Passive electrical properties of the neuron

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Fall 2004
The following books were used for the lecture:

“From neuron to brain” Nicholls et al., 3rd Ed.

“Ion channels of excitable membranes” B. Hille, 3rd Ed.

“Fundamental neuroscience” Zigmond et al.

“Principles of neuroscience” Kandel et al., 4th Ed.
Review of basic concepts

Experiment 1

Cell

[NaCl] in 10 mM

[NaCl] out 100 mM

Cell membrane

◆ Concentration Gradient
NaCl 10 mM in
NaCl 100 mM out

Measure membrane potential

Concentration Gradient

V = ??
Measure membrane potential

- Concentration Gradient
- membrane is equally permeable to both Na and Cl

\[ V = ?? \]
Membrane is *selectively* permeable to Sodium
Membrane is selectively permeable to Sodium

Concentration Gradient

in

out
Membrane is **selectively** permeable to Sodium

Excess of positive ions

Potential difference

Concentration Gradient

Excess of negative ions
Membrane potential

- Concentration Gradient across the membrane
- Membrane is selectively permeable to ions

\[ V = \frac{RT \log [\text{out}]}{F \frac{[\text{in}]}{[\text{out}]}} \]

\[ V_{Na} = +58 \text{ mV} \]

\[ V_{Cl} = -58 \text{ mV} \]
Membranes as capacitors

- **Internal conducting solution** (ions)
- **External conducting solution** (ions)
- **Thin insulating layer** (membrane, 4nm)
Membranes as Resistors

Internal conducting solution (ions)

External conducting solution (ions)

Ion channels
Voltage-gated, NT-gated etc.
Electrical model of the cell membrane

Ion channel
RESISTOR

Membrane
CAPACITOR
Capacitor

\[ C = \frac{Q}{V} \quad \text{Coulomb/Volt or Farads (F)} \]

\[ C = \varepsilon_0 \cdot \frac{A}{d} \quad \varepsilon_0 \text{ electrostatic permittivity} \]

\[ \uparrow A = \uparrow C \]

\[ \uparrow d = \downarrow C \]
Capacitor

rubber membrane

Water pressure

Release of pressure

Small Capacitance

Capacitors in parallel add larger Capacitance
Resistance

A = Area

l = length

Ohm's law

\[ R = \frac{V}{I} \text{ Ohms (Ω)} \]

\[ R = \rho \frac{l}{A} \]

\[ \rho \text{ resistivity} \]

\[ \uparrow l = \uparrow R \]

\[ \uparrow A = \downarrow R \]

For the same current, a larger R produces larger V
Resistors

For ion channels is better to think in terms of conductance:

\[ R_1 = \frac{1}{g_1} \]

As the # of Rs in parallel increases RT decreases!

\[ \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \]

More (open) channels in the membrane more conductance:

\[ g_T = g_1 + g_2 \]

\[ R_T = R_1 + R_2 \]

Long, thin parts of a neuron have large resistance!
Some useful equations

Current

\[ I = \text{Coulombs/second Amperes (A)} \]

Ohm’s law

\[ V = IR \]

Capacitance

\[ C = \frac{Q}{V} \text{ Coulombs/Volts (F)} \]

Voltage across capacitor

\[ V = \frac{Q}{C} \]

Changing the voltage in a capacitor

\[ \Delta V = \frac{\Delta Q}{C} \]

We change the charge by passing current

\[ I_c = \frac{\Delta Q}{\Delta t} \]

The change in \( V \) depends on the duration of \( I_c \)

\[ \Delta V = I_c \cdot \frac{\Delta t}{C} \]
Also remember...

Current likes to flow through the path with less resistance

\[ R = 100 \ \Omega \]

\[ R = 1 \ \Omega \]

And

\[ I_T = I_1 + I_2 \]
Electrical model of the cell membrane

Membrane current $I_m = I_i + I_c$
Effects of passing current on circuits containing R and C

V changes **instantaneously** with I

V changes **linearly in time** with I

V changes **exponentially** with a time constant = RC
For a rising exponential

\[ V = V_0 \ast (1 - e^{-t/RC}) \]
Experiment 2

Passing current and recording the membrane potential from a paramecium

Negative current makes the membrane potential more negative hyperpolarization

Positive current makes the membrane potential more positive depolarization

“electrotonic potential”
Linear relationship between current and voltage

\[ V = I \times R_{\text{in}} \]

Input Resistance \( R_{\text{in}} = 100 \, \text{M}\Omega \)
Specific membrane resistance

cross section of a cell

To compare cell with different sizes

The specific membrane resistance
(resistance per area)

\[ R_M = \Omega \cdot \text{cm}^2 \]

depends on the # of channels per cm\(^2\)

More channels make \( R_M \) smaller

For a spherical cell

\[ R_{in} = \frac{R_M}{4\pi a^2} \quad a = \text{radius} \]

\( R_{in} \) determines how much the cell depolarizes in response
to a steady current
Example

same \( R_m = 2000 \ \Omega \cdot \text{cm}^2 \)

