A Robust Online Parametric Identification Method for Non-Deteriorating and Deteriorating Distributed Element Models with Viscous Damping

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Reprint of a paper published in the Int. Jo of Nonlinear Mechanics
International Journal of Non-Linear Mechanics
Volume 42, Issue 10, December 2007, Pages 1194-1203
http://dx.doi.org/10.1016/j.ijnonlinmec.2007.09.003

24 August 2007 1:15

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ABSTRACT

The online parametric identification of deteriorating and non-deteriorating distributed element models (DEMs) with viscous damping is studied using a generalization of Masing model to provide the proper framework for identification. The approach renders the hysteretic response of the DEM into a time-independent single-valued mapping from equivalent displacement values into equivalent force values, while considering the effect of damping as a parallel element. This approach allows for parametric identification of this nonlinear rate-dependent hysteretic behavior to be performed using nonlinear optimization techniques. A changing objective function, defined as a norm of force estimation error over a shifting window of recent data, is employed so that classic nonlinear optimization techniques can be used for the online identification problem. A variation of the steepest descent method is used with significant modifications. Special measures are taken to guarantee robustness of the results in presence of noise. The results show that the proposed identification method exhibits a very good performance in identifying the correct values of the parameters in real time, and is robust in dealing with noise. The proposed method can be applied to many other types of hysteretic behavior as well.

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1 Introduction and Motivation

The modeling of nonlinear hysteretic behavior in structural and mechanical systems has been an area of considerable interest for the purpose of modeling the source of energy dissipation and to track the oft-observed deterioration that accompanies the hysteresis. Several researchers including authors Ashrafi and Smyth have explored a variety of techniques to identify parameters in assumed models from nonlinear hysteretic vibration response data. Representative studies of this problem are reported in the works of Masri and Caughey (1979), Spanos (1981), Toussi and Yao (1983), Andronikou and Bekey (1984), Vinogradov and Pivovarov (1986), Iwan and Cifuentes (1986), Jayakumar (1987), Yar and Hammond (1987), Misawa (1989), Roberts and Spanos (1990), Masri and Caughey (1991), Peng and Iwan (1992), Loh and Chung (1993), Masri (1994), Benedettini et al. (1995), Chassiakos et al. (1995, 1998), Sato and Qi (1998), Smyth et al. (1999), Kosmatopoulos et al. (2001), Lin et al. (2001), Rüdinger and Krenk (2001), Smyth et al. (2002), Yuen and Beck (2003), Yang and Ma (2003), Peifer et al. (2003), Saadata et al. (2004), Lin and Betti (2004), Yang et al. (2004), Li et al. (2004), Yang et al. (2006), Ashrafi et al. (2006), Ashrafi and Smyth (2006), and Ashrafi (2006). Use of time-variant parameters to model the changes in behavior of a system can be very effective in tracking the response during measured transient response (Smyth et al., 1999). However, while such a result is extremely useful from the standpoint of robust adaptive control, where the principal requirement is accurate prediction of the nonlinear restoring force of the structure, it has distinct shortcomings when one seeks to re-use the identified model to perform numerical simulation with a new and different excitation. This shortcoming is due to the fact that any identified parameter variation, say loss of stiffness, is unique to the specific response time-history, and would not match the parameter time variation for another arbitrary excitation. This difficulty stemmed from the use of assumed models that are intrinsically invariant in time (not exhibiting deterioration) and using time variation in the identified parameters to indicate deterioration or change. A preferable class of models would exhibit deterioration with a set of time-invariant model parameters.

This led authors Ashrafi and Smyth (2007) to recently propose a new generalized Masing model approach which is highly efficient in capturing deterioration and whose parameters are physically meaningful. This latter point is of great importance to modeling in general, but in particular when performing model identification for the purposes of health monitoring and damage detection. This approach provides a direct analytical link from the model parameters for a distributed element model (DEM) to the model restoring force vs. displacement curve. In addition, the single-valued mapping from the defined equivalent displacement values to equivalent force values provides a framework for efficient parametric identification of the hysteretic behavior. Successful offline and online identification methods for this model were presented in Ashrafi et al. (2006) and Ashrafi and Smyth (2006). However, the studied behavior in these papers was rate-independent. In this paper, online identification of the rate-dependent hysteretic behavior of the DEM with viscous damping is studied. The emphasis is on the more challenging deteriorating model although the same formulation applies to the non-deteriorating model as well. This study includes introduction of a new method for online identification which is very robust in dealing with noise, and adjusts its behavior based on the observed parameter changes as well as different characteristics of the data. The proposed method can be applied to other hysteretic models as well, which is a subject of further investigation by the authors.

2 Modeling the Deteriorating DEM with Damping

The generalized Masing model proposed in Ashrafi and Smyth (2007) builds on the work of others using DEM, but with several critical distinguishing features. The distributed element model, as shown schematically in Figure 1, is composed of a set of parallel elements which consist of in-

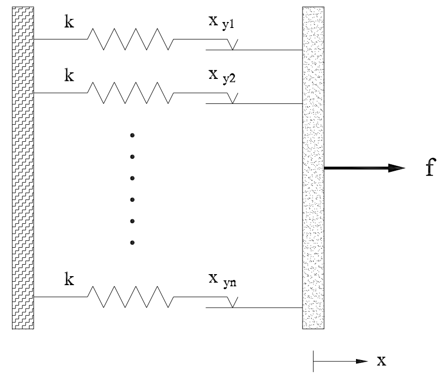

Fig. 1. Distributed Element Model. Element have the same stiffness k but different yield displacement thresholds x_{yi}.

series springs and sliders which slide (or yield) at prescribed yield displacement thresholds. The non-deteriorating DEM was introduced in Iwan (1966) and is also called the Iwan model. The distribution of the element yield displacements of the model may take a variety of shapes. The uniform distribution was the example element yield force distribution originally used by Iwan. Cifuentes and Iwan (1986) expanded the Iwan model to incorporate deterioration as well. Figure 2 shows a schematic view of the deteriorating Iwan model. The model consists of three types of elements in parallel. One elastic element, one elastoplastic element, and a series of elastoplastic elements breaking after their respective displacement limits. The breaking of each element happens when it exceeds a deformation of βx_{yi}, where β is the ductility, and x_{yi} is the corresponding element yield displacement threshold. The result is deterioration in both the strength and stiffness of the model. New deterioration happens only when exceeding the past maximum amplitude of displacement. The ductility is assumed to be the same for all the elements. The effect of adding the first two elements to the model is to preserve a bilinear behavior at the end when all other elements have broken. This well-known bilinear behavior can be added to the response of the rest of elements. Hence, the behavior of a system consisting only of the elements with the capacity to break was considered in Ashrafi and Smyth (2007). This model was referred to as the deteriorating DEM rather than deteriorating Iwan model.

