A non-local gradient-enhanced damage model for viscoelastic materials

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Abstract

Predicting the direction, growth rate and damage zone is crucial in order to model correctly the degradation of viscoelastic materials. While local damage models have been popular in the literature, they are all similar in that they lack a length scale that will regularize the solution and lead to mesh independent results.

Based on an equivalent stress measure concept and apply it to a generalized viscoelastic Maxwell model with a Murakami type damage-rate law. Viscoelastic behavior is achieved by a semi-analytical integration of the constitutive law assuming time dependent behavior of the deviatoric component and purely elastic response of the volumetric part. The scheme leads to a coupled set of nonlinear equations which are solved simultaneously using a monolithic Newton framework to obtain displacement and damage fields as function of time. The Jacobian matrix of the Newton scheme is formulated analytically.

Mesh-insensitive behavior is demonstrated for one and two dimensional problem.

Viscoelastic Constitutive Model

Viscoelastic properties are obtained from a generalized Maxwell model in which the material modulus is presented in a Prony-series form. Additive decomposition of the effective stress $\tilde{\sigma}$ into its deviatoric and volumetric components is used,

$$\tilde{\sigma} = \tilde{\sigma}^{vol} + \tilde{\sigma}^{dev} = 3K\,\mathrm{tr}(\varepsilon) + 2G(t)\varepsilon^{dev}(t) \qquad (1)$$

Where K is the Bulk modulus, $G(t)$ is the time dependent shear modulus and $\mathrm{tr}(\cdot)$ is the trace operator. Material viscoelastic behavior is achieved by changes in shape rather than volume, thus the shear modulus is written in a Prony-series form while the volumetric stress is purely elastic.

$$G(t) = G_0\left(\mu_0 + \sum_{i=1}^{n}\mu_i e^{-\frac{t}{\lambda_i}}\right) \qquad (2)$$

Where λ_i is the characteristic time, G_0 is the time-independent shear modulus and μ_i satisfies $\sum_{i=0}^{n}\mu_i = 1$. Which yields the deviatoric stress,

$$\tilde{\sigma}^{dev} = 2G_0\int_{-\infty}^{t}\left[\mu_0 + \sum_{i=1}^{n}\mu_i e^{\left(\frac{-(t-\tau)}{\lambda_i}\right)}\right]\dot{\varepsilon}^{dev}(\tau)d\tau \qquad (3)$$

Continuum Damage Model

Gradual accumulation of microcracks and microcavities are accounted for through the concept of the effective stress in which the total stress σ is obtained by applying the damage projector M to the effective stress $\tilde{\sigma}$ as,

$$\sigma(t) = M^{-1}\tilde{\sigma}(t) \qquad (6)$$

A non-local damage projector M is proposed following the isotropic damage model by Murakami (1981) in which,

$$M^{-1} = \boldsymbol{I} - \int_{-\infty}^{t}\dot{D}^{NL}(\tau)d\tau\boldsymbol{I} \qquad (7)$$

and the Murakami's Damage rate is obtained from the non-local equivalent stress χ^{NL}

$$\dot{D}^{NL} = B\frac{\langle\chi^{NL}\rangle^{r}}{(1-D)^{k}} \qquad (8)$$

where B, r and k are material parameters and D is the damage variable with values from 0.0 (undamaged) to 1.0 (fully damaged). Where the Macaulay brackets $\langle\cdot\rangle$ are defined by,

$$\langle\cdot\rangle = \frac{\mathrm{abs}(\cdot)+(\cdot)}{2} \qquad (9)$$

The non-local damage rate is constrained by the value of χ^{NL} being larger than a stress-equivalent history threshold χ_{th} whose initial value is updated during simulation as

$$\dot{D}^{NL} = \begin{cases}\dot{D}^{NL} & if\ \max(\chi^{NL}) \geq \chi_{th}\\ 0 & if\ \max(\chi^{NL}) < \chi_{th}\end{cases} \qquad (10)$$

Following the derivation for local-damage model by Londono et al (2016), the model proposed is also thermodynamically consistent.