Cell diameter is 50 m

\[ a = 25 \ \text{m} = 25 \times 10^{-4} \ \text{cm} \]

\[ R_{\text{in}} = R_m / (4\pi a^2) \]

\[ R_{\text{in}} = 2000 \ \Omega \cdot \text{cm}^2 / (4\pi (25 \times 10^{-4} \ \text{cm})^2) \]

\[ R_{\text{in}} = 25 \ \text{M\Omega} \]

Cell diameter is 5 m

your numbers here...

\[ R_{\text{in}} = 637 \ \text{M\Omega} \]

\( R_{\text{in}} \) is larger in a smaller cell
Specific membrane capacitance of biological membranes

$C_M = 1 \ \text{F/cm}^2$

For a cell at $-80 \text{ mV}$ how many ions is this?

$C_M = \frac{Q}{V}$

$Q = 10^6 \frac{C}{V} \times 0.08 \text{ V}$

$Q = 8 \times 10^{-8} \frac{C}{\text{cm}^2}$

Faraday constant $\approx 10^5 \text{ Coulombs/mole}$

Avogadro’s number $= 6.02 \times 10^{23} \text{ mole}^{-1}$

Then this is $4.8 \times 10^{11} \text{ ions/cm}^2$ Is this a lot???
Let’s assume the cell is 50 µm in diameter

\[ a = 25 \text{ m} = 25 \times 10^{-4} \text{ cm} \]

The surface area of a sphere \( A = 4\pi a^2 \)

\[ A = 7.85 \times 10^{-5} \text{ cm}^2 \]

4.8 \times 10^{11} ions in 1 cm²

So total is 4 \times 10^7 ions

The volume of this cell is 6.55 \times 10^{-8} \text{ cm}^3

Then this number of ions is \( \sim 10^{-6} \text{ M} \)

If KCl inside is 120 mM this means that only

\( \sim 1/120,000 \) ions is in excess!
For a spherical cell, the input capacitance

\[ C_{in} = C_M \times 4\pi a^2 \quad a = \text{radius} \]

More charge (current) is required to change the voltage across a large cell
In summary

\[ R_{in} = \frac{R_M}{4\pi a^2} \]
\[ C_{in} = C_M \times 4\pi a^2 \]

The product of input Capacitance and Resistance \((\tau)\) determines the time it takes change the potential.

Notice that \((\tau)\) is not affected by \(a\)
Real Neurons

How are signals affected by the passive properties of the membrane?
Experiment 3

Local potentials are graded

resting potential

Almost no potential change is observed, why?
Current in axons and dendrites

$\Gamma_m$ Section of axon or dendrite of determined length ($x$). In this case 1 cm.

$r_a$ axial resistance ($\Omega/cm$)

$r_m$ membrane resistance ($\Omega*cm$)
Axial resistance increases with distance \( (x) \)

Total axial resistance

\[ r_x = r_a \times x \quad (x = 4) \]

(remember RT = R1 + R2)

Near the site of injection, the current flows

\( V_o \) through \( r_m \) (less resistance)

Then \( V_o = I_m \times r_m \)
What is the value of $V$ at increasing distances from the site of current injection?

$$V = V_0 e^{-x/\lambda}$$

$\lambda$ is the length constant\(^*$

$$\lambda = \sqrt{\frac{r_m}{r_a}} \text{ (cm)}$$

Increasing $r_m$ increases $\lambda$

Decreasing $r_a$ increases $\lambda$

i.e. $V$ is closer to $V_0$
For 1 cm of cytoplasm (dendrites or axon)

\[ \rho \quad (\Omega \cdot \text{cm}) \quad \text{resistive property of } 1 \text{ cm}^3 \text{ of cytoplasm (dendrites or axon)} \]

\[ r_a = \frac{\rho}{(\pi a^2)} \]

\[ r_m = \frac{R_m}{(2\pi a)} \]

\[ \lambda = \sqrt{\left( \frac{r_m}{r_a} \right)} \]

\[ \lambda = \sqrt{\frac{R_m \cdot a}{2\rho}} \quad \text{If } R_m \text{ and } \rho \text{ are constant} \]

\[ \lambda = \sqrt{K \cdot a} \]

The length constant is proportional to the square root of the radius of the process

For neurons is usually 0.1 to 1 mm
Effect of length constant

For a dendrite or axon with the same diameter as the length constant increases the potential decreases less with distance.
Effect of diameter

For a dendrite or axon with increasing diameters the length constant increases and the potential decreases less with distance.
The passive properties of membranes and axon diameter affect the speed of conduction of action potentials

The speed of conduction of action potentials is inversely related to $r_a \times c_m$

The speed of conduction is increased by increasing the diameter of the axon which decreases $r_a$

The giant axon of the squid 1 mm!
Myelination, the alternative to increasing the diameter of the axon.

Glial cell wrap around axons many times (20-160 times) this like adding 320 membranes (in series). This increases Rm and decreases Cm.