By studying the response of DEMs to arbitrary excitations, one can begin to understand how the memory is stored in the system response. Rather than requiring the storage of the entire previous

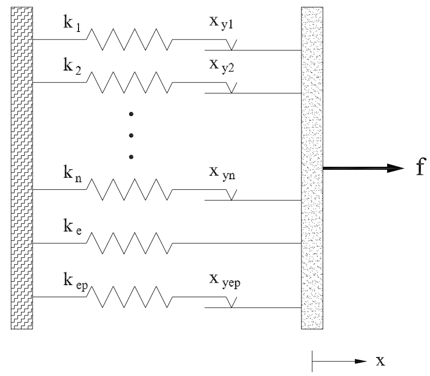

Fig. 2. Deteriorating Iwan model. There is one elastic element with stiffness k_e and one element with elastic stiffness k_{ep} and yield displacement threshold x_{yep}. The other elements have different stiffness and yield displacement thresholds and break when the total displacement exceeds the product of ductility and their yield displacement thresholds.

excitation and response history to model future responses, it turns out that the model only needs information on a few discrete milestones in its history in order to predict response behavior in the immediate future. These milestones are actually a particular sequence of maxima and minima in the recent history of the input. This sequence was called the Sequence of Dominant Alternating Extremes (SDAE) in Ashrafi and Smyth (2007). The recent history is defined by the time from the previous global maximum or minimum in the input. Most importantly, by keeping track of these relatively few overall response displacement maxima/minima, one circumvents the need to keep track of the state of each element within the DEM. This concept is valid for both the non-deteriorating and deteriorating responses. More details about this approach and how to exploit the SDAE can be found in Ashrafi and Smyth (2007). n_{SDAE}, the number of points in the SDAE, is evolving in time. At each time, x_{yt}, the yield displacement threshold of the element that would just start to yield, is also determined based on the SDAE.

It has been shown that the non-deteriorating DEMs fall under a class of extended Masing models (Jayakumar, 1987). Thyagarajan (1990) showed that the degrading behavior can be described using an extension of Masing rules. Chiang (1992) also generalized the Masing model rules to handle the response to arbitrary transient excitation with deterioration. The resulting computational model for the response was however relatively complex requiring continuous updating for each loading and unloading cycle. In addition, the deterioration was introduced in a comparatively non-specific damage index. Then, when the Iwan deterioration hypothesis (Iwan and Cifuentes, 1986) is used, the response is only given in terms of the slope for the case of stable cyclic loading. In contrast, in Ashrafi and Smyth (2007) the concept of the observed SDAE, which entirely describes the system memory, is used to classify the restoring force versus displacement response curves for the same physical model. A more clear and rigorous formulation is presented of how to capture the memory and determine the current response based on the SDAE compared to Thyagarajan (1990). In addition, new generalized deteriorating Masing rules are proposed which result in a computationally efficient modeling of the hysteretic behavior and a time-independent formulation for different segments of the hysteretic force-displacement response. These rules are:

Rule1: When the current displacement is maximum in amplitude, the response is on the virgin loading curve, called Curve 1.

Rule2: For unloading and reloading when Rule 1 is not governing, the response curve is a piecewise smooth function defined by different segments of Curve 2, the virgin curve stretched by a factor of two in both directions. At each moment, $X = 2x_{yt}$, is the position on Curve 2 defining the response. x_{yt}, the yield displacement threshold of the element that would just start yielding, is a piecewise continuous function of time calculated using the SDAE.

The response of the model is on virgin curve when the current displacement has the maximum amplitude in history. The virgin curve can be obtained by applying a monotonic increase or decrease to the model displacement. Equivalent displacement values X and equivalent force values F are defined at each time. X is the position on Curve 1 or Curve 2 which is used to represent the response, while F is the corresponding force value on either curve. $X = x_{yt} = |x|_{max}$ when on Curve 1 while $X = 2x_{yt}$ when on Curve 2. The equivalent displacement and force values are always positive. However, it is possible for noisy experimental data to yield some negative values for F. There is a similar set of rules for the non-deteriorating model as well which are not presented here. These rules are equivalent to the rules for the extended Masing model by Jayakumar (1987), but can also be considered as a special case of the above rules when $\beta \to \infty$.

Curves 1 and 2 were shown to be directly related analytically to the ductility and initial stiffness of the model, as well as the probability distribution function for element yield displacement thresholds of the DEM. The details of this relationship are presented in Ashrafi and Smyth (2007), but it is important here to stress the relative simplicity of the result. There are only two root curves which define the response. Curve 1 represents the response to virgin loading, while Curve 2 represents the response to non-virgin loading. Curve 2 is analogous to Curve 1 enlarged by a factor of two, except that the effect of the elements broken during virgin loading is excluded from it. At each time, an equivalent displacement X and equivalent force F determines the location of the response on Curve 1 or Curve 2. As one simulates the response for a given distribution, one simply switches back and forth between the two curves. Depending on the changes in the global extreme displacement $|x|_{max}$, Curve 2 is occasionally updated. An advantage of the proposed formulation is that it provides a single-valued mapping from equivalent displacement values to equivalent force values which is not time-dependent. This is in contrast to the fact that there is no such single-valued mapping from displacement values to force values in hysteretic models. This property opens the door to using classic optimization techniques for identification of the hysteretic response.

