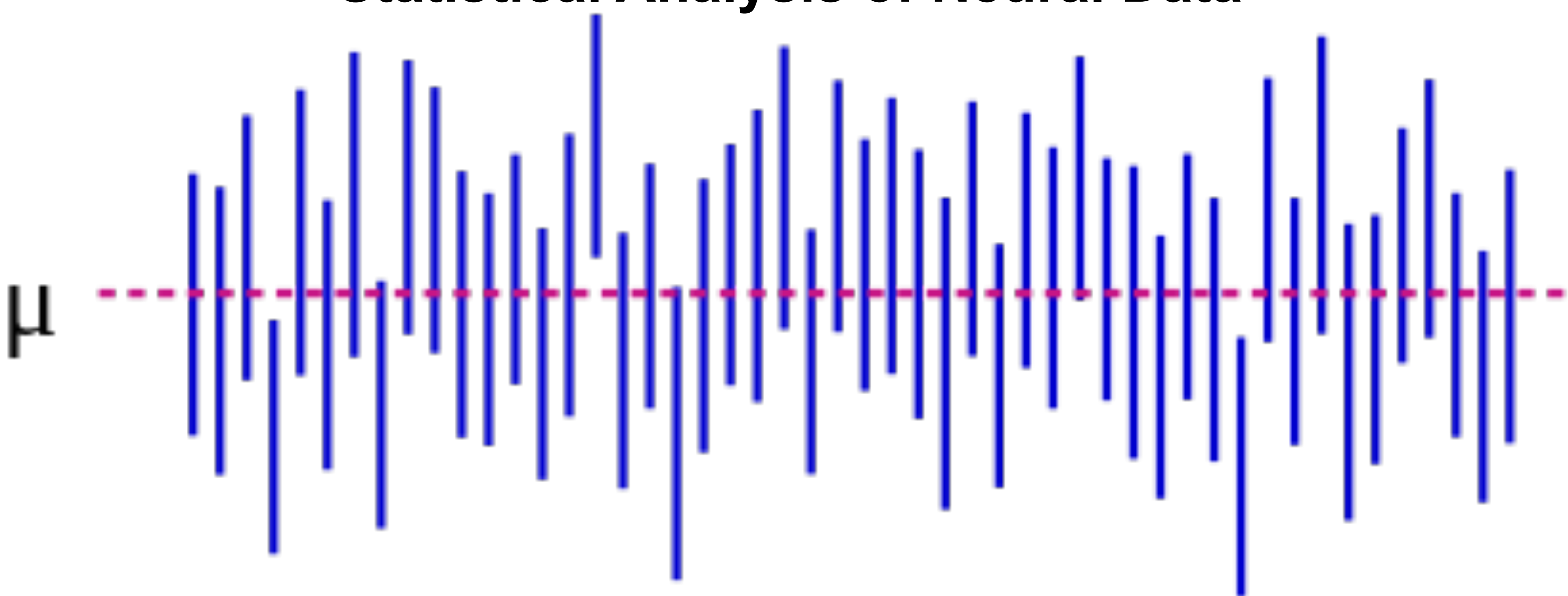
Applied Neuroscience

Columbia
Science
Honors
Program
Spring 2017

Statistical Analysis of Neural Data

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So many books, So little time.
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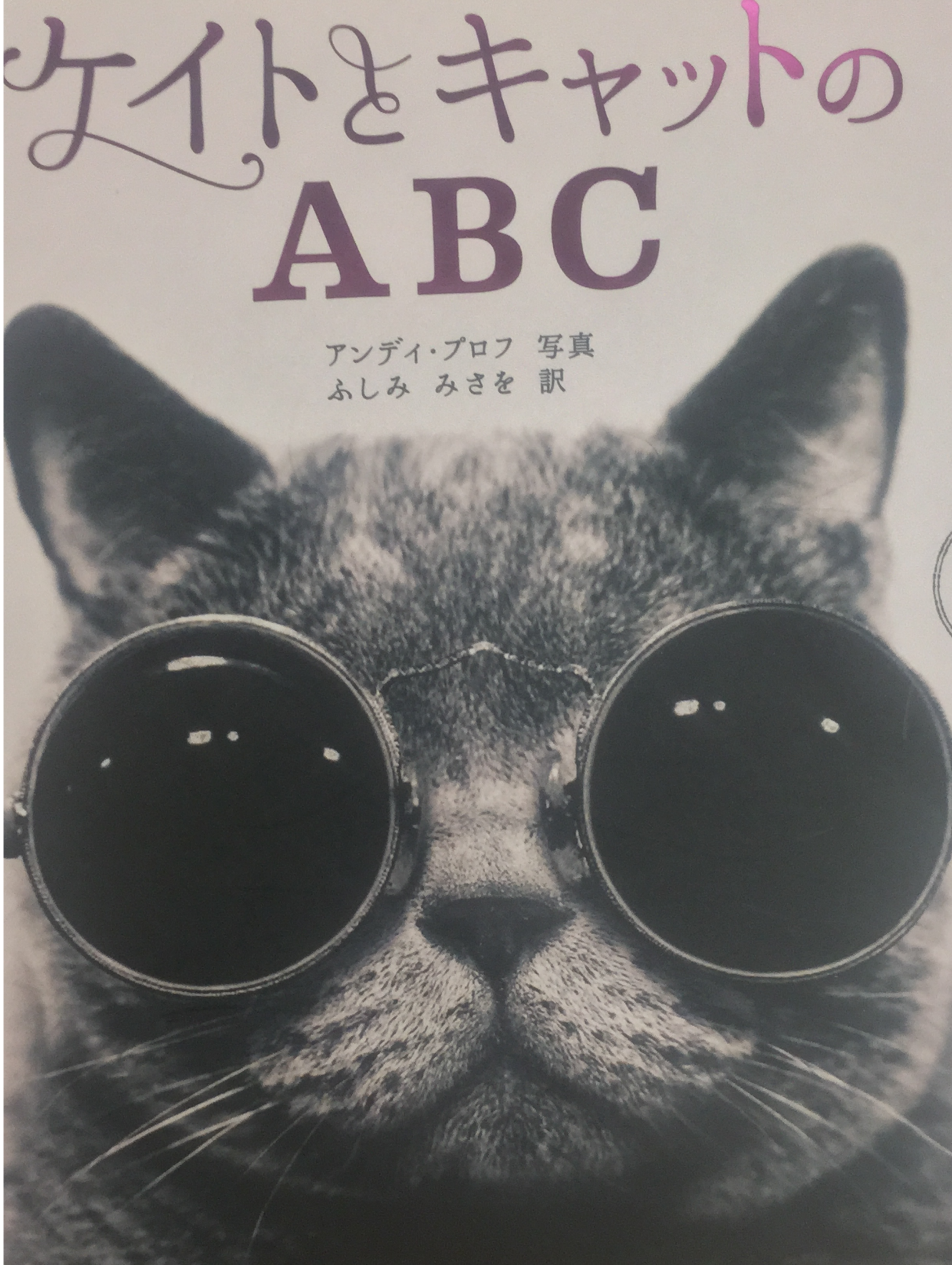
Neural Data Analysis

Objective: Statistical Analysis of Neural Data

Agenda:

1. Review
   Q and A
2. Guest Lecture
   Scott Linderman
3. Linear Algebra
   Matrices
   Vectors



Matrix Dimensions

A matrix is a rectangular array of numbers between brackets

Examples:

$$A = \begin{bmatrix} 0 & 1 & -2.3 & 0.1 \\ 1.3 & 4 & -0.1 & 0 \\ 4.1 & -1 & 0 & 1.7 \end{bmatrix}, \qquad B = \begin{bmatrix} 3 & -3 \\ 12 & 0 \end{bmatrix}$$

Dimension is always given as (numbers of) rows x columns

- *A* is a 3 x 4 matrix, *B* is 2 x 2
- The matrix *A* has four columns; *B* has two rows

m x n matrix is called *square* if $m = n$, *fat* if $m < n$, *skinny* if $m > n$

Matrix Coefficients

Coefficients (or entries) of a matrix are the values in the array

Coefficients are referred to using double subscripts for row, column

