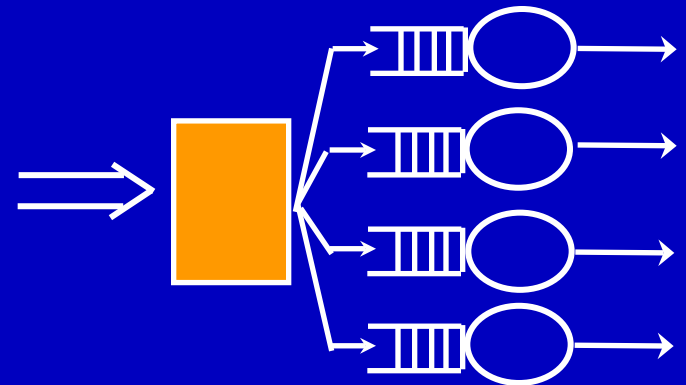


Surprising results on task assignment for high-variability workloads

Mor Harchol-Balter, CMU, Comp. Sci.

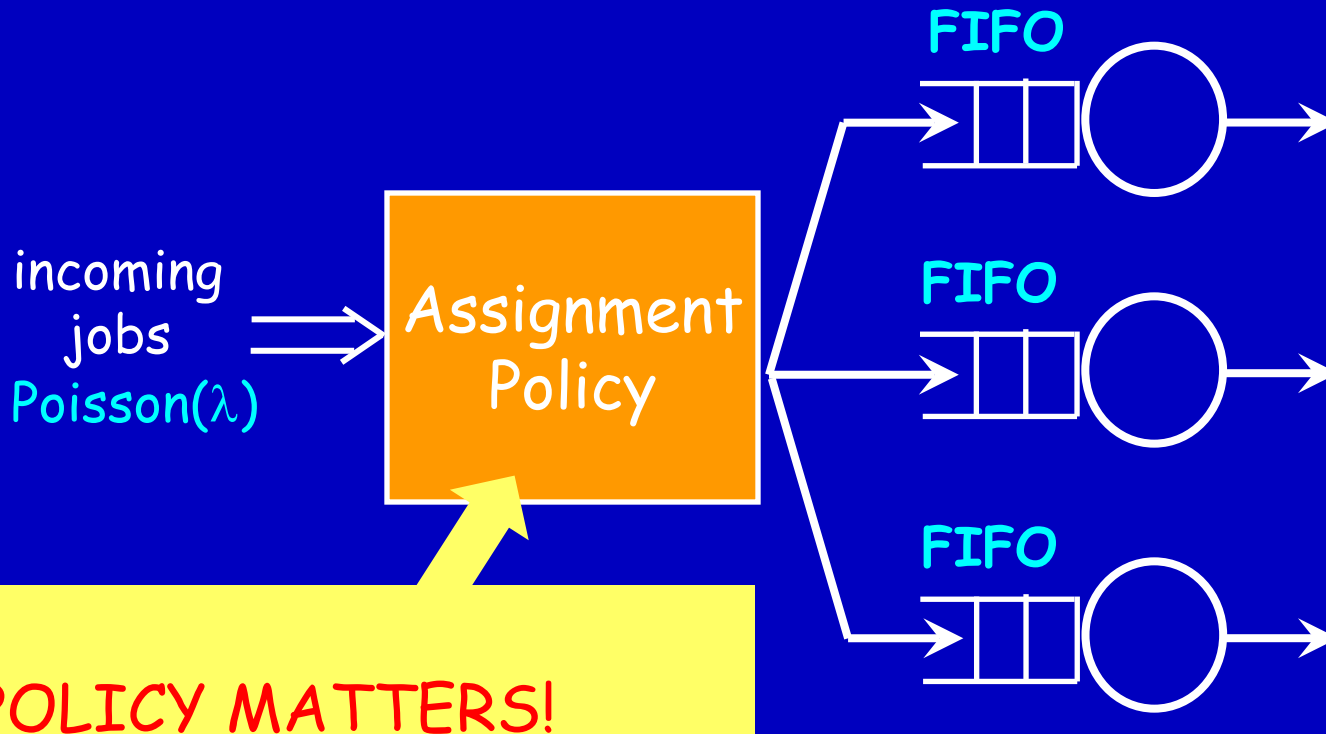
Alan Scheller-Wolf, CMU, Tepper Business