For the more general case of the deteriorating DEM, where $\phi(x)$ is distribution function of the element yield displacement thresholds with $\int_0^\infty \phi(x)dx = 1$, β is the ductility, and all of the elements have stiffness $\bar{k}$ such that the initial stiffness of the model is equal to $\int_0^\infty \bar{k}\phi(x)dx = \bar{k}$, the equivalent force trajectory is defined by the following curves:

$$F = Curve_1(X) = \bar{k}\int_{\frac{X}{\beta}}^{X} \bar{x}\phi(\bar{x})d\bar{x} + \bar{k}X\int_{X}^{\infty} \phi(\bar{x})d\bar{x} \tag{1}$$

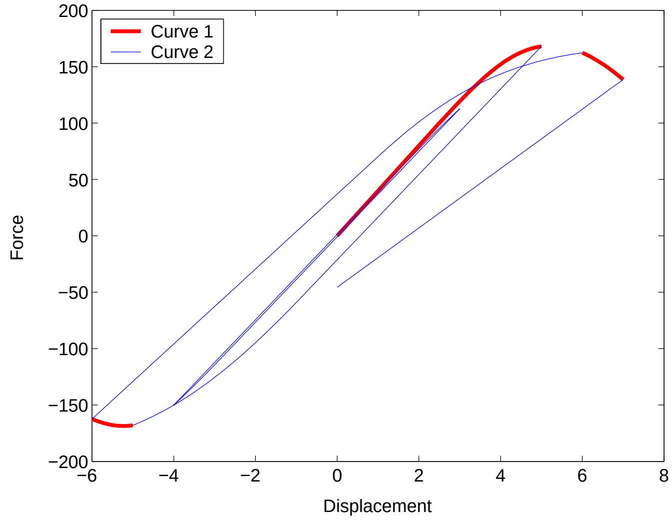

Fig. 3. A sample deteriorating restoring force vs. displacement response trajectory highlighting the portions corresponding to $Curve_1(X)$ (the thicker line) and $Curve_2(X, |x|_{max})$.

$$F = Curve_2(X, |x|_{max}) =$$

$$= \begin{cases} \bar{k}X \int_{\frac{|x|_{max}}{\beta}}^{\infty} \phi(\bar{x})d\bar{x} \quad \text{for} \quad X < \dfrac{2|x|_{max}}{\beta} \\ \\ 2\bar{k} \int_{\frac{|x|_{max}}{\beta}}^{X/2} \bar{x}\phi(\bar{x})d\bar{x} + \bar{k}X \int_{X/2}^{\infty} \phi(\bar{x})d\bar{x} \quad \text{for} \quad \dfrac{2|x|_{max}}{\beta} \leq X \leq 2|x|_{max} \end{cases} \tag{2}$$

The curves are highlighted in a simulated response shown in Figure 3. Figures 4 shows two displacement and force histories along with the equivalent displacement and force values that are used in this study. In Ashrafi and Smyth (2007), the corresponding equations were also written explicitly for the cases of the uniform, the Rayleigh, and the Weibull distributions. The Rayleigh and Weibull distributions are used in the current study and are defined as:

$$\phi(x) = \frac{\pi}{2}\frac{x}{\alpha^2}\exp(-\frac{\pi x^2}{4\alpha^2}) \qquad \text{Rayleigh distribution} \tag{3}$$

$$\phi(x) = \frac{a}{b}\left(\frac{x-x_0}{b}\right)^{a-1}\exp\left[-\left(\frac{x-x_0}{b}\right)^{a}\right] \quad \text{for} \quad x \geq x_0 \qquad \text{Weibull distribution} \tag{4}$$

The DEM has rate-independent hysteresis. However, rate-dependence is characteristic of many physical systems. To be able to use the considerable modeling efficiency of the proposed model in modeling rate-dependent behavior, an element with rate-dependent behavior can be added in

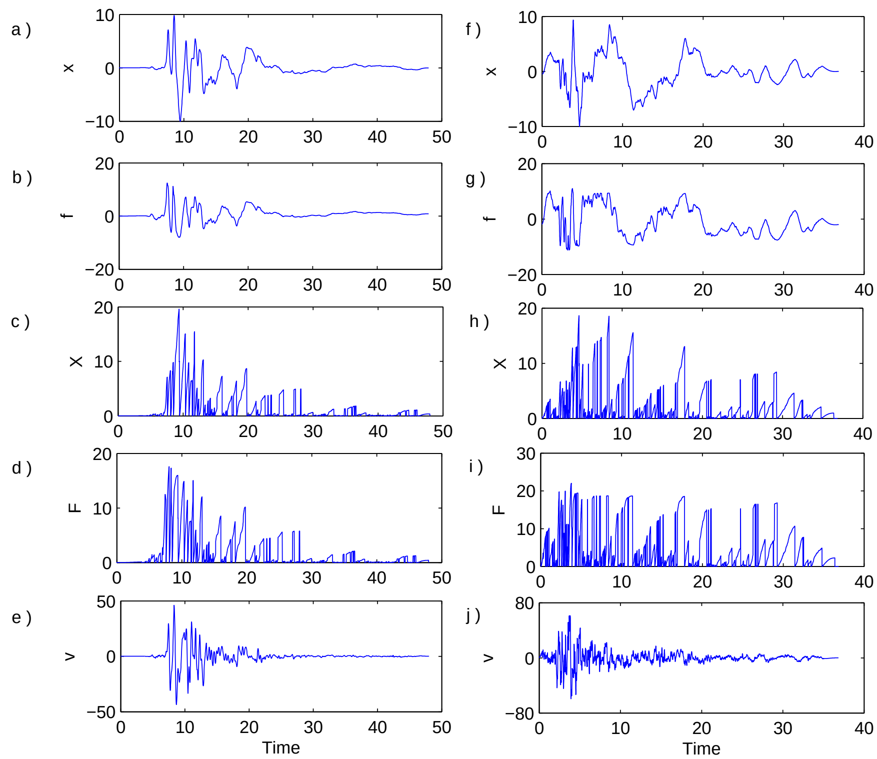

Fig. 4. The time histories used for this study. The left side panels show the response of a model with Rayleigh distribution of parameters with $\beta = 2, \bar{k} = 4$ and $\alpha = 4$ to the scaled KOBE/KJM000D record from the Kobe earthquake on 16/1/1995. The right side shows the response of a model with $\beta = 4, \bar{k} = 4$ and $\alpha = 3$ to the record SFERN/L04111 from the San Fernando earthquake on 9/2/1971. The earthquake records are obtained from the PEER Strong Motion Database (PEER, 2003). Different rows show the displacement, restoring force, equivalent displacement and force, and velocity time histories.

parallel to the DEM. In Iwan and Cifuentes (1986) and Cifuentes and Iwan (1989), a small viscous damping was added to the deteriorating Iwan model. The same approach is used here by adding a viscous damping element in parallel to the deteriorating DEM. The corresponding model is called the Distributed Element Model with Viscous Damping (DEMWVD) and is shown in Figure 5. The added damping has the additional advantage that it preserves some damping in the model for small values of displacement, where the response of the deteriorating DEM would be linear otherwise.