Gradient-enhanced damage

Following the gradient enhanced model proposed (Mühlhaus H.B, 1991, Peerlings RHJ, et al., 1996), a second order gradient equation is proposed as

$$\chi^{NL} - c\nabla^2\chi^{NL} = \langle\chi\rangle \qquad (11)$$

where c is a measure of the characteristic length of the material and the local Hayhurst's equivalent stress measure χ is obtained from

$$\chi = \alpha\tilde{\sigma}^{(1)} + \beta\sqrt{3J_2} + (1-\alpha-\beta)\mathrm{tr}(\tilde{\sigma}) \qquad (12)$$

where α and β are material-dependent weights, $\tilde{\sigma}^{(1)}$ is the maximum principal stress and $J_2 = \frac{1}{2}\tilde{\sigma}^{dev}_{mn}\tilde{\sigma}^{dev}_{mn}$.

Strong Form

Mathematically the problem is described by

Equilibrium Equation	$\nabla\cdot\sigma - f^{ext} = 0$	$in\ \Omega$
Gradient damage equation	$\chi^{NL} - c\nabla^2\chi^{NL} = \langle\chi\rangle$	$in\ \Omega$
Viscoelastic Constitutive law	$\sigma(t,D,\varepsilon) = M^{-1}(t,D,\varepsilon)\tilde{\sigma}(t,\varepsilon)$	$in\ \Omega$
Damage Constitutive law	$\dot{D} - B\frac{\langle\chi^{NL}\rangle^{r}}{(1-D)^{k}} = 0$	$in\ \Omega$
Prescribed Tractions	$\sigma\cdot n = t_{tr}$	$in\ \Gamma_t$
Prescribed Displacements	$u = u_{BC}$	$in\ \Gamma_{BC}$
Prescribed flux of $\overline{\chi}$	$\nabla\chi^{NL}\cdot n = 0$	$in\Gamma_{\chi}$

Numerical results in 1D

Mathematically the problem is described by

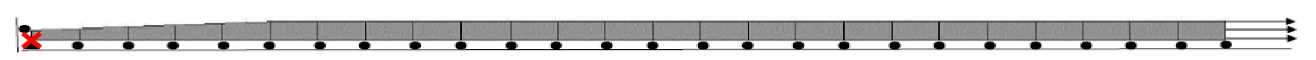

Fig.1 One dimensional symmetric view for gradient-enhanced damage regularization. Coarse mesh shown