Andrew Young, Morgan Stanley



Server farm model

Goal: Minimize mean response time: $E[T]$



n servers

general i.i.d.
job sizes $\sim X$

$$C^2 = \frac{\text{var}(X)}{E[X]^2}$$

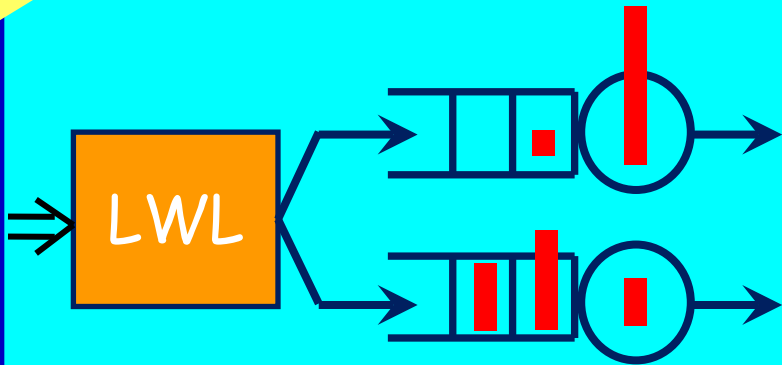
$$\rho = \lambda E[X] \leq n$$

Good Answers

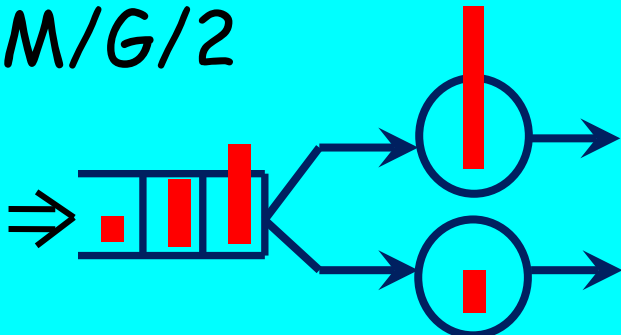
+High throughput

LWL (Least Work Left)

Select job to host with least remaining work.



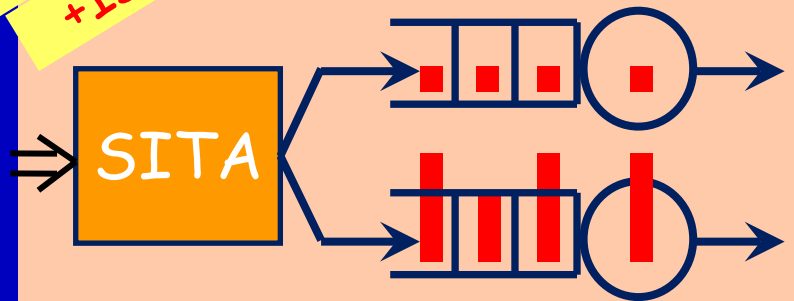
M/G/2



+Protects against high variability
+Isolation for smalls

SITA (Short Interval)

Split on size



Prior Work on SITA

SITA in Practice

- Supercomputing Centers
[Hotovy, Schneider, O'Donnell 96]
[Schroeder, Harchol-Balter 00]
- Manufacturing Centers
[Buzacott, Shanthikumar 93]
- File Server Farms
[Cardellini, Colajanni, Yu 01]
- Supermarkets

Optimizing SITA cutoffs

- [Harchol-Balter, Crovella, Murta 98]
- [Bachmat, Sarfati 08]
- [Sarfati 08]
- [Harchol-Balter, Vesilo 08]

SITA variants

- [Harchol-Balter 00]
- [Harchol-Balter 02]
- [Thomas 08]
- [Tari, Broberg, Zomaya, Baldoni 05]
- [Fu, Broberg, Tari 03]

SITA vs. LWL

- [Broberg, Tari, Zeepongsekul 04]
- [Harchol-Balter 02]
- [Ciardo, Rieck 04]
- [Fu, Broberg, Tari 03]
- [Harchol-Balter, Crovella, Murta 99]
- [Sarfati, Broberg, Tari, Zeepongsekul 04]
- [Tari, Broberg, Zomaya, Baldoni 05]
- [Thomas 08]

All conclude SITA far superior for high variability

In search of a proof of SITA's total dominance.

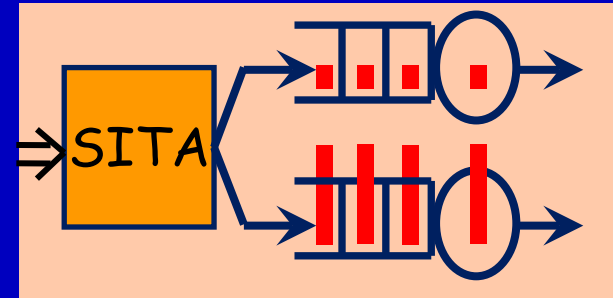
OK, so not optimal, but definite win for high variability.

Should at least beat all commonly used policies when variability is high enough.

Months later

Years later

Can't prove anything because it's not true!





