Proceedings of IMECE2005:
2005 ASME International Mechanical Engineering Congress and Exposition
November 5-11, 2005, Orlando, Florida USA

IMECE2005-79810

STRING STABLE CONTROL AND POSITION ASSIGNMENT FOR UNMANNED AIR VEHICLES IN FORMATION

Michael Kaplun
Christopher P. Simon
Anouck R. Girard, Professor
Columbia University
Department of Mechanical Engineering
220 S.W. Mudd Bldg, MC 4703
500 West 120th Street
New York, NY 10027
United States
Email: {mk2452,cps2105,ag2363}@columbia.edu

ABSTRACT

This paper presents a method for string stable formation control and formation position assignment for several unmanned air vehicles. The model addresses two main issues of formation control; (a) stability analysis of the interconnected system, (b) vehicle position assignment and switching. The vehicles move along a given trajectory in a specified formation shape. This allows us to decouple the formation shape problem from the motion of the group along a given path. Each vehicle maintains its position relative to an inertial waypoint, and its position relative to its neighbors. Thus, we are developing a leaderless formation control scheme. A specific leader is not desirable, because it may lead to instability of the interconnected system in the case of an error associated with the leader vehicle, and is not fault tolerant. We include an analysis of the error dynamics of the system for a V-shaped formation. This specific shape was selected as birds use it frequently and NASA has shown that this formation is energy efficient. Our system is mesh stable and spacing errors do not exacerbate downstream of the origin.

When planes fly in V-formation, the front plane uses more energy and, therefore, more fuel. The trailing planes benefit from the vortex created by the wings of the plane in front of them and use less energy to maintain a cruising speed. For the purposes of maximizing fuel efficiency in an entire team of UAVs, the model uses a simple proximity algorithm to assign the vehicles' positions and determine when and which vehicles to switch. The only physical variables that need to be observed are the positions of each plane relative to an arbitrary virtual leader position, and the velocities and fuel reserves of each plane, and a small number of other indicial variables. The fuel burnt by the front plane determines when to signal for a switch in position and the geometry of the formation in combination with the fuel levels of the remaining planes controls the decision making process.

The different aspects of the model are all meant to work together and individually. Matlab simulation plots of path motion and error convergence are shown as proof of concept.

Keywords: formation control, string stability, unmanned air vehicles, fuel efficiency

INTRODUCTION

Here, we present an analysis for a string stable controller of unmanned air vehicles in a V-shape formation. Past research has dealt with the issue of mesh stability for formations [2] for look-ahead interconnected systems. We consider 2-dimensional motion of a V-shaped formation of UAVs for simplicity's sake. A stability analysis is provided for a leaderless formation control law for the vehicles. Each vehicle has a positioning error associated with its location with respect to some desired inertial position, as well as a relative positioning error with respect to the vehicles immediately surrounding it. A leaderless formation is robust because it does not lend itself to failure should the leader become disengaged from the formation. Thus, the theory and controller developed in this paper are based on decentralized control where each vehicle maintains its own stability within the mesh. The reasoning for the V-formation is taken from the natural shape in which flocks of birds fly.

The first section of this paper will present a review of string stable controllers for formation of vehicles. We will briefly show how a system of loosely connected vehicles can remain string stable. Here we present a leaderless control scheme for a triangular (V-shape) formation, and mesh stability of such a leaderless control scheme is analyzed.

The second half of this paper deals with a switching algorithm, built on top of the controller from the first section that alternates which vehicle is in the front position of the formation. This algorithm assumes centralized decision making, which can be eliminated by designating a team leader. The controlling computer uses the positions and remaining fuel of each vehicle to determine which of the trailing planes is the best choice for the new leader. Then, a new command is fed into the controller. This helps to show how the controller performs stably in a practical application.

Finally, simulation results are shown.

