## Cost of Capital

Risk, Return and Valuing Projects

## Capital Budgeting

- To find value of a project, we estimate the expected free cash flows (expected, nominal, after tax, operating cash flows) and discount with an appropriate discount rate
- What is the appropriate discount rate?


## Cost of capital

- An alternative to investing in a real project is to invest money in a portfolio of securities with the same "risk" as the project. To take on a project, the returns must be at least as good as those offered in the market


## Cost of Capital

- When a firm invests in a project, it is using shareholder and debt holder money. A project should be taken only if the return on the project leads investors to voluntarily provide funds.
- The project should provide enough cash to satisfy claimants required rate of return


## Required rate of return

- The required rate of return embodies
- returns that can be earned in the market
- the time value of money (the interest rate)
- the "riskiness" of the project


## Risk and Return

Consider the following gamble


## Risk and Return

The expected value is 10,000
Would you pay 10,000 to take this gamble?
Now consider 10,000 flips of a coin, each flip paying 2 if heads and 0 if tails
With over $99 \%$ probability, your payoff will be between 9800 and 10,200
Diversification

## Diversification

- By investing in 10,000 independent gambles, almost all of the risk is diversified away--diversifiable risk


## Systematic Risk

A different gamble:
An initial flip of the coin which pays 1 if heads and -1 if tails on each and every one of a subsequent 10,000 independent coin flips
payoff is 9800 to 10200 almost for sure plus or minus 10,000 --the initial coin flip, which is common to all subsequent flips cannot be diversified away

## Systematic Risk

More generally, we could have an initial flip that pays of $b$ or -b on each and every subsequent independent flip

Outcome $=10,000 * \mathrm{~b} * \mathrm{X}+$ independent coin
flips

## Components of Risk

- The first component of the gamble is not diversifiable
- b measures the sensitivity of the outcome to the non-diversifiable risk
- The second component of risk (independent coin tosses) is diversifiable


## Components of Risk

Non-diversifiable risk
Systematic risk market risk
diversifiable risk unsystematic risk
firm-specific risk

## Examples

- Systematic risk
- Economy wide fluctuations
- inflation fluctuations
- large technology changes
- Firm specific risks
- fire destroying plant
- new brand competition
- CEO dies


## Systematic Risk

- $\beta$ (beta) measures a securities sensitivity to non-diversifiable risk
- $\beta$ is a measure of relative systematic risk
- Relative to the overall market rate of return
- S\&P 500
- World Market Portfolio


## Risk and Return

- A securities required rate of return (expected rate of return, discount rate) is a function of
- time value of money (interest rate)
- risk that investors care about
- Interest rate is given by rf (return on short term treasury bills
- Risk is measured by systematic risk


## CAPM

- The Capital Asset Pricing Model provides the relation between risk and expected (or required) rate of return
$\mathrm{rj}=\mathrm{rf}+\beta \mathrm{j}(\mathrm{rm}-\mathrm{rf})$
$=$ risk free rate + risk premium
$=$ risk free $+($ relative risk $)(m k t$ risk prem. $)$


## Using CAPM

- To use CAPM to get a required equity rate of return, we need
- rf (risk free rate)
- $\beta$ (beta--measure of relative systematic risk)
- rm-rf (market risk premium)


## Interest rate and Inflation

- Think of interest rate (rf) as a required real rate of return plus expected inflation
- Short term interest rate reflects short term expectations of inflation, long term (30 yr) government bond reflects long term inflation
- investing for short period in long term bond is risky--interest rate changes


## Risk free Rate

- Long bond rate $=$ short term real rate + risk premium + expected long term inflation
- If we are using CAPM to arrive at a cost of capital to evaluate a long term project, we need for the risk free rate a short term real rate plus expected long term inflation
- rf = r30 - risk premium
- risk premium is probably $1-2 \%$


## Beta

- Beta is obtained from services that provide this information on publicly traded securities
- Value Line
- Wilshire Associates
- Where do they get betas?
- Regression analysis
- models


## Market Risk Premium

- A very controversial part of the CAPM
- Historically--9.2\%, on average
- large stock return - Tbill return arith. Avg.
- Some would argue should be $3-4 \%$
- The risk premium changes through time
- For Our Purposes we shall say the market risk premium is $8 \%$ (consensus)


## CAPM application

- $\mathrm{rj}=\mathrm{r} 30-1 \%+8 j * 8 \%$
- Example
- current long bond yield 5.3\%
$-\mathrm{Bj}=1.2$
- $\mathrm{rj}=.053-.01+1.2^{*} .08$

$$
=.043+.096
$$

$$
=13.9 \%
$$

## Firm's Cost of Capital

- A firm's cost of capital is the expected rate of return that must be provided so that investors (debt holders and equity holders) voluntarily provide funds
- If the firm provides less than this, firm value falls
- If the firm provides more than this, firm value increases