Alternative Title:

The TRUTH about SITA,
under very high job size variability

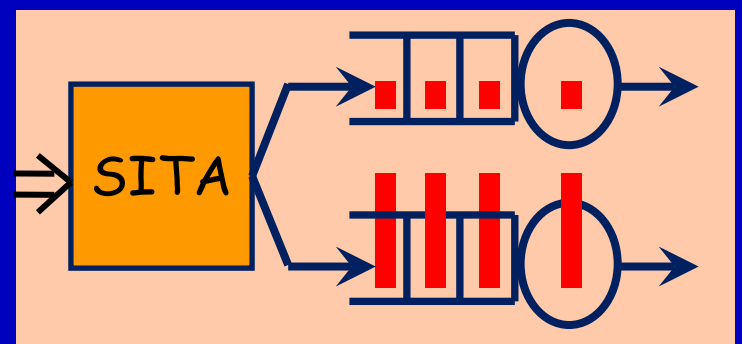
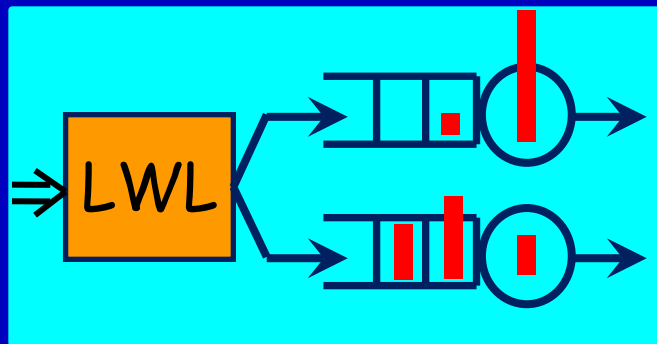
$$C^2 = \frac{\text{var}(X)}{E[X]^2} \rightarrow \infty \quad \text{while} \quad E[X]: \textit{fixed}$$

Q: In this talk we will show ...

as $C^2 \rightarrow \infty$

- a) SITA diverges & LWL diverges?
- b) SITA converges & LWL diverges?
- c) SITA diverges & LWL converges?
- d) SITA converges & LWL converges?

A: All of the above




Q: In this talk we will show ...

as $C^2 \rightarrow \infty$

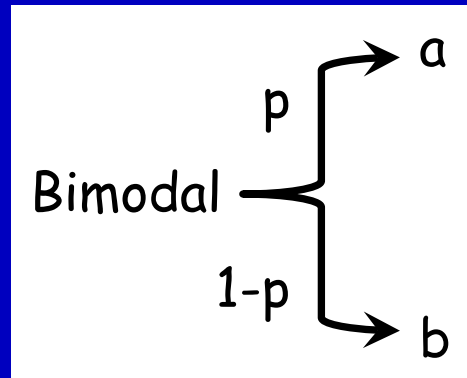
	Convergent LWL	Divergent LWL
Convergent SITA	✓	✓
Divergent SITA	✓	✓

Looking for simple job size distributions to illustrate each.

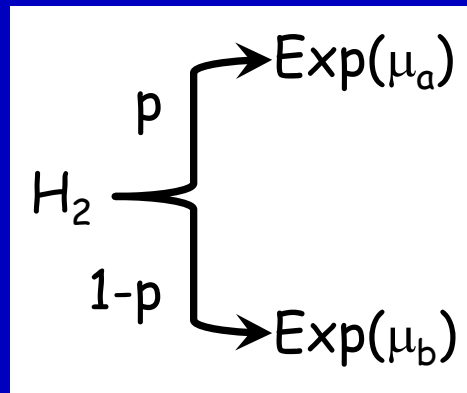


Results (2 server system)

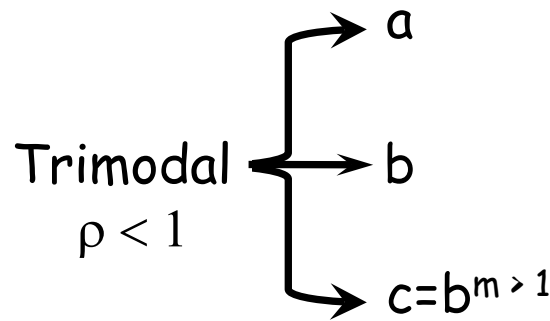
	Conv. LWL	Diverg. LWL
Conv. SITA		<div style="text-align: center;"> <div style="border: 1px solid black; padding: 5px; display: inline-block;">depends on pa & $(1-p)b$</div> ↑ ↓ </div>
Diverg. SITA		