NOMENCLATURE

η: Vehicle position
a: Vehicle acceleration
v: Vehicle velocity
u: System input (desired acceleration)
K: gain
i,j: reference position
V: relative air velocity
V′: air velocity after rotation
W: rotation vector
α: angle of attack
D: initial drag force
D′: rotated drag force
D_{FF}: resultant drag force
L: initial lift force
L′: rotated lift force
L_{FF}: resultant lift force
c: cost function
x_i: x-position of plane i
y_i: y-position of plane i

BACKGROUND

I

Analysis for mesh stability for a loosely connected system, is presented by Pant et al [1]. A loosely connected mesh stable system is one in which the maximum position/spacing error decreases as it propagates down the formation. A sliding surface control law is implemented to relate the error of the i^{th} position vehicle to that of its immediate neighbors, i+1 as well as to a desired position. By analyzing the norm of the error propagation response we can determine mesh stability of a system. The norm of the error dynamics transfer function is representative of the error amplification. Thus, if this norm is less than one, error amplification will ultimately go to zero, indicating decrease an error. Such a system is mesh stable. If the norm is greater than one the system is not mesh stable.

In [2] Pant et al offer a mesh stability analysis for look-ahead interconnected systems. The control scheme is based on a specified leader and the formation moves accordingly to changes in the lead vehicle and immediate neighbors.

Researchers working on platooning of passenger vehicles as a means to increase highway capacity without building new highways [12] first noticed that strings of automatically controlled vehicles exhibited "string instabilities", i.e., disturbances in the front of the platoon were amplified as they were propagated upstream. Linear transfer function analysis [17, 12, 15] showed that these instabilities could be eliminated by the introduction of a common reference trajectory for all of the vehicles. If all of the vehicles in the platoon have knowledge of the lead vehicle's absolute velocity [17], then "weak string stability" can be achieved, i.e., no disturbance would ever be amplified as it traveled upstream in the platoon. Also, if all of the vehicles in the platoon had knowledge of the relative position error between themselves and the lead vehicle, then "strong string stability" could be achieved, i.e., all downstream disturbances could be geometrically attenuated as they traveled upstream in the platoon [17]. The lead vehicle information needs to be communicated to all of the vehicles via a wireless communication link.

The concept of string stability is extended to 3-D configurations in [2] and the term "mesh stability" is used to denote the property of disturbance attenuation in multi-dimensions. The paper concentrates on minimizing the communication requirements to achieve mesh stability and analyses systems with "look-ahead" sensor information that could be communicated or sensed directly.

Other possible approaches include virtual structures [11, 20, 13], rigid graphs [14] and potential methods [13, 21]. Nearest neighbor rules, Lyapunov theory, graph techniques and non-smooth control results are used to study how multiple agents eventually move in the same direction despite the absence of centralized coordination and despite the fact that each agent's set of nearest neighbors changes with time as the system evolves [18, 19]. A design for fixed wing aircraft can be found in [16].

II

A great amount of work is currently being done on autonomous air vehicles as the field of robotics gets larger and larger. Often, aircrafts need to travel much longer distances than ground vehicles. In order to accomplish this more efficiently, we take an example from Mother Nature. When birds fly long distances, they usually fly in formation; while in formation, they take turns flying in front. This turn taking or switching of leaders is actually a means of energy conservation. Experiments putting numbers on exactly how much energy birds save when flying in formation were only performed within the last 5 – 10 years. It turns out that "wingbeat frequency decreased from the leader to the bird in the fourth position." And, "When in formation, birds had a heart rate that was 11.4%-14.5% lower than in birds flying alone." As a summary, the "results provide empirical evidence that, compared with solo flight, formation flight confers a significant aerodynamics advantage which allows birds to reduce their energy expenditure while flying at a similar speed."[3]. The question becomes whether or not aircrafts in formation flight would receive the same aerodynamic benefit as their natural counterparts. A study, conducted cooperatively between NASA Dryden Flight Research, NASA Ames Research Center, the Boeing Company, and the University of California, Los Angeles, known as the Autonomous Formation Flight Project or AFF Project, took on the primary goal of "demonstrat[ing] a sustained 10-percent reduction in the consumption of fuel by a trailing airplane during cruise."[4] The actual reports were better than even that 10-percent target. "The AFF Project reports … flights showing fuel flow reductions of up to 20 percent when the trailing aircraft flew 0.56 Mach at 25,000 feet."[5]. And as high as 0.86 Mach and 36,000 feet the fuel benefit was 15 percent. This result would end up saving significant money on fuel. The predicted monetary benefits included an "annual per-trailing airplane reductions of \$0.5 x 10^6 (year 200 average prices) in the cost of fuel."[6] The results were not just about saving fuel and money. The reduction in fuel burned also had a positive impact on the environment. "Emissions of carbon dioxide and nitrous oxide greenhouse gasses could be reduced by 10% and 15%, respectively."[7]