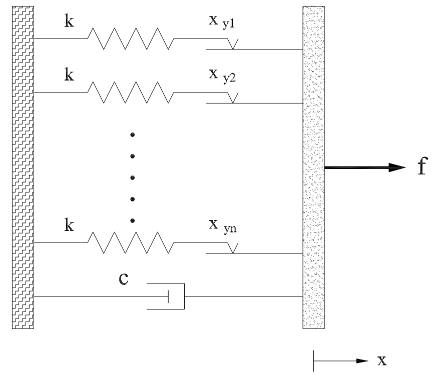

Fig. 5. Distributed Element Model with viscous damping. DEM elements have the same stiffness k but different yield displacement thresholds x_{yi}. The viscous damping element has damping value of c.

3 Online Identification Method for the DEM with Viscous Damping

The presented formulation for equivalent force-equivalent displacement curves is highly nonlinear in parameters. Hence, accurate identification of the parameters requires a method that can cope with any general nonlinear behavior. Many of the identification schemes widely used render the problem linear in parameters and then identify them (Ljung, 1999). Such an approach could require making approximations and putting limitations on the kind of nonlinearity which in turn would reduce the generality and accuracy of the model. Therefore, it is more suitable to use optimization approaches to find the parameters that minimize some norm of the error of the predicted results. The time-independence of the presented formulation opens the door to using classic nonlinear optimization techniques for parametric identification of the hysteretic response. This allows the identification method to be as independent as possible from the model and be applicable to very general cases. The use of nonlinear optimization for offline and online parametric identification of the DEM response has been studied in Ashrafi et al. (2006) and Ashrafi and Smyth (2006).

Although for the DEM and DEMWVD, the parameters do not change in time, the online identification method should be able to track any change in parameters. This adaptive capability is needed for modification of the parameters from their inaccurate initial estimates. However, online nonlinear optimization is a very challenging problem. The typical optimization methods are developed for when there is one known objective function with known data. But in the online problem, the optimization should be performed as new data becomes available. Furthermore, timely processing of the data is essential. Hence, it was proposed in Ashrafi and Smyth (2006) that the optimization be performed on a changing objective function defined as a norm of the force estimation error over a shifting window of the recent data. A variation of the steepest descent method was proposed with significant modifications for this purpose. The underlying assumption was that application of the nonlinear optimization approach at each time step to the corresponding objective function should result in convergence to the neighborhood of the true global optimum values. This assumption and its validity and limitations are discussed in Ashrafi (2006). Successful application of this approach to the real-time identification of the parameters was only made possible because of the efficiency of the proposed generalized Masing model in capturing the hysteretic response. The identification scheme could essentially treat the hysteretic response as sum of a few time-independent mappings of equivalent displacement values to equivalent force values, which was very efficient computationally. In the absence of this property, by changing the system parameters during identification, the whole past response would have to be recalculated to obtain the current response. Instead, based on this formulation, the equivalent force is determined by only one function evaluation and the total force is determined by n_{SDAE} function evaluations. This enormous computational cost reduction made real-time identification of the parameters possible and is a very important advantage of the proposed model.

However, in Ashrafi and Smyth (2006), the study was limited to the DEM which has a rate-independent behavior. Although the same philosophy of online nonlinear optimization will be used here for identification of the DEM with viscous damping, an entirely new identification method is suggested. Several changes are made here to the method introduced in Ashrafi and Smyth (2006) to apply it to the DEMWVD. One important change is that the objective function should be defined based on the total force rather than equivalent force values. This need stems from the fact that only the total response of the model can be measured, and the portion of it representing the generalized Masing response is not known a priori. Therefore, it is not possible to determine equivalent force values based on the measured response. Definition of the objective function based on total response might be more advantageous for dealing with noise as well, per studies in Ashrafi (2006). Two objective functions are used in this study which are defined as:

$$J_p(\hat{\boldsymbol{\theta}}) = \sum_{t=p-i+1}^{t=p} (f(t, \hat{\boldsymbol{\theta}}) - \check{f}(t))^2 \qquad \text{Objective function 1} \tag{5}$$

$$J_p(\hat{\boldsymbol{\theta}}) = \sum_{t=p-i+1}^{t=p} \sum_{\hat{t}_1}^{\hat{t}_{n_{SDAE}}} \{(f(\hat{t}, \hat{\boldsymbol{\theta}}) - \check{f}(\hat{t}))^2 / n_{SDAE}\} \qquad \text{Objective function 2} \tag{6}$$

where $\hat{\boldsymbol{\theta}}$ is the estimated parameter vector, $f(t, \hat{\boldsymbol{\theta}})$ and $f(\hat{t}, \hat{\boldsymbol{\theta}})$ are the estimated total forces at time steps t and $\hat{t}$, and $\check{f}$ is the measured restoring force. The first objective function only uses the total force at the current time, while the second uses the total force at all the points in the SDAE. l is the selected window length while $i = \min(p, l)$ is the actual window length. The choice of window length l should be such that the selected window provides reliable information about the response. l should be large enough that it captures representative nonlinear response of the model when it happens. Also, assuming the presence of zero-mean noise, the window length should be such that the portion of noise inside the window has almost zero mean value. Thus, choice of l should be such that the data does not include considerable noise with frequency less than $\frac{1}{l\Delta t}$, where Δt is the data time increment. However, choice of very large windows increases the computation cost and is an obstacle to online identification. In addition, for systems with time-variant parameters, it will also result in slower identification of the changes in parameters. One solution that will reduce the computational cost is to use every n-th point in the window, which reduces the cost by a factor of n. Therefore, using a window which is n times as big will take the same time. Furthermore, the shifting window which advances by only one time step at a time will, in the following steps, use the points that were not previously used. Therefore, the high frequency data will not be lost. The proposed solution is also useful for keeping the computational cost down when the time sampling of the data is too fine.