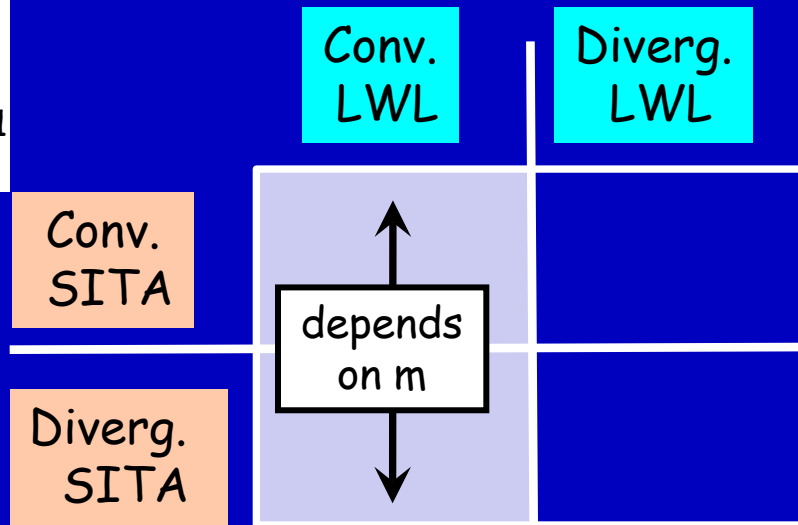
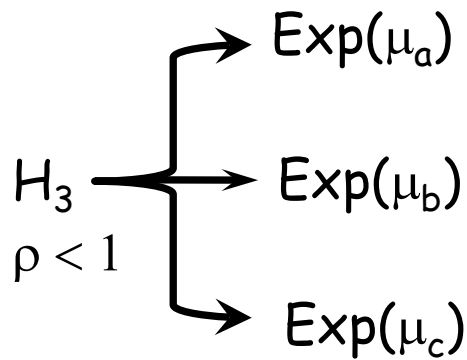
or



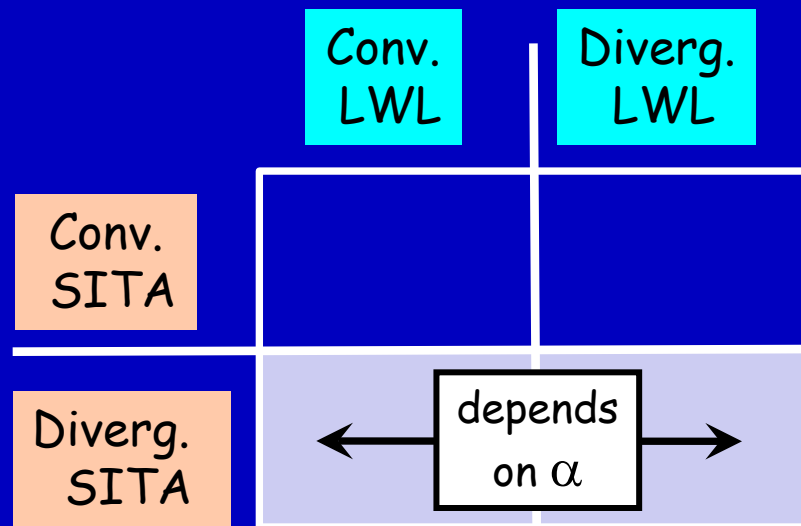
Results (2 server system)



or

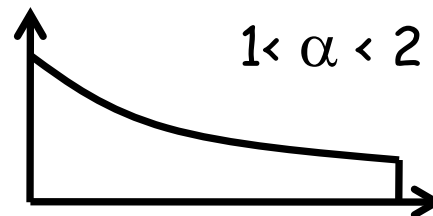


Results (2 server system)



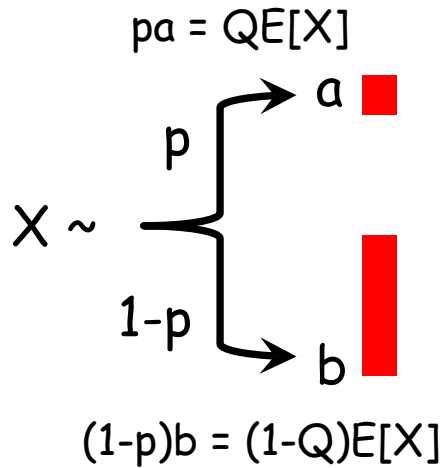
Bounded Pareto(α)

$$1 < \alpha < 2$$



Bimodal Results

	Conv. LWL	Diverg LWL
Conv. SITA		↑ depends ρ_a & ρ_b ↓
Diverg SITA		

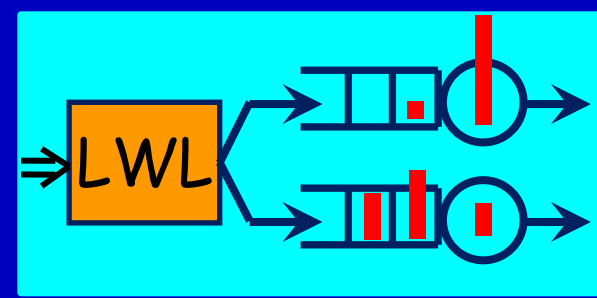


Lemma: As $C^2 \rightarrow \infty$, but $E[X]$, Q : const,
 a 's get little smaller $\rightarrow QE[X]$
 b 's get much bigger $\rightarrow \infty$
 $p \rightarrow 1$

THM: If $\rho_a < 1$ & $\rho_b < 1$
 \rightarrow Convergent SITA

THM: LWL always diverges.

Understanding LWL



Isn't LWL always bad for high C^2 ?

It depends ...