Physically the planes (or birds) in any one of the trailing positions are benefiting from the vortex created in the wake of the previous plane. This vortex creates an updraft which effectively changes the angle of attack of the trailing plane [8].

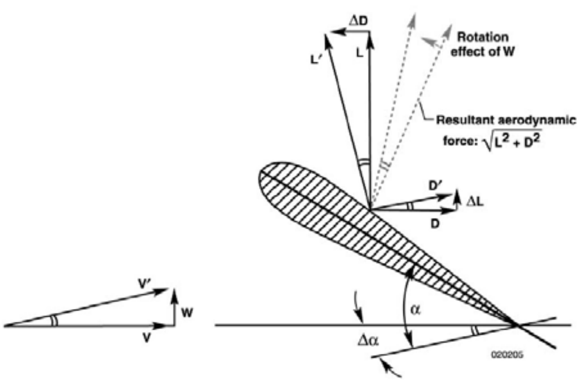

Figure 1: Resultant Aerodynamic Forces [9]

In this picture V is the direction of the wind as if the plane were flying alone, V′ is the direction of the airflow coming off of the previous plane. L and D represent the solo flight lift and drag respectively. The rotation W of the air velocity generates a change in the angle of attack, Δα, which increases the overall angle of attack to α. This new angle of attack effectively rotates the airfoil, thereby generating the new aerodynamic forces L′ and D′, lift and drag. D′ has a component ΔL in the positive lift direction, and L′ as component ΔD in the negative drag (or positive thrust) direction, both helping to make the flight more efficient.

$$D_{FF} = D'\cos(\Delta\alpha) - \Delta D \quad (1)$$

$$\Delta D = \sin(\Delta\alpha) L \quad (2)$$

$$L_{FF} = L'\cos(\Delta\alpha) + \Delta L \quad (3)$$

$$\Delta L = \sin(\Delta\alpha) D \quad (4)$$

The resultant aerodynamic force is $\sqrt{L_{FF}^2 + D_{FF}^2}$.

Since formation flight creates such a benefit it would be nice if it could be used for autonomous aircraft as well as manned air vehicles. Since these effects only benefit trailing planes the plane in the front of the formation is not flying any more efficiently than if it were flying alone. This means that if the team of planes as a whole is going to save energy flying together, and if they all have the same initial capabilities, then they must take turns flying in the lead.

STABILITY ANALYSIS

In this section we will perform a stability analysis to show that a formation control law for a V-shaped vehicle formation is mesh stable.