A new method inspired by the steepest descent method is proposed here for optimization of the objective function. The original method is shown in Equations 7 and 8 (Haftka et al., 1990). $\boldsymbol{s}$ is the unit vector in the direction where the objective function has the steepest descent. ε is determined either proportional to norm of ∇J or through a line search such that the function J is minimized along $\boldsymbol{s}$. These equations are repeatedly applied until convergence to the optimum value of the fixed objective function is achieved.

$$\hat{\boldsymbol{\theta}}_{p+1} = \hat{\boldsymbol{\theta}}_p + \varepsilon_p \boldsymbol{s}_p \tag{7}$$

$$\boldsymbol{s}_p = -\frac{\nabla J_p}{\|\nabla J_p\|} \tag{8}$$

In this study, several changes are made to this method. These modifications are necessary considering the changing definition of the objective function over time, as well as the requirement for the identification to be fast. In a nonlinear response, it is quite possible that the set of optimum or near-optimum parameters for different parts of the response is not unique. Also, model inaccuracies and presence of noise might result in bias in parameters that minimize the error function. Furthermore, the optimum parameters calculated based on a window of data might not be the same as those for the entire data (Ashrafi, 2006). Hence, instead of fully minimizing the objective function at each time step through several iterations, only one iteration is performed at each time step. This will also be helpful by drastically decreasing the identification time. To ensure that each segment of data has a limited effect on the result and cannot cause divergence, or convergence to the wrong values, the norm of change of parameters at each time step is restricted. Furthermore, success of the optimization requires the parameters to be normalized and be roughly of the same order. The proposed adaptive law is:

$$\hat{\boldsymbol{\theta}}_{p+1} = \hat{\boldsymbol{\theta}}_p + \varepsilon_p \boldsymbol{s}'_p \tag{9}$$

$$\boldsymbol{s}_p = -\frac{\nabla J(\hat{\boldsymbol{\theta}}_p)}{\|\nabla J(\hat{\boldsymbol{\theta}}_p)\|} \tag{10}$$

$$\boldsymbol{s}'_p = \boldsymbol{s}_p + \boldsymbol{r}_p \tag{11}$$

$$\delta\theta_{max} = (2 \ \sim 5)\frac{\delta\theta_{est}}{t^*} \tag{12}$$

$$\varepsilon_p = \begin{cases} \lambda_x(p) * \lambda_v(p) * \lambda_f(p) * \lambda_\theta(p) * \delta\theta_{max} & \text{if } J(\hat{\boldsymbol{\theta}}_{p+1}) \leq J(\hat{\boldsymbol{\theta}}_p) \\ 0 & \text{if } J(\hat{\boldsymbol{\theta}}_{p+1}) > J(\hat{\boldsymbol{\theta}}_p) \end{cases} \tag{13}$$

$$\lambda_\theta(p) = \begin{cases} 1 & \text{for } p \leq l \\ \lambda_\theta(p-1) * \delta_d \geq \dfrac{\delta\theta_{min}}{\delta\theta_{max}} & \text{for } \varepsilon_p \varepsilon_{p-1} \boldsymbol{s}'_p . \boldsymbol{s}'_{p-1} < 0 \\ \lambda_\theta(p-1) * \delta_i \leq 1 & \text{for } \varepsilon_p \varepsilon_{p-1} \boldsymbol{s}'_p . \boldsymbol{s}'_{p-1} > 0 \;\;\&\;\; \varepsilon_p \varepsilon_{p-2} \boldsymbol{s}'_p . \boldsymbol{s}'_{p-2} > 0 \\ \lambda_\theta(p-1) & \text{otherwise} \end{cases} \tag{14}$$

Identification of the parameters starts from the l-th time step. The index p in Equations 9 to 14 represents the current time step, for which only one iteration is performed on an objective function whose definition is changing in time. On the contrary, p in Equations 7 and 8 represented the iteration number for optimization of an objective function whose definition did not change in time. To help elevate the slow convergence shortcoming of the steepest descent method, a unit random vector $\boldsymbol{r}_p$ is used in Equation 11 if it results in further reduction in J. Otherwise, $\boldsymbol{r}_p$ is set to zero. In this paper, three random vectors are generated for $\boldsymbol{r}_p$ at each time step for possible use in Equation 9. ε, the norm of change in parameters at each time step, has an upper limit $\delta\theta_{max}$ which is further reduced at each time step by four reduction functions λ_x, λ_v, λ_f and λ_θ. ε is set to zero if the amount of change in the parameters is so much that it results in increase in J. $\delta\theta_{max}$ is determined based on $\delta\theta_{est}$, a reasonable upper estimate of the norm of error in parameters, and t^*, the number of time steps in which the method is to have the capacity to achieve that modification. The choice of t^* depends on the length of the nonlinear data and its level of nonlinearity, how fast the modification is required, and how noisy the data is (or how large a window of data is reliable). It is suggested that t^* be at least as large as the window of data used for evaluation of J. The resulting $\delta\theta_{max}$ should not be so small that it prohibits timely modification of parameters or so large that it leads to divergence or convergence to wrong parameters based on limited data. Also, considering the sensitivity of the force-displacement function to the parameters, the change of parameters in one step should not be so large that it causes fundamental change in model properties or so small that the change in the model is negligible. $\delta\theta_{min}$ is the lower limit of ε before application of the first three reduction functions. The changes in ε are to allow for the parameters to be modified rapidly when far from the optimum solution, while being able to converge gradually toward the correct solution in its vicinity. $\delta\theta_{min}$ should be selected small enough that the final error in the estimated values is small. However, with choice of a very small value, it will take a long time for ε to rise to reasonable levels again when needed, which is undesirable for online identification. Also, especially in presence of noise, it is pointless to choose $\delta\theta_{min}$ very small.