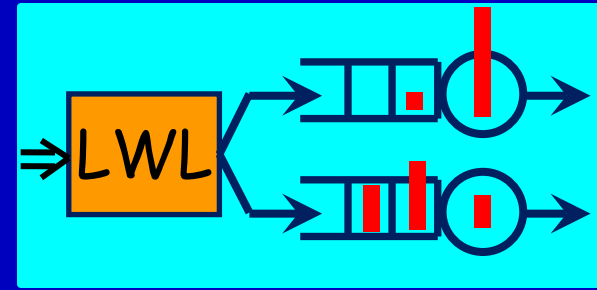
But shorts stuck behind longs, so $E[T] \rightarrow \infty$

Need 2 longs for this to be a problem!

So we need: $\Pr\{2 \text{ longs}\} * E[T | 2 \text{ longs}]$?

Suffices to just look at $E[X^{3/2}]$.

Understanding LWL



Thm: [Scheller-Wolf, Sigman 97], [Scheller-Wolf, Vesilo 06] (2 SERVERS)

$$\text{If } E[X^{3/2}] < \infty \ \& \ \rho < 1 \Rightarrow E[T]^{LWL} < \infty$$

(\Leftarrow usually)

1 spare server

$C^2 \rightarrow \infty$

Thm:

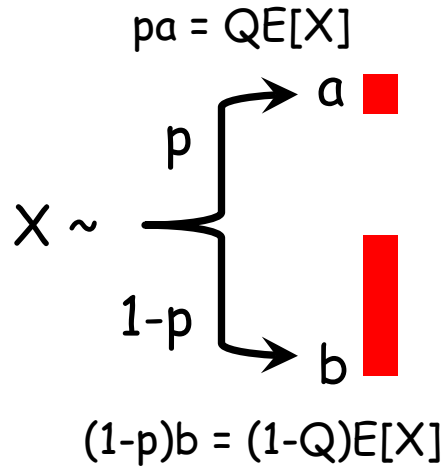
$$\text{If } \left\{ \begin{array}{l} E[X^{3/2}] : \text{bounded} \\ \text{while } C^2 \rightarrow \infty \end{array} \right\} \ \& \ \rho < 1 \Rightarrow \text{LWL converges}$$

(\Leftarrow usually)

I can make both happen!



Bimodal Results



Lemma: As $C^2 \rightarrow \infty$, but $E[X]$, Q : const,
 $a \rightarrow QE[X]$, $b \rightarrow \infty$, $p \rightarrow 1$

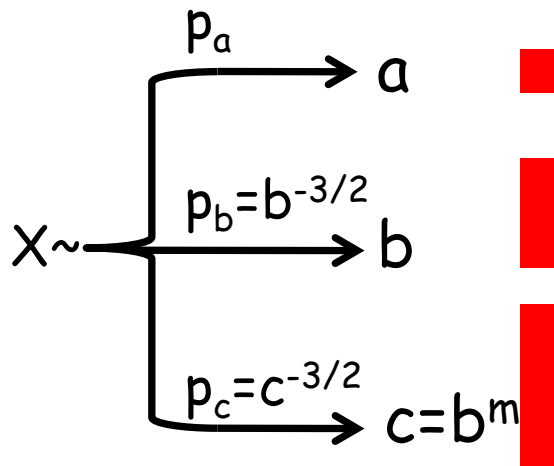
THM: LWL always diverges.

$$\begin{aligned}
 E[X^{3/2}] &= pa^{3/2} + (1-p)b^{3/2} \\
 &= QE[X]\sqrt{a} + (1-Q)E[X]\sqrt{b} \\
 &\rightarrow \infty \text{ (as } C^2 \rightarrow \infty)
 \end{aligned}$$

THM: If $\rho_a < 1$ & $\rho_b < 1$
 \rightarrow Convergent SITA

	Conv. LWL	Diverg LWL
Conv. SITA		\uparrow depends ρ_a & ρ_b \downarrow
Diverg SITA		

Trimodal Results



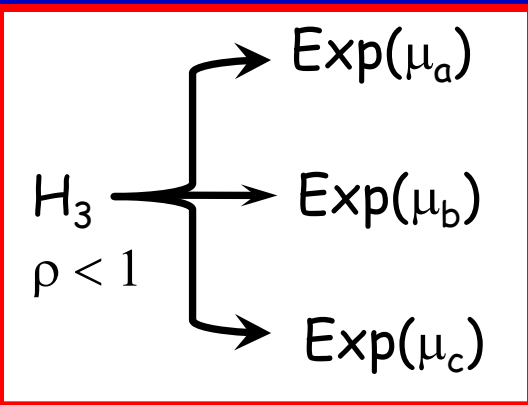
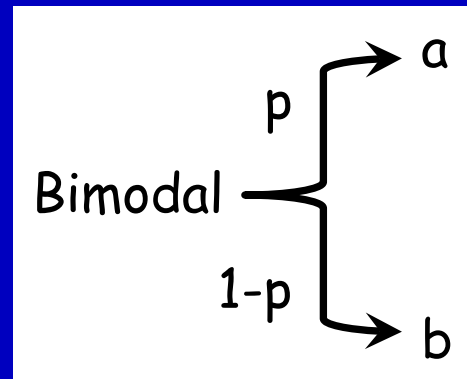
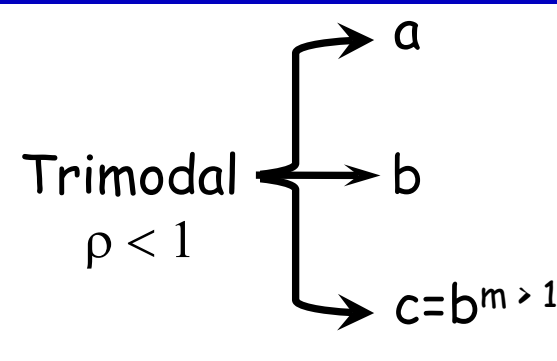
Lemma: As $C^2 \rightarrow \infty$, but $E[X]: \text{const}$,
 $a \rightarrow E[X]$
 $b \rightarrow \infty, c \rightarrow \infty$
 $p_a \rightarrow 1$