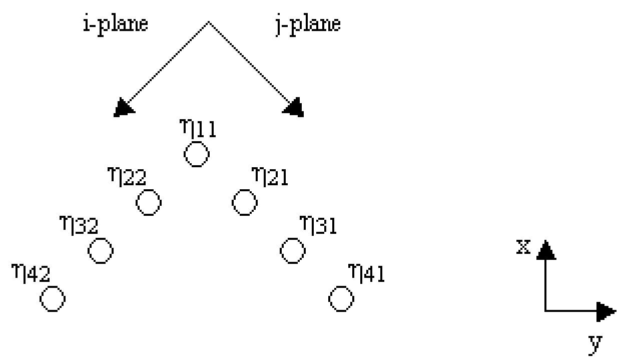

Figure 2: V-Shape Formation

Each vehicle has an error associated with respect to its desired inertial position, as well as a relative error defined by its position with respect to neighboring vehicles (the vehicle directly in front of it, the vehicle directly behind it, and the vehicle on the same horizontal row). For example, η_{32} has neighbors η_{22}, η_{42} and η_{31}. To generalize, $\eta_{i,j}$ has neighbors $\eta_{i-1,j}$, $\eta_{i+1,j}$ and $\eta_{i,j-1}$ (the j plane consists of two rows, 1 and 2. If we are on the first row we look at $\eta_{i,j+1}$). We can define error as below. The Kr gain allows us to adjust the importance of the inertial term and the formation shape terms.

$$e_{i,j} = \eta_{i,j} - \eta^{d}_{i,j} + K_r(\eta_{i,j} - \eta_{i-1,j}) + K_r(\eta_{i,j} - \eta_{i+1,j}) + K_r(\eta_{i,j} - \eta_{i,j-1}) \quad \textbf{(5)}$$

$$\eta = \begin{pmatrix} x \\ y \end{pmatrix} \quad \textbf{(6)}$$

The control variables are not immediately visible in eq. (5). By differentiating eq. (5) twice we develop an equation where the control variable, $\ddot{\eta}_{i,j}$, is immediately visible.

We apply a sliding control law is applied to drive the error to zero. The sliding surface is defined as follows:

$$S_{i,j} = \dot{e}_{i,j} + q_1 e_{i,j} \quad \textbf{(7)}$$

We must force the control law to converge to zero in order to achieve our control objective. The following control input will cause the sliding controller to converge to zero.

$$u_{i,j} = \frac{1}{2}[-KS_{i,j} - q_1 \dot{e}_{i,j} + K_a(\ddot{\eta}_{i-1,j} + \ddot{\eta}_{i+1,j} + \ddot{\eta}_{i,j-1})] \quad \textbf{(8)}$$

Here we do not account for time delays due, for example, to actuator dynamics. The input acceleration is $u_{i,j}$, thus we will use this notation in place of $\ddot{\eta}_{i,j}$. Differentiating equation (5) twice, and substituting (8) into (5), we obtain a differential equation which relates the error dynamics of a particular vehicle with respect to its neighbors.

$$\ddot{e}_{i,j} + \dot{e}_{i,j}\left(\frac{K + 3K_rK + q_1 + 3K_rq_1}{2}\right) + e_{i,j}\left(\frac{Kq_1 + 3K_rKq_1}{2}\right) =$$
$$K_a\left(\frac{\ddot{e}_{i-1,j} + \ddot{e}_{i+1,j} + \ddot{e}_{i,j-1}}{2}\right) + (K_rq_1 + K_rK)\left(\frac{\dot{e}_{i-1,j} + \dot{e}_{i+1,j} + \dot{e}_{i,j-1}}{2}\right) +$$
$$K_rKq_1\left(\frac{e_{i-1,j} + e_{i+1,j} + e_{i,j-1}}{2}\right) \quad \textbf{(9)}$$

Taking the Laplace transform we get:

$$E_{i,j} = \begin{bmatrix} H(s) & 0 & 0 \\ 0 & H(s) & 0 \\ 0 & 0 & H(s) \end{bmatrix} * \left[\frac{E_{i-1,j}(s) + E_{i+1,j}(s) + E_{i,j-1}(s)}{2}\right] \quad \textbf{(10)}$$

$$H(s) = \frac{K_as^2 + (K_rq_1 + K_rK)s + K_rKq_1}{s^2 + \left(\frac{K + 3K_rK + q_1 + 3K_rq_1}{2}\right)s + \left(\frac{Kq_1 + 3K_rKq_1}{2}\right)} \quad \textbf{(11)}$$

For mesh stability the norm of $\|H(s)\|_\infty$ must be less than or equal to 1. Each subsystem within the mesh must be string stable in order for the overall structure to be mesh stable [2]. Thus, we can now specify acceptable ranges and conditions on our gains K, K_r and q_1.