## Firm's WACC

- What does it mean that the firm's investors voluntarily provide capital?
- The equity holders have a required rate of return, re, the debt holders have a required rate of return, rd
- The debt holders are investing D , the market value of debt
- The equity holders are investing E, the market value of equity


## WACC

- Thus, the firm must provide re on the amount E provided by equity holders, and rd on the amount D provided by debt holders
- However, since the interest payments are tax deductible, the firm has to earn
$-\mathrm{kd}=(1-\mathrm{t})$ rd in order to provide the return rd where $t$ is the corporate tax rate


## WACC

- Defining $\mathrm{V}=\mathrm{D}+\mathrm{E}$, the firm's cost of capital is given by:
$\mathrm{WACC}=(\mathrm{E} / \mathrm{V}) \mathrm{re}+(\mathrm{D} / \mathrm{V}) \mathrm{kd}$ $\mathrm{WACC}=(\mathrm{E} / \mathrm{V}) \mathrm{re}+(\mathrm{D} / \mathrm{V}) \mathrm{rd}(1-\mathrm{t})$


## Calculating WACC

- $\mathrm{E}=(\text { market price })^{*}($ number of shares)
- $\mathrm{D}=$ total value of debt
- $\mathrm{V}=\mathrm{D}+\mathrm{E}=$ total market value of the firm
- re $=$ required equity rate of return
- use CAPM
- use dividend growth $r=$ future div yield +g
- rd = market yield on debt
- $\mathrm{t}=$ effective corporate tax rate (fed + local $)$


## Example

Price $=30$, number of shares $=100 \mathrm{~m}$ total debt $($ short + long $)=1 b$ market yield on debt $=8 \%$
tax rate $=40 \%$
$B=.9$
$\mathrm{r} 30=5.3 \%$

## Example

- $\mathrm{Re}=.053-.01+.9^{*} .08$

$$
=11.5 \%
$$

- $\mathrm{kd}=.08^{*}(1-.4)=4.8 \%$
- $\mathrm{E}=30 * 100 \mathrm{~m}=3 \mathrm{~b}$
- $\mathrm{D}=1 \mathrm{~b}$
- $V=4 b$
- WACC $=.75^{*} .115+.25^{*} .048=9.825 \%$


## Valuation principle

- The value of the firm $(\mathrm{V})$ is the present value of expected future free cash flows, discounted at the firm's WACC
- The net present value of a project with the same risk (systematic) as the firm, and with the same capital structure as the firm is the present value of future free cash flows at the firm's WACC less the initial investment


## Project cost of capital

- Projects may have
- different leverage ratios
- different risks
- What do we do?


## WACC and leverage

- In a Modigliani and Miller world, without taxes, changes in leverage do not affect the WACC--firm value $=p v$ of fcf's at WACC, fcf's are unaffected by leverage, value does not change with leverage, so WACC must not change with leverage


## Leverage

- As add debt, more weight is put on "cheaper" debt, but the equity gets riskier, and hence the required equity return gets larger


## Leverage and cost of equity

- The following shows the relation between levered equity expected returns, rl, and unlevered equity expected returns, ru

$$
\mathrm{rl}-\mathrm{rd}=(\mathrm{ru}-\mathrm{rd})[1+(\mathrm{D} / \mathrm{E})]
$$

D/E debt equity ratio in market value terms

## Relevering

- Suppose you observe rll associated with debt to equity ratio 11 , you wish to find the cost of equity, rl2, at a new debt to equity ratio 12 . You also observe rd.
- $\mathrm{Rl} 2=\mathrm{rd}+(\mathrm{rl} 1-\mathrm{rd})^{*}(1+\mathrm{l} 2) /(1+11)$


## example

- R11 $=15 \%, r d=9 \%,(\mathrm{D} / \mathrm{E}) 1=.5,(\mathrm{D} / \mathrm{E}) 2=$ 1.2
- $\mathrm{ru}-\mathrm{rd}=(\mathrm{rll}-\mathrm{rd}) /(1+11)=(.15-.09) /(1+.5)$

$$
=.06 / 1.5=.04
$$

- rl2 - rd $=(\mathrm{ru}-\mathrm{rd}) *(1+12)=.04 * 2.2=.088$

$$
\mathrm{rl} 2=17.8 \%
$$

$$
\mathrm{rl2}=\mathrm{rd}+(\mathrm{rl1}-\mathrm{rd}) *(1+12) /(1+11)
$$

$$
=.09+.06 * 2.2 / 1.5=17.8 \%
$$

## Project cost of capital

- Determine leverage appropriate for the project
- Determine WACC for project
- Where do we get the required returns?


## Comparables

- Find firms, with publicly traded equity that are similar to the project under consideration
- similar in terms of lines of business
- similar in terms of unlevered systematic risk


## comparables

- Estimate unlevered costs of equity of comparables
- Take "judgmental" average
- find project cost of debt
- lever the average unlevered required return at the project's debt equity ratio
- calculate WACC


## example