	Conv. LWL	Diverg LWL
Conv. SITA	↑ depends on m ↓	
Diverg SITA		

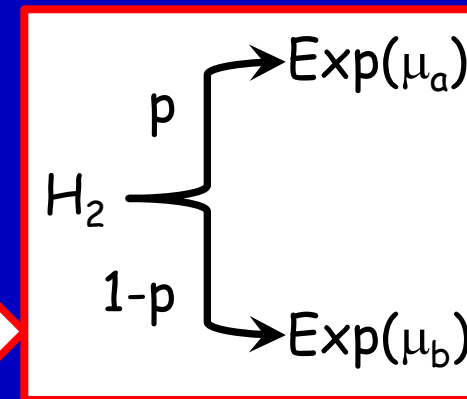
THM: If $m \leq 3$, SITA converges
 If $m > 3$, SITA diverges

THM: LWL always converges for $p < 1$
 $E[X^{3/2}] = p_a a^{3/2} + p_b b^{3/2} + p_c c^{3/2}$
 $\rightarrow E[X]^{3/2} + 1 + 1$

Results (2 server system)



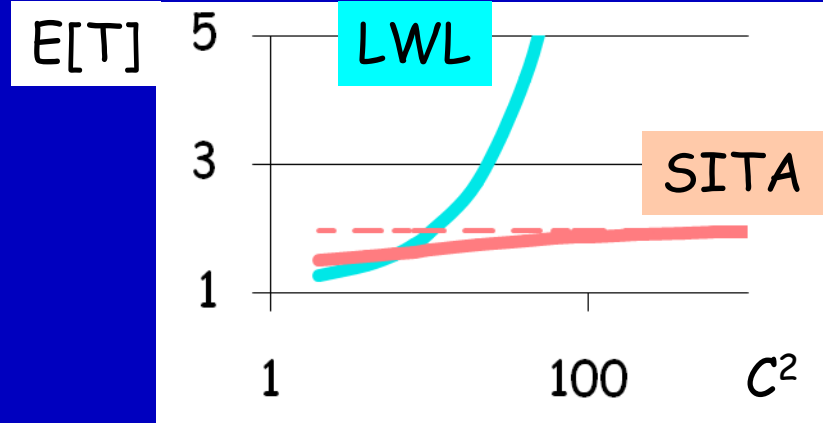
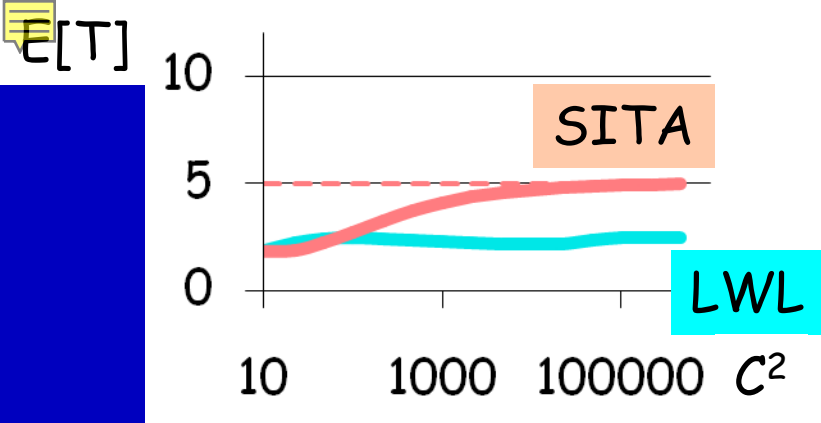
	Conv. LWL	Diverg. LWL
Conv. SITA	✓✓	✓✓
Diverg. SITA	✓✓	✓✓