Let h(t) be the impulse response of H(s).

$$\| h(t) \|_1 = \sup_w | H(iw) | = H(0) = \frac{2K_r}{1 + 3K_r} \quad \textbf{(12)}$$

H(0) < 1 if $K_r > 0$. At high frequencies H(jw) ≈ K_a. String stability dictates that $K_a < 1$. However, in the case of low frequencies $K_a < 1$ would drive H(jw) ≥ 1. Thus, in order to guarantee string stability $K_a = 1$ (weak string stability).

VEHICLE COORDINATION

For stability analysis we have assumed second-order dynamics for our UAVs. Here we consider a (nonlinear) kinematic model for the controller design law.

The reference frames are shown below.

$$\dot{x} = u_1 \cos\psi + V_{wx} \quad \textbf{(13a)}$$
$$\dot{y} = u_1 \sin\psi + V_{wy} \quad \textbf{(13b)}$$
$$\dot{\psi} = u_2 \quad \textbf{(13c)}$$

The input variables are the speed, u_1, and turn rate, u_2, of the airplane. We are considering aircraft flying in a 2D plane. For the purpose of controller design, we set the velocities due to wind, V_{wx} and V_{wy}, to zero.

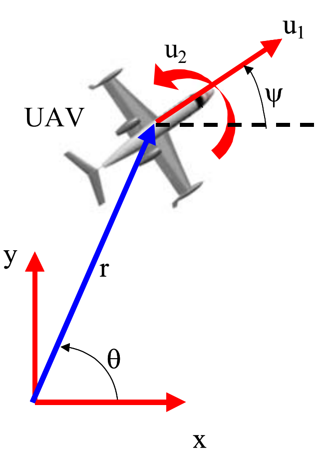

Figure 3: Variable Definition for Kinematic, 2D UAV Model.

We start by defining our position vector, η, and our desired position vector, η_d.

$$\eta = \begin{pmatrix} x \\ y \end{pmatrix} \quad \textbf{(14a)}$$

$$\eta^d = \begin{pmatrix} x_d \\ y_d \end{pmatrix} \quad \textbf{(14b)}$$

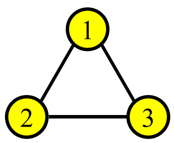

Figure 4: Three-Vehicle Formation

The error expressions for each vehicle, flying in the formation shown in figure 4, are shown below. Λ_R is a positive definite matrix. If Λ_R is zero we revert to the single vehicle case. If Λ_R is large, more emphasis is placed on relative terms than on absolute position and the shape is tightly controlled.

$$e_1 = \eta_1 - \eta_1^d + \Lambda_R(\eta_1 - \eta_2 - \eta_{12}^d) + \Lambda_R(\eta_1 - \eta_3 - \eta_{13}^d) \quad \textbf{(15a)}$$

$$e_2 = \eta_2 - \eta_2^d + \Lambda_R(\eta_2 - \eta_1 - \eta_{21}^d) + \Lambda_R(\eta_2 - \eta_3 - \eta_{23}^d) \quad \textbf{(15b)}$$

$$e_3 = \eta_3 - \eta_3^d + \Lambda_R(\eta_3 - \eta_1 - \eta_{31}^d) + \Lambda_R(\eta_3 - \eta_2 - \eta_{32}^d) \quad \textbf{(15c)}$$

