Variance

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Variance is a statistic that is used to measure deviation in a probability distribution. **Deviation** is the tendency of outcomes to differ from the expected value. Studying variance allows one to quantify how much variability is in a probability distribution.

Definition

If X is a numerical discrete random variable with distribution $p(x)$ and expected value $\mu = E(X)$, the **variance** of X, denoted as σ^2 or $Var[X]$, is

$$\sigma^2 = Var[X] = E[(X-\mu)^2] = \sum_x (x-\mu)^2 p(x)$$

Based on this definition, we know that the variance is always non-negative. We also know that if the variance is equal to 0, then the random variable is equal to the expected value μ.

Theorem

The variance of random variable X is

$$Var[X] = E[X^2] - \mu^2$$

Standard Deviation

The **standard deviation** of a random variable, denoted σ, is the square root of the variance:

$$\sigma(X) = \sqrt{Var(X)}$$