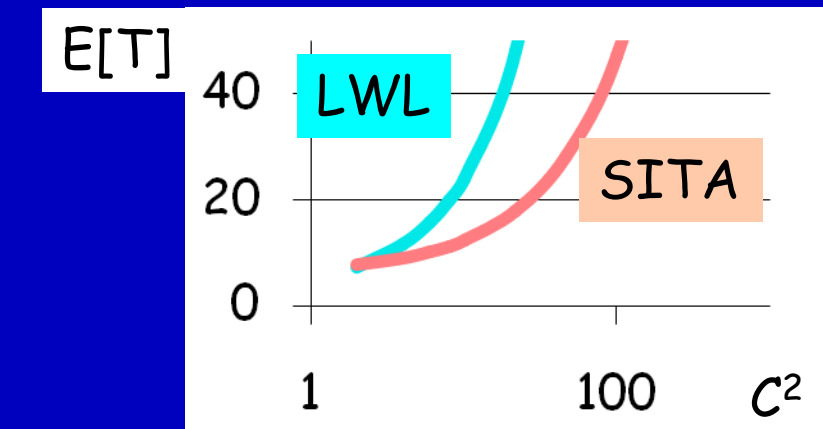
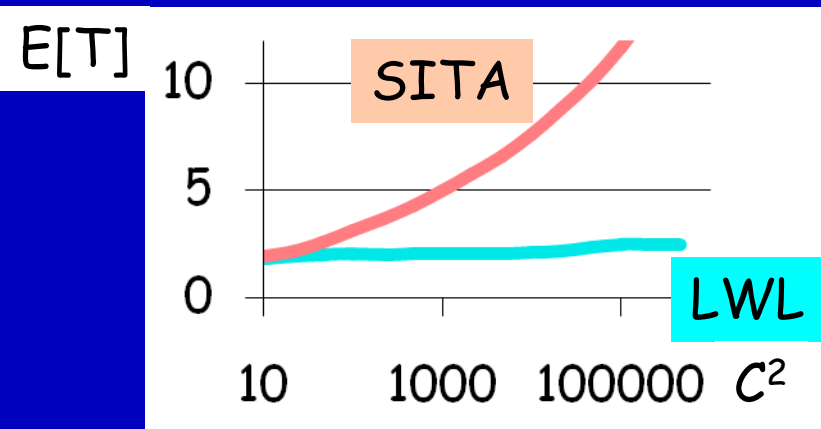
Way more complex, because job types overlap!



"Separation in the limit"



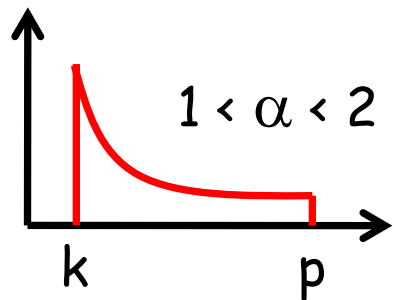
	Conv. LWL	Diverg. LWL
Conv. SITA	✓	✓
Diverg. SITA	✓	✓



Bounded Pareto (2 server system)

	Conv. LWL	Diverg LWL
Conv. SITA		
Diverg SITA	← depends on α →	

$X \sim$ Bounded
Pareto (k, p, α)



Lemma: As $C^2 \rightarrow \infty$, but $E[X]$, α : const,
 $k \rightarrow (\alpha - 1)/\alpha \cdot E[X]$
 $p \rightarrow \infty$

THM: SITA always diverges.

THM: If $\alpha > 3/2$ and $\rho < 1$, then LWL converges. Else LWL diverges.

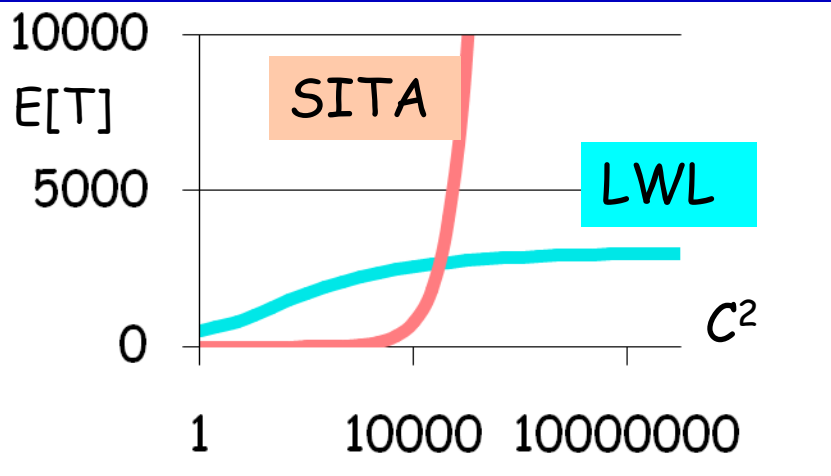
Extends to $n > 2$ servers when $\rho < n - 1$