We can then define $S_i = \dot{e}_i + Ke_i$ where "i" is the vehicle index. For example, for the first vehicle, after some reorganizing of the terms:

$$S_1 = (I + 2\Lambda_R)\dot{\eta}_1 + (-\dot{\eta}_1^d + \Lambda_R(-\dot{\eta}_2^d - \dot{\eta}_{12}^d - \dot{\eta}_3 - \dot{\eta}_{13}^d) + K(\eta_1 - \eta_1^d + \Lambda_R(2\eta_1 - \eta_2 - \eta_{12}^d - \eta_3 - \eta_{13}^d))) \quad \textbf{(16)}$$

We can then set $V = \dot{\eta}_1$ and:

$$V_R = (I + 2\Lambda_R)^{-1}[(-\dot{\eta}_1^d + \Lambda_R(-\dot{\eta}_2 - \dot{\eta}_{12}^d - \dot{\eta}_3 - \dot{\eta}_{13}^d) + K(\eta_1 - \eta_1^d + \Lambda_R(2\eta_1 - \eta_2 - \eta_{12}^d - \eta_3 - \eta_{13}^d))] \quad \textbf{(17)}$$

Reorganizing, we can express the sliding surface, S, as:

$$S = V - V_R \quad \textbf{(18)}$$

We now need to find a strategy to make the velocity, V go to V_R. If $V=V_R$, then S equals zero, which drives the position error to zero, which is our goal. We will use a strategy called dynamic extension to that aim. We have:

$$S = \begin{pmatrix} u_1 \cos\psi \\ u_1 \sin\psi \end{pmatrix} - (I + 2\Lambda_R)^{-1}[(-\dot{\eta}_1^d + \Lambda_R(-\dot{\eta}_2 - \dot{\eta}_{12}^d - \dot{\eta}_3 - \dot{\eta}_{13}^d) + K(\eta_1 - \eta_1^d + \Lambda_R(2\eta_1 - \eta_2 - \eta_{12}^d - \eta_3 - \eta_{13}^d))] \quad \textbf{(19)}$$

We take a derivative of our sliding surface, S.

$$\dot{S} = \begin{pmatrix} \dot{u}_1 \cos\psi - u_1 \sin\psi\,\dot{\psi} \\ \dot{u}_1 \sin\psi + u_1 \cos\psi\,\dot{\psi} \end{pmatrix} - (I + 2\Lambda_R)^{-1}[(-\ddot{\eta}_1^d + \Lambda_R(-\ddot{\eta}_2 - \ddot{\eta}_{12}^d - \ddot{\eta}_3 - \ddot{\eta}_{13}^d) + K(\dot{\eta}_1 - \dot{\eta}_1^d + \Lambda_R(2\dot{\eta}_1 - \dot{\eta}_2 - \dot{\eta}_{12}^d - \dot{\eta}_3 - \dot{\eta}_{13}^d))] \quad \textbf{(20)}$$

Rewriting the above equation in matrix form, and remembering that $\dot{\psi} = u_2$:

$$\dot{S} = \begin{bmatrix} \cos\psi & -\sin\psi u_1 \\ \sin\psi & \cos\psi u_1 \end{bmatrix} \begin{bmatrix} \dot{u}_1 \\ u_2 \end{bmatrix} - (I + 2\Lambda_R)^{-1}[(-\ddot{\eta}_1^d$$
$$+ \Lambda_R(-\ddot{\eta}_2 - \ddot{\eta}_{12}^d - \ddot{\eta}_3 - \ddot{\eta}_{13}^d) \quad \textbf{(21)}$$
$$+ K(\dot{\eta}_1 - \dot{\eta}_1^d$$
$$+ \Lambda_R(2\dot{\eta}_1 - \dot{\eta}_2 - \dot{\eta}_{12}^d - \dot{\eta}_3 - \dot{\eta}_{13}^d))]$$

Setting some notation:

$$A = \begin{bmatrix} \cos\psi & -\sin\psi u_1 \\ \sin\psi & \cos\psi u_1 \end{bmatrix} \quad \textbf{(22)}$$

$$\nu = \begin{bmatrix} \dot{u}_1 \\ u_2 \end{bmatrix} \quad \textbf{(23)}$$

Notice that the A matrix is invertible for $u_1 \neq 0$, and this is not a constraint for controller design as aircraft cannot fly below a certain speed, so that $u_1 = 0$ is not a possibility for any kind of realistic flight. We then have:

$$\dot{S} = A\nu - V_R \quad \textbf{(24)}$$

For convenience purposes we avoid writing out the entire expression for V_R.