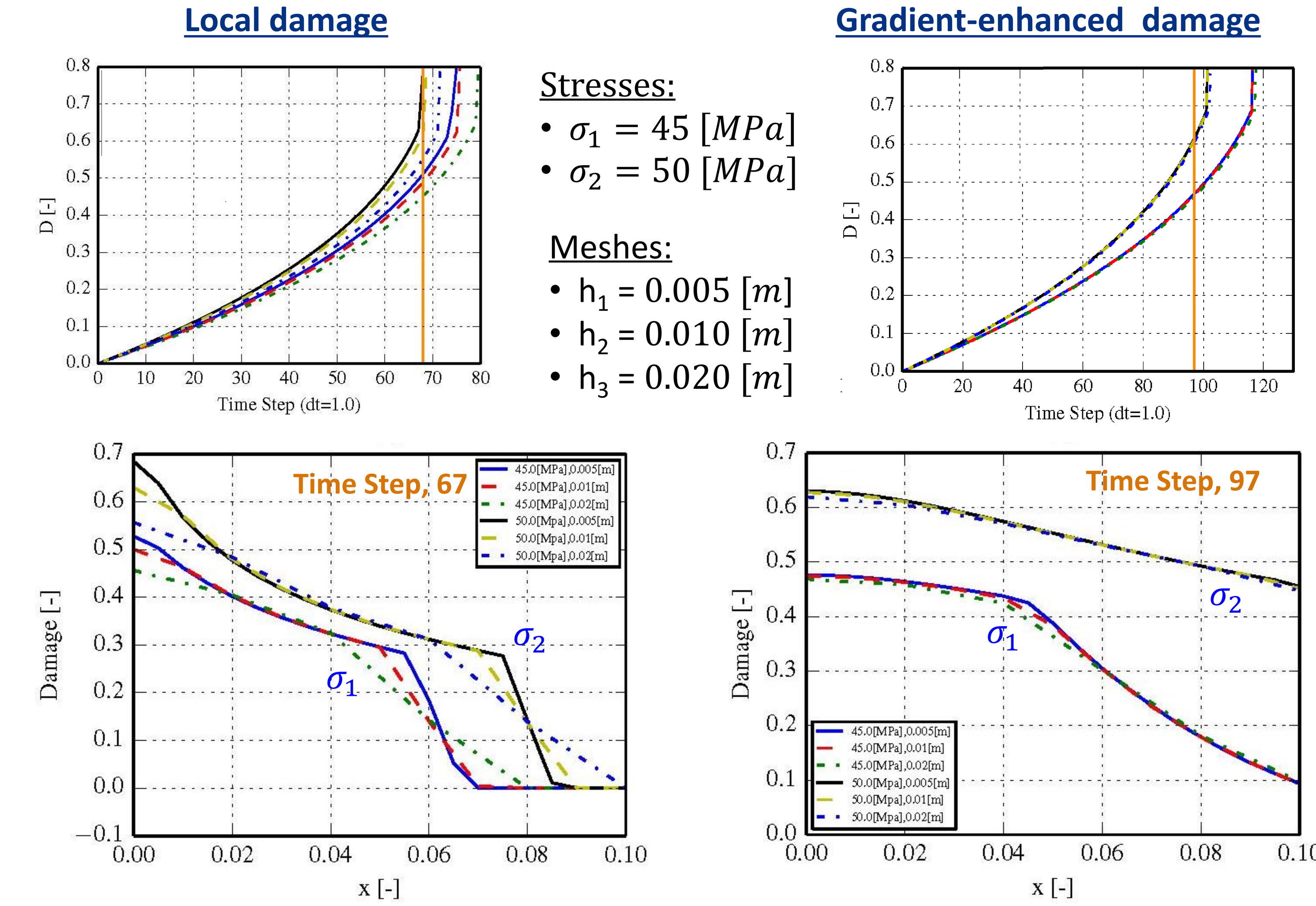

Fig.2 Local (Left) and Nonlocal (right) damage for 1D test under two stress values (creep) and three meshes used

Numerical results in 2D

In 2D, mesh sensitivity is tested in the Kalthoff problem, in which, the symmetric part of a plate subjected to a constant velocity impact is applied to a pre-notched plane-strain plate. Mesh sensitivity and the effect of the Hayhurst's weight parameters on the crack path are studied

Mesh sensitivity test

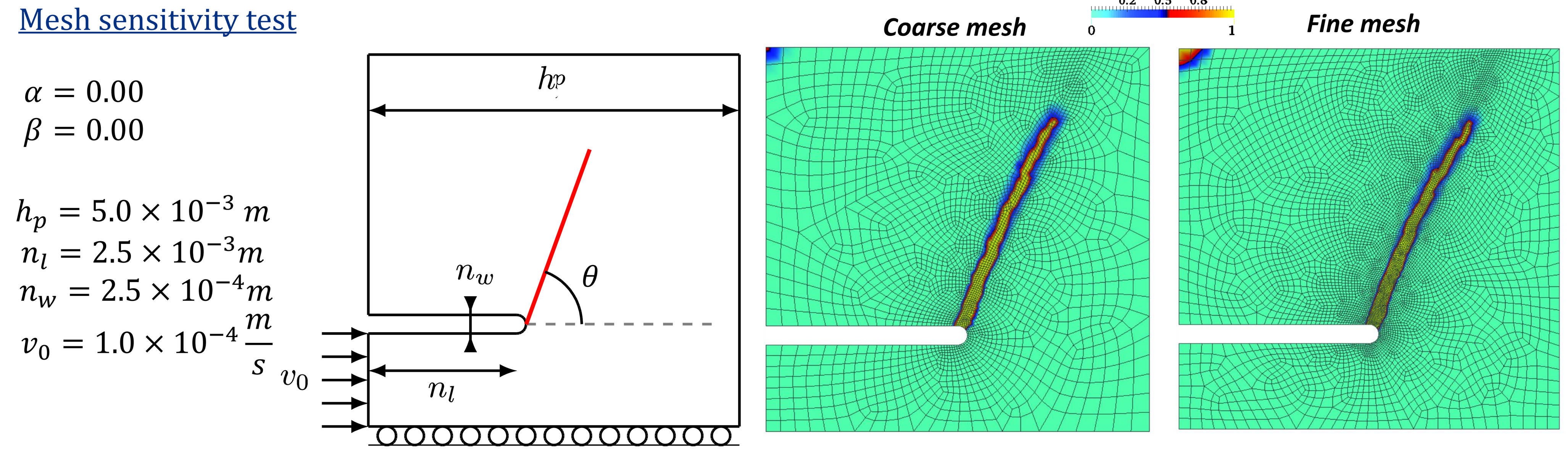

Fig.3 Kalthoff problem scheme (left). Results of the coarse mesh with 3492 elements (center) and fine mesh with 7269 elements (right)