Bounded Pareto Results

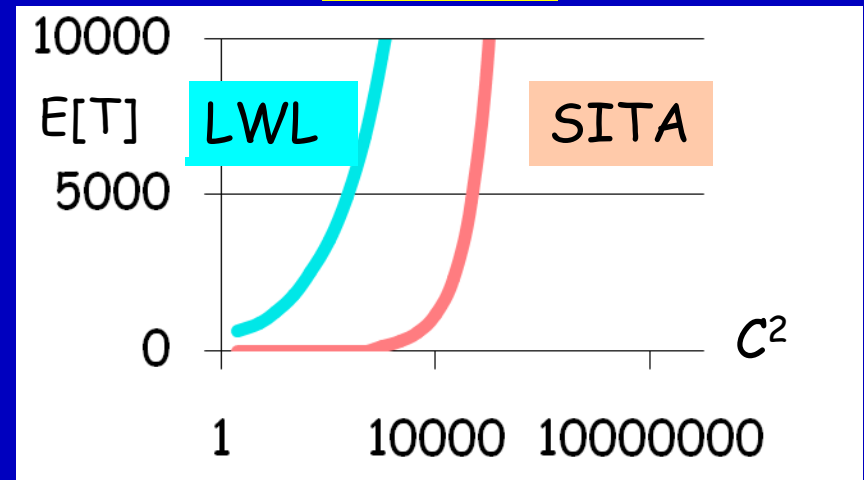
Why was this not noticed?



$\alpha = 1.6$

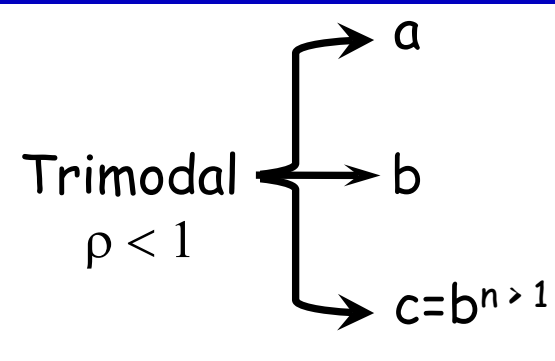


$\alpha = 1.4$

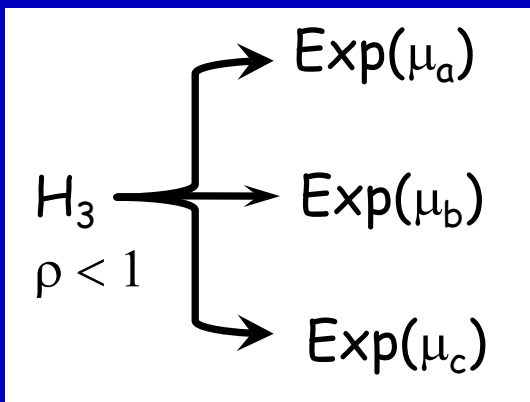


	Conv. LWL	Diverg. LWL
Conv. SITA		
Diverg. SITA	✓	✓

Summary



or



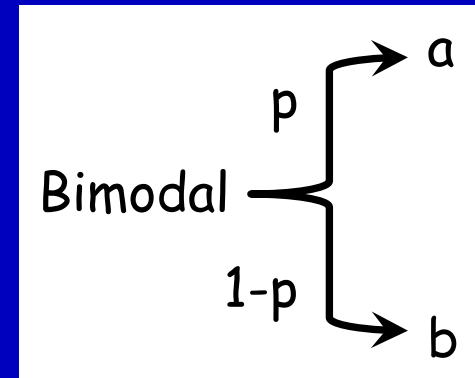
Conv.
LWL

Diverg.
LWL

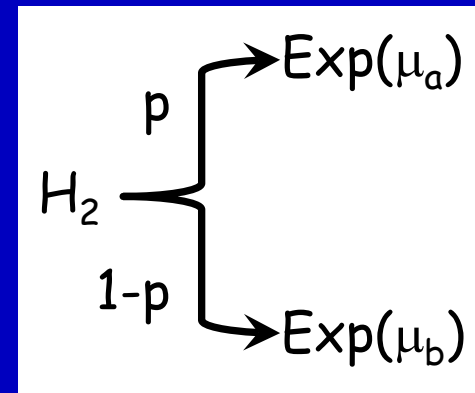
Conv.
SITA

Diverg.
SITA

	Conv. LWL	Diverg. LWL
Conv. SITA	✓✓	✓✓
Diverg. SITA	✓✓✓	✓✓✓

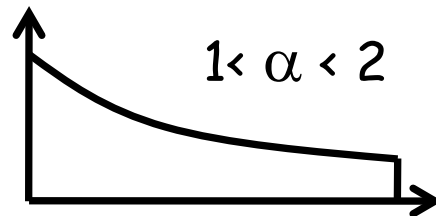


or



Bounded Pareto(α)

$$1 < \alpha < 2$$



Old Nursery Rhyme

	Conv. LWL	Diverg. LWL
Conv. SITA	✓	✓
Diverg. SITA	✓	✓



When SITA is good, it is very, very good
But when it is bad, it is horrid.