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DEVELOPMENT AND PARALLEL IMPLEMENTATION OF ALGEBRAIC MULTIGRID SOLUTION ALGORITHMS FOR EXTENDED FINITE ELEMENT METHODS TO MODEL THREE-DIMENSIONAL CRACK PROBLEMS

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(Host: Prof. Haim Waisman)

Algebraic Multigrid (AMG) does not work very well when directly applied to fracture mechanics problems modeled with extended finite element methods (XFEM) because of several factors. For instance, the XFEM enrichment functions, having a local support near the crack at original fine mesh, cannot be directly or accurately represented on the coarser discretizations generated by AMG. The value of XFEM for modeling fracture mechanics problems lies in its ability to directly incorporate discontinuities in the shape functions. These shape functions span seamlessly across the elements containing the discontinuities, so that the mesh itself need not conform to them. However, this results in graph patterns of the tangent stiffness matrix that contain strong couplings that cross crack discontinuities. The different number of degrees of freedom in XFEM, for regular mesh nodes and enriched nodes, also poses a difficulty for AMG.

In the authors' earlier works, AMG methods were adapted to work for XFEM and applied to solve two dimensional crack problems. In one approach, the Schur complement of the XFEM linear system (in which the enriched degrees of freedom were condensed out) was used to develop a Hybrid-AMG method such that crack-conforming aggregates were formed. Another alternative approach involves transforming the original XFEM linear system into a modified system that is amenable for a direct application of AMG. It has been shown that if only Heaviside-enrichments are present, a simple transformation decouples the linear system along the discontinuities and retains only the regular degrees of freedom in the AMG mesh hierarchies. A back-transformation of the resulting solution vector provides the solution to the original linear system.

This approach has been further extended for three dimensional crack problems modeled with XFEM using user element subroutines developed in the nonlinear finite element program FEAP. These user elements have been further adapted to work with the parallel version of the program ParFEAP. The proposed AMG solution algorithms are implemented using the ML and MueLu packages on the Trilinos framework developed by Sandia National Laboratories. The solution subroutines within ParFEAP have been modified to work with the Trilinos framework. This allows for the proposed XFEM and AMG methods to be scaled up for solving large problems on parallel computing clusters with distributed memory architectures. Various numerical examples are presented to verify the accuracy of the resulting solutions and the convergence properties of the AMG algorithm. The parallel scalability performance of the implementation is also discussed.