It was shown in Ashrafi (2006) that in presence of noise, the parameters that minimize the error function can be different for different segments of the data, and be different from the true parameters of the model. The optimizations were done offline and hence, this observation is a mathematical property independent from the details of any online identification scheme. This adverse effect on the accuracy of the identified parameters is specifically present when the data has low nonlinearity and a high level of noise. Hence, full minimization of the error function does not yield robust accurate estimates of the parameters. For accuracy and robustness of the identification, the changes in the parameters should be limited where noise would have considerable effects. This is particularly necessary for the low-frequency noise whose period is larger than the time span used for evaluating J. The reduction functions λ_x, λ_v, and λ_f, as shown in Figure 6, are used to achieve this goal. λ_x reduces ε when $x_{yt,avg}$, the average value of x_{yt} over the used window of data, is small. This value which can indicate the level of nonlinearity can be also seriously reduced by presence of noise, and hence is a useful parameter. λ_v and λ_f have the same function using the standard deviations of v and f over the same window of data. This will effectively limit the change of parameters where noise has a significant contribution to the data. These values are calculated at each time step p. The parameters x_l, x_h, v_l, v_h, f_l and f_h are selected considerably larger than the expected contribution of noise. These values have significant effect on robustness of the identification method. Since the data is richest for high levels of excitation and force, cutting more of the low-amplitude

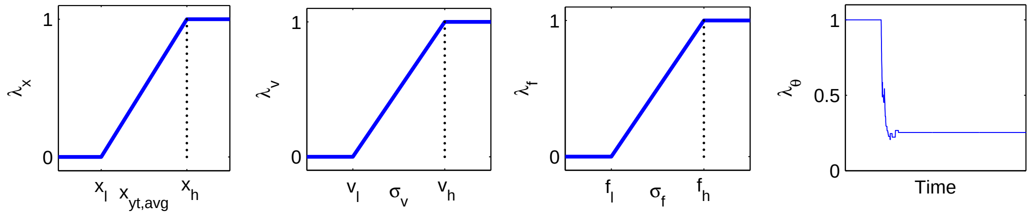

Fig. 6. The reduction functions used in Equations 13 and 14 of the adaptive law to regulate the change of parameters at each time step. The function λ_θ depends on the history of the changes in parameters in any particular case. Therefore, the function λ_θ shown here is only an example of how λ_θ could change in time between 1 and $\delta\theta_{min}/\delta\theta_{max}$.

data should not have a serious effect on richness of the data, but can drastically diminish the effect of noise. Therefore, these values should be selected conservatively high. It should be added that depending on the characteristics of the model and the data used for identification, it is also possible to use other reduction functions to achieve the goal of robustness.

A new feature of the proposed adaptive law is the use of the λ_θ function to regulate ε at each time step. This function reduces when $\boldsymbol{s'}$ is reversing in direction in a general sense, which is what happens when zigzagging about a solution. On the other hand, when $\hat{\boldsymbol{\theta}}$ is modified and there is no such zigzag behavior for at least two consecutive time steps, λ_θ starts to increase. However λ_θ always remains within the defined bounds. Therefore, ε can adjust while being guaranteed not to die out or increase without bounds. This allows for the parameters to be modified rapidly when the solution is far from the optimum solution. At the same time, the parameters will be modified gradually in the neighborhood of the solution in a manner that elevates the zigzag problem. An advantage of this approach is that it adjusts the identification behavior based on the observed characteristics of the objective function and the changes in the identified parameters and does not require any a priori assumptions. Two parameters δ_i and δ_d determine the rate at which λ_θ can increase or decrease respectively. $\delta_i > 1$ while $\delta_d < 1$. In this study, $\delta_i = 1.2$ and $\delta_d = 0.95$ are used.

4 Identification Results

The proposed identification scheme is applied to the response of two DEMWVDs to earthquake records. The left side of Figure 4 shows the response of a model that experiences strong nonlinearity and deterioration. The right side of Figure 4 shows the response of a model that undergoes less degradation. Zero-mean Gaussian white noise is added to displacement, velocity, and force values to study the effects of noise on parameter estimation. The standard deviation of the noise varies depending on the desired noise to signal ratio. The corresponding values of displacement, velocity, and force are $\check{x}, \check{v}$, and $\check{f}$ respectively. The parameter vector is defined as $\boldsymbol{\theta} = [\beta \;\; \bar{k} \;\; \alpha \;\; c^*]^T$ for the Rayleigh distribution and $\boldsymbol{\theta} = [\beta \;\; \bar{k} \;\; a \;\; b \;\; x_0 \;\; c^*]^T$ for the Weibull distribution. Considering the expected range of values for damping, $c^* = 10c$ is used for identification so that its order of magnitude is one. c is the damping coefficient. The studies are performed mostly using the Rayleigh distribution, both as the true model of the system, and the model used for identification. However, to study the effects of model inaccuracy on the identified results, use of Rayleigh distribution for identification of a model with Weibull distribution is also investigated.

Two different strategies should be adopted depending on the level of nonlinearity and deterioration in the response. For highly nonlinear response, shorter intervals of data are needed for convergence to the correct parameters since the convergence is faster. Therefore, higher cut-off values can be chosen for the reduction functions, such that the parameters are modified only when the response is expected to be highly nonlinear. The choice of high cut-off values results in more robustness of the identification results in presence of high levels of noise. On the other hand, for response with low nonlinearity, the convergence to the correct parameters is slower, and hence, identification has to be performed using longer parts of the data which requires choosing lower cut-off values for the reduction functions. However, this results in less robustness to presence of noise.

Table 1 and Figure 7 present the results of online identification for the models in Figure 4 with $c^* = 1$, and contaminated with different noise levels. For the response of the model to the Kobe earthquake, the values $\delta\theta_{max} = 0.25, \delta\theta_{min} = 0.025, x_l = 2, x_h = 2.5, v_l = 8, v_h = 12, f_l =$

1.5, and $f_h = 2.5$ are used in the identification. For the San Fernando earthquake, the values $\delta\theta_{max} = 0.05, \delta\theta_{min} = 0.01, x_l = 1, x_h = 1.2, v_l = 4, v_h = 5, f_l = 0.6$, and $f_h = 0.8$ are used instead. Notice that for the San Fernando earthquake, lower $\delta\theta_{max}$ is used because of less expected error in parameter estimation (closer initial guess), and lower cut-off values for reduction functions are used because of the lower expected level of degradation. For the cases studied, the proposed identification method was very fast and proved capable of performing the identification in real time. It can be observed that the parameters are determined with very good accuracy in presence of noise. Given the slow convergence of the steepest descent method close to the solution (Haftka et al., 1990), and the restrictions on optimization to limit bias in parameters in presence of noise and to limit the computational cost, some minimal error in the final identified results is accepted, even in absence of noise. Within this accepted minimal error range, it is sometimes possible to get less error for more noisy data. There is a trade-off between reducing the minimal error and having a fast and robust method. However, given high nonlinearity in the response, very good convergence can be achieved with high levels of noise and initial parameters far from the correct values. It can be seen that even with as high as 100% error in the initial estimate for some parameters and high levels of noise, the identified parameters are very accurate. Both of the objective functions used gave satisfactory results.