A_{ij} is the value in the i^{th} row, j column of A; also called i, j entry of A
i is the *row index* of A_{ij}; j is the column index of A_{ij}

(Here, A is a matrix; A_{ij} is a number)

Example:

$$A = \begin{bmatrix} 0 & 1 & -2.3 & 0.1 \\ 1.3 & 4 & -0.1 & 0 \\ 4.1 & -1 & 0 & 1.7 \end{bmatrix}$$

A_{23} = - 0.1, A_{22} = 4, but A_{41} is meaningless

The row entry of the index with value -2.3 is 1; its column index is 3

Column and Row Vectors

A matrix with one column, *i.e..*, size *n x 1,* is called a ***column vector***

A matrix with one row, *i.e.,* size *1 x n,* is called a ***row vector***

'vector' alone usually refers to column vector

We give only one index for column and row vectors, and call entries ***components***

$$v = \begin{bmatrix} 1 \\ -2 \\ 3.3 \\ 0.3 \end{bmatrix} \qquad w = \begin{bmatrix} -2.1 & -3 & 0 \end{bmatrix}$$

v is a 4-vector (or 4 x 1 matrix); its third component is $v_3 = 3.3$
w is a row vector (or 1 x 3 matrix); its third component is $w_3 = 0$

Matrix Equality

A = *B* means:

- *A and B* have the same size
- The corresponding entries are equal

For example:

