A simple derivation of the Nernst Equation

The goal of this handout is to help you avoid taking notes during the lecture. I hope this derivation of the pervasive Nernst equation helps give you a feel for the thinking behind its development as well as some inroad into practically applying the equation to problems in Neuroscience.

In my mind the derivation starts when we observe the formal similarity between molar flux for diffusive properties in terms of the diffusion coefficient (D_s) and the local concentration (c_s):

$$M'_s = -D_s \frac{dc_s}{dx} \tag{1}$$

The molar flux of solute due to electrophoretic effects is defined in terms of the concentration of s, the electrical mobility (u_S), and the local potential (ψ):

$$M'_s = -u_s c_s \frac{d\psi}{dx} \tag{2}$$

Now, the fundamental insight of Nernst and Planck was to realize that these two effects are fundamentally additive, thus:

$$M_s = -D_s \frac{dc_s}{dx} - u_s c_s \frac{d\psi}{dx} \tag{3}$$

Two other important relationships allow us to do the work of deriving the Nernst equation. The first step is to recall the Nernst-Einstein relationship:

$$D_s = \frac{u_s RT}{z_s F} \tag{4}$$

Also, we should realize that what we care about in figuring out the potential across the membrane is the current, not the molar flux. In order to get from the molar flux to the current we need only multiply by the valence and Faraday's constant (i.e. $I_s = M_s z_s F$). For the derivation we first multiply equation 3 by $z_s F$ to obtain an expression for I_s:

$$I_s = -z_s F D_s \frac{dc_s}{dx} - z_s F u_s c_s \frac{d\psi}{dx} \tag{5}$$

By substitution with equation 4 we get:

$$I_s = -z_s F D_s \frac{dc_s}{dx} - z_s F \frac{D_s z_s F}{RT} c_s \frac{d\psi}{dx} \tag{6}$$

Collecting terms we obtain a common formulation of the Nernst-Planck Equation:

$$I_s = -z_s F D_s \left[\frac{dc_s}{dx} + \frac{z_s F}{RT} c_s \frac{d\psi}{dx} \right] \tag{7}$$

Finally, what we are interested in is the equilibrium condition, i.e. the point when the net flux is equal to zero. By setting $I_s = 0$ and rearranging terms we obtain:

$$\frac{d\psi}{dx} = -\frac{RT}{z_s F}\frac{1}{c_s}\frac{dc_s}{dx} \tag{8}$$

Integrating with respect to x across the width of the membrane (from inside (i) to outside (o); c_s has been substituted with $[s]$ to make the notation easier to read) we readily obtain the famous Nernst equilibrium expression:

$$\psi_i - \psi_o = -\frac{RT}{z_s F}\ln\frac{[s]_i}{[s]_o} \tag{9}$$

or, in the canonical form, for ion X:

$$V_m = \frac{RT}{z_x F}\ln\frac{[X]_o}{[X]_i} \tag{10}$$