Table 1
The results of online identification for the Distributed Element Model with viscous damping. Different levels of noise are added to the displacement, velocity, and force values to assess robustness of the identification scheme. Normalized noise levels are defined as $\frac{|\breve{x}-x|}{|x|}$, $\frac{|\breve{v}-v|}{|v|}$, and $\frac{|\breve{f}-f|}{|f|}$. The correct parameters are $\hat{\boldsymbol{\theta}} = [2\ \ 4\ \ 4\ \ 1]^T$ for the Kobe record, and $\hat{\boldsymbol{\theta}} = [4\ \ 4\ \ 3\ \ 1]^T$ for the San Fernando record.

Case	record	Objective function	noise level	Initial $\hat{\boldsymbol{\theta}}$	Final $\hat{\boldsymbol{\theta}}$	$\frac{\lvert\hat{\boldsymbol{\theta}}-\boldsymbol{\theta}\rvert}{\lvert\boldsymbol{\theta}\rvert}$
1	Kobe	1	0%	$[3\ \ 3\ \ 6\ \ 2]^T$	$[2.02\ \ 3.80\ \ 4.06\ \ 1.00]^T$	0.034
2	Kobe	1	1%	$[3\ \ 3\ \ 6\ \ 2]^T$	$[2.00\ \ 3.78\ \ 4.09\ \ 1.00]^T$	0.039
3	Kobe	1	5%	$[3\ \ 3\ \ 6\ \ 2]^T$	$[2.03\ \ 3.70\ \ 4.09\ \ 0.99]^T$	0.051
4	Kobe	1	10%	$[3\ \ 3\ \ 6\ \ 2]^T$	$[1.99\ \ 3.89\ \ 4.10\ \ 1.02]^T$	0.025
5	Kobe	2	0%	$[3\ \ 3\ \ 6\ \ 2]^T$	$[2.01\ \ 3.71\ \ 4.10\ \ 1.00]^T$	0.050
6	Kobe	2	1%	$[3\ \ 3\ \ 6\ \ 2]^T$	$[2.01\ \ 3.70\ \ 4.11\ \ 1.00]^T$	0.052
7	Kobe	2	5%	$[3\ \ 3\ \ 6\ \ 2]^T$	$[2.01\ \ 3.71\ \ 4.12\ \ 1.00]^T$	0.051
8	Kobe	2	10%	$[3\ \ 3\ \ 6\ \ 2]^T$	$[2.06\ \ 3.33\ \ 4.27\ \ 1.05]^T$	0.119
9	SF	1	0%	$[3\ \ 4.5\ \ 2\ \ 1.5]^T$	$[3.86\ \ 4.26\ \ 2.93\ \ 1.00]^T$	0.046
10	SF	1	1%	$[3\ \ 4.5\ \ 2\ \ 1.5]^T$	$[3.80\ \ 4.35\ \ 2.92\ \ 1.03]^T$	0.064
11	SF	2	0%	$[3\ \ 4.5\ \ 2\ \ 1.5]^T$	$[3.89\ \ 4.18\ \ 2.96\ \ 1.02]^T$	0.033
12	SF	2	1%	$[3\ \ 4.5\ \ 2\ \ 1.5]^T$	$[3.87\ \ 4.22\ \ 2.95\ \ 1.05]^T$	0.042

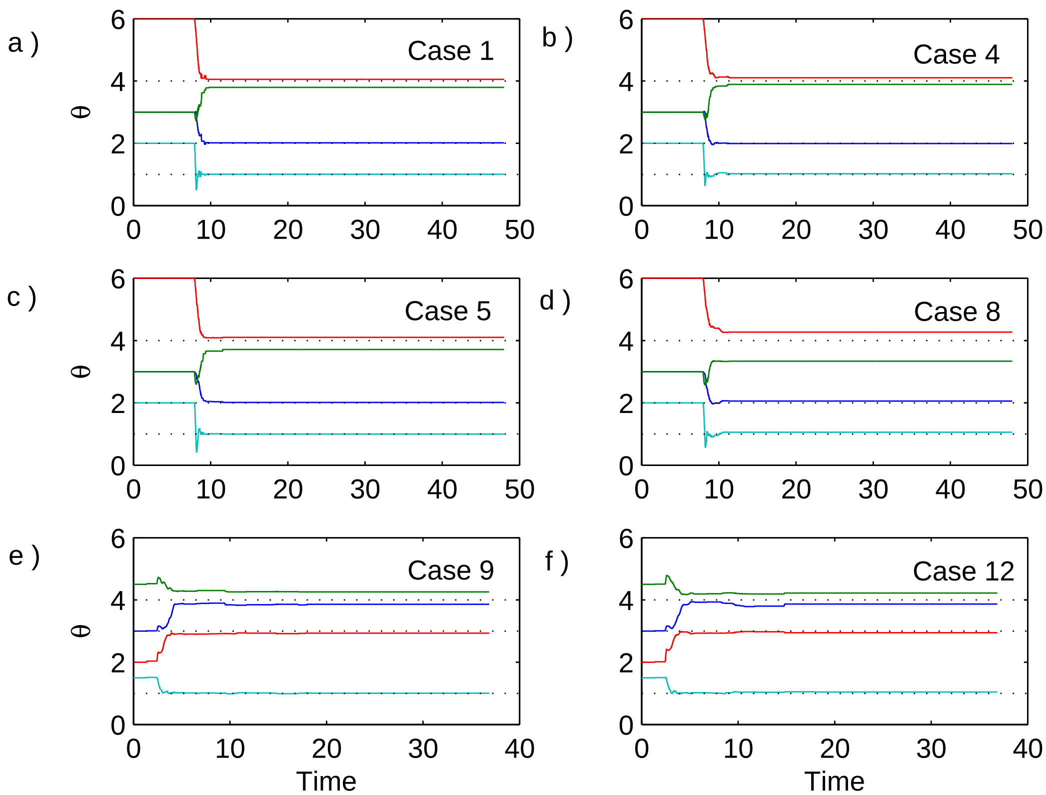

Fig. 7. Change of parameters during identification for several cases of Table 1.