- $\begin{bmatrix} -2 \\ 3.3 \end{bmatrix} \neq \begin{bmatrix} -2 & -3.3 \end{bmatrix}$ since the dimensions don't agree
- $\begin{bmatrix} -2 \\ 3.3 \end{bmatrix} \neq \begin{bmatrix} -2 \\ 3.1 \end{bmatrix}$ since the 2nd components don't agree

Zero and Identity Matrices

$0_{m \times n}$ denotes the *m x n* **zero matrix**, with all entries zero

I_n denotes the *n x n* **identity matrix**, with

$$I_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$$0_{2\times3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$0_{n \times 1}$ called *zero vector;* $0_{1 \times n}$ called *zero row vector*

Unit Vectors

e_i denotes the i^{th} unit vector: its i^{th} component is one, all others zero

The three unit 3-vectors are:

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \qquad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \qquad e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Unit vectors are the columns of the identity matrix I

Some authors use **1** (or e) to denote a vector with all entries one, called a **ones vector**

The ones vector of dimension 2 is $1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Matrix Transpose

Transpose of $m \times n$ matrix A, denoted A^T *or* A', *is* $n \times m$ *matrix*

$$\left(A^T\right)_{ij} = A_{ji}$$

Rows and columns of A are transposed in A^T

Example:
$$\begin{bmatrix} 0 & 4 \\ 7 & 0 \\ 3 & 1 \end{bmatrix}^T = \begin{bmatrix} 0 & 7 & 3 \\ 4 & 0 & 1 \end{bmatrix}$$

- Transpose converts row vectors to column vectors, vice versa
- *See:*

$$\left(A^T\right)^T = A$$

Matrix Addition and Subtraction

If A and B are both m x n, we form $A + B$ by adding corresponding entries

Example:
$$\begin{bmatrix} 0 & 4 \\ 7 & 0 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 9 & 3 \\ 3 & 5 \end{bmatrix}$$

Row or column vectors can be added in the same way (but never to each other)

Matrix subtraction is similar:

$$\begin{bmatrix} 1 & 6 \\ 9 & 3 \end{bmatrix} - I = \begin{bmatrix} 0 & 6 \\ 9 & 2 \end{bmatrix}$$

(Here we had to figure out that I must be 2 x 2)

Properties of Matrix Addition

- Commutative: $A + B = B + A$
- Associative: $(A + B) + C = A + (B + C)$, so we can write as $A + B + C$
- $A + 0 = 0 + A = A$; $A - A = 0$
- $(A + B)^T = A^T + B^T$

Scalar Multiplication

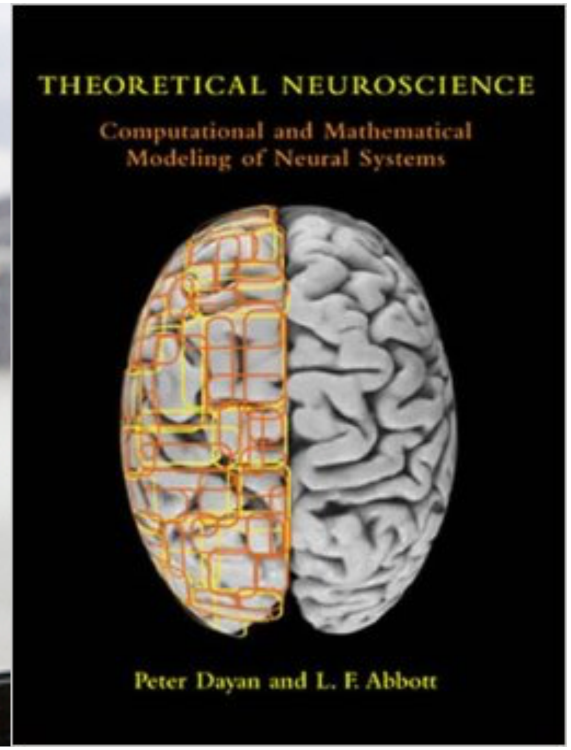
We can multiply a number (*scalar*) by a matrix by multiplying every entry of the matrix by the scalar

This is denoted by juxtaposition, with the scalar on the left:

$$(-2)\begin{bmatrix} 1 & 6 \\ 9 & 3 \\ 6 & 0 \end{bmatrix} = \begin{bmatrix} -2 & -12 \\ -18 & -6 \\ -12 & 0 \end{bmatrix}$$

- $(\alpha + \beta)A = \alpha A + \beta A$; $(\alpha\beta)A = (\alpha)(\beta A)$
- $\alpha(A + B) = \alpha A + \alpha B$
- $0 \cdot A = 0$; $1 \cdot A = A$

Next Time:



Guest Lecture by Professor Larry Abbott
"Computational Brain Models"
Department of Theoretical Neuroscience
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