

Supplementary Material

- S0.mp4: Spectrum of A for the DSB at $T = 0$ with a varied hierarchy depth. Model parameters: $g = 0.3, \omega_0 = 1, J = 0.2$.
- S1.mp4: Spectrum of A for the 2-site Holstein model at $T = 0$ with a varied hierarchy depth. Model parameters: $g = 0.3, \omega_0 = 1, J = 0.1$.
- S2.mp4: Spectrum of A for the 3-site Holstein model at $T = 0$ with a varied hierarchy depth. Model parameters: $g = 0.3, \omega_0 = 1, J = 0.2$.
- S3.mp4: Spectrum of A for the Holstein model at $T = 0$ and $L = 2$ with a varied number of sites. Model parameters: $g = 0.3, \omega_0 = 1, J = 0.2$.
- S4.mp4: Spectrum of A for the CSB at $T = \frac{1}{\pi}$ and $K = 0$ with a varied hierarchy depth. Model parameters: $\lambda = 0.15^2, \gamma = 0.5, J = 0.2$.
- S5.mp4: Spectrum of A for the CSB at $T = \frac{1}{\pi}$ and $K = 1$ with a varied hierarchy depth. Model parameters: $\lambda = 0.3^2, \gamma = 0.5, J = 0.2$.