Table 2 shows the identification results when there is inaccuracy in the model. The actual model to be studied has a Weibull distribution with $a = 2.1, b = 7$, and $x_0 = 0.2$. The Rayleigh distribution, used here for identification, is a subclass of Weibull distributions when $a = 2, b = 2\alpha\sqrt{\pi}$, and $x_0 = 0$. Since any Rayleigh distribution will have some error in estimating the correct distribution, Rayleigh distribution can be used in identification to study the effects of model inaccuracies. As observed, the identification results are rather consistent and show much improvement in the parameters. Success of the identified parameters in simulating the response of the model to arbitrary excitation requires good accuracy of the identified ductility and initial stiffness parameters (Ashrafi et al., 2006). Therefore, the results, especially for the first two rows of Table 2 should provide good approximations of the response of the model to arbitrary input. This can be verified by the excellent match between the response of the identified model and the true model to another earthquake, as seen in Figure 8. Therefore, if the model used for identification can provide a reasonably close representation of the actual system, the identified model will also be a close physical approximation of the actual system.

Table 2
The results of online identification of the DEMWVD using inaccurate distribution model. The response of a model with Weibull distribution and parameter vector $\boldsymbol{\theta} = [3 \;\; 4 \;\; 2.1 \;\; 7 \;\; 0.2 \;\; 1]^T$ to the Kobe record in Figure 4a is identified using a Rayleigh distribution. The last column shows the normalized error in the identified distribution.

Case	Obj. func.	noise level	Initial $\hat{\boldsymbol{\theta}}$	Final $\hat{\boldsymbol{\theta}}$	$\int_o^\infty \frac{(\phi(x) - \hat{\phi}(x))^2}{\phi(x)^2}$
1	1	0%	$[3.5 \;\; 4.5 \;\; 6 \;\; 1.5]^T$	$[3.02 \;\; 4.16 \;\; 6.32 \;\; 1.02]^T$	0.06
2	1	10%	$[3.5 \;\; 4.5 \;\; 6 \;\; 1.5]^T$	$[2.96 \;\; 4.11 \;\; 6.45 \;\; 1.06]^T$	0.08
3	2	0%	$[3.5 \;\; 4.5 \;\; 6 \;\; 1.5]^T$	$[3.55 \;\; 3.97 \;\; 6.40 \;\; 1.11]^T$	0.07
4	2	10%	$[3.5 \;\; 4.5 \;\; 6 \;\; 1.5]^T$	$[3.36 \;\; 4.03 \;\; 6.32 \;\; 1.16]^T$	0.06

5 Conclusions

This paper presents a new method for online parametric identification of the non-deteriorating and deteriorating nonlinear hysteretic behavior of the Distributed Element Model with Viscous Damping (DEMWVD). The modeling is based on the generalized Masing model approach introduced

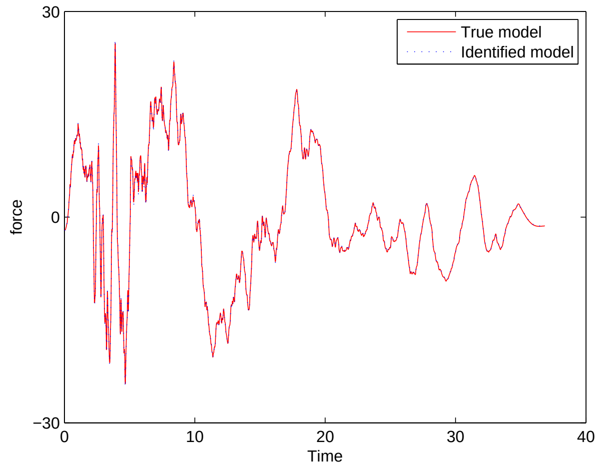

Fig. 8. Verification of the accuracy of the identified model in Case 2 of Table 2. The response of the true and identified models are simulated in response to the record from the San Fernando earthquake.

in Ashrafi and Smyth (2007). The offline identification and online identification of the DEM have been studied in Ashrafi et al. (2006) and Ashrafi and Smyth (2006). For the online identification, adaptive capability is needed for modification of the parameters from their inaccurate initial estimates. Due to the very general nonlinear nature of the model, nonlinear optimization methods are used for online identification. However, online nonlinear optimization is a challenging problem. Typical optimization methods are developed for when there is one known objective function with known data. But in the online problem, timely optimization should be performed as new data becomes available. It is proposed here that the optimization be performed on a changing objective function defined as a norm of the response estimation error over a shifting window of the recent data. Several modifications are applied to the steepest descent method, including allowing one iteration and limited change of parameters per time step.

It is shown that the proposed online identification method is successful in accurate parameter identification for this rate-dependent hysteretic behavior in presence of high levels of noise. Mathematically, the level of nonlinearity in the data should be sufficiently high that the parameters of the nonlinear system can affect the results such that an almost unique optimum solution may be obtained. Several reduction functions are introduced to regulate the norm of the change in parameters such that the effect of noise on the results is reduced and the speed of the change in parameters is adjusted based on the distance form the optimum solution. The identification for the examples studied here is achieved in real time. Successful application of this approach to the real-time identification of the parameters is only possible because of the efficiency of the proposed generalized Masing model in capturing the hysteretic response. The DEM behavior can be determined using time-independent mappings of the corresponding equivalent displacement to equivalent force values, which is very efficient computationally. The nonlinear optimization approach used in this paper for online identification can be applied to much wider types of nonlinear rate-dependent hysteretic behavior for models with both time-invariant and time-variant parameter. These applications are currently being studied by the authors.

Acknowledgments

The authors acknowledge partial support for this research by National Science Foundation.

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