Crack path ($\alpha = 0.00$):

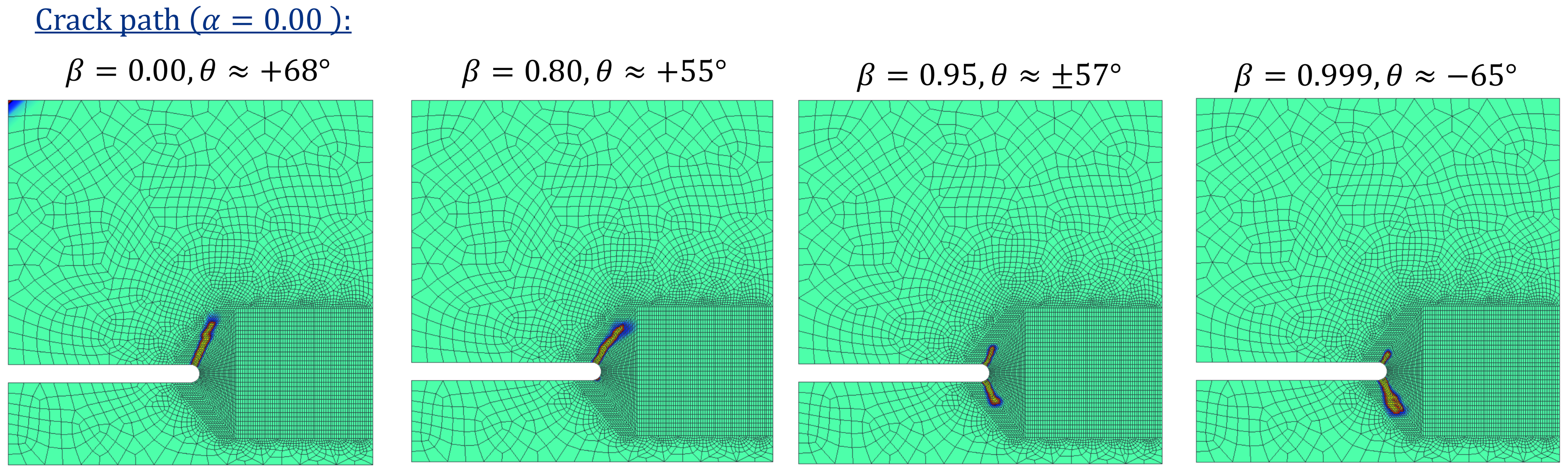

Fig.4 Different crack path direction for different Hayhurst's weight parameters under the same boundary conditions and mesh (5700 elements)

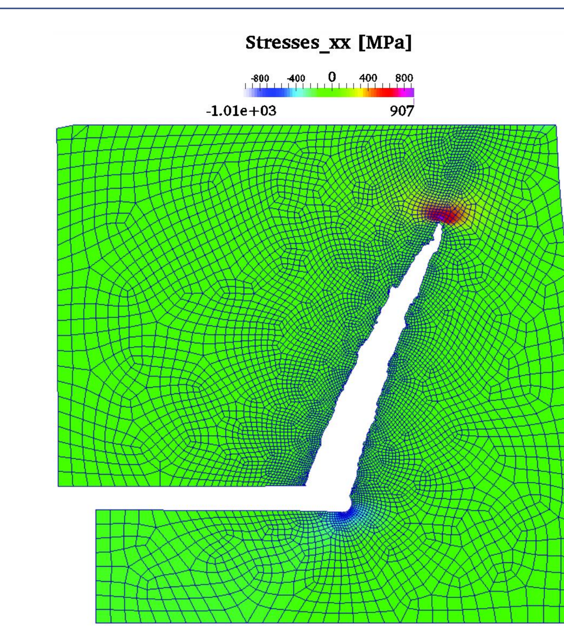

Fig.5 Deformation shape of Kalthoff problem with a scale factor of 20. Elements with Damage greater than 0.96 are removed. Fine mesh shown (7269 elements).

Conclusions

- A nonlocal damage model is proposed for viscoelastic solids by introducing a coupled non-linear equation for displacement and an equivalent stress measure
- Gradient-enhanced damage model display mesh insensitive results for 1D and 2D numerical tests
- In 2D, the selection of material parameters in the source term of the gradient equation allows for different crack paths and damage rates
- Selection of the parameters α and β to generate different crack paths need to be studied further for each material. In any case, the model proposed yield mesh insensitive results

References

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