We would like to have exponential dynamics for our sliding surface: $\dot{S} = -\Lambda S$ where Λ is a positive definite matrix. Our desired behavior is then:

$$\dot{S} = A\nu - V_R = -\Lambda S \quad \textbf{(25)}$$

We can then get the vector ν from the equation:

$$\nu = A^{-1}[V_R - \Lambda S] \quad \textbf{(26)}$$

The vector ν gives us values for u_2 and $\dot{u}_1$. What we really need are values for u_1 and u_2. We end up passing the ν vector through a system that integrates the first field, to obtain the required values.

SWITCHING ALGORITHM

This section develops a fuel efficiency technique based upon the controller developed above. The algorithm it uses is a immediate application of the controller and therefore accesses it directly.

Starting from a random position, three planes must decide how to position themselves in V-shaped formation. The algorithm determines the geometric centroid of the randomly distributed planes. Then the plane farthest in the positive x direction relative to that point becomes the lead plane and the remaining two are assigned to follow behind that lead plane. Of those two, the plane whose position is the farthest in the positive y direction is in the uppermost of the two trailing formation assignments and the other plane the lowermost. There are some instances for which special consideration had to be put into the program. If there are two planes that are equidistant in the x direction from the centroid and they are the farthest in that direction as well, the plane closest to the centroid becomes the lead plane and the other takes one of the trailing positions. In the event that there are two planes whose random positions are both equally farthest in front of the centroid and equally above and below it, the third plane would have to lie on the same constant y line as the centroid. Since the planes would already be symmetric in the y direction it becomes more efficient to make that third plane the leader and to have the other two fall back to follow it. The following graph shows the positions of the three planes with respect to time based on random starting positions.

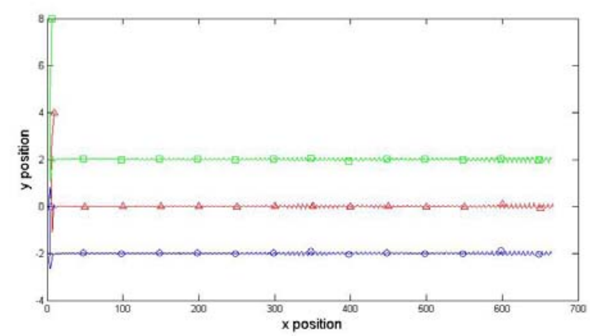

Figure 4: x-y positions of all three airplanes

As the planes move forward in the x direction, they are moving forward in time. In this particular case the random initial positions of the plane are (5,0), (9,4) and (6,8). The plane flying in the center with respect to the y direction ends up 2 units ahead of the other two planes. There is some slight chatter in the trajectory of each plane but that is to be expected with a sliding controller and could be smoothed out.

Once the planes have actually flown enough to have gotten into formation and burned enough fuel, it becomes time for a mid-flight shift while in formation. The decision making process here must account not only for distances but for the amount of fuel left in each plane, since it is necessary to make sure that all of the planes are being used to their full potential to prolong the distance traveled. The decision is made based off of a simple cost function. Using distance as measure of cost for aircraft in flight is a common technique. In [10] Chandler, et al.

use the proximity of a plane to a target as the benefit function for that plane to take on that target for assignment. In our algorithm, the cost for any plane to assume the lead position is its distance from the front of the formation divided by its remaining fuel.

$$c = D / (\% fuel) \tag{27}$$

$$D = \sqrt{(x_i - x_l)^2 + (y_i - y_l)^2} \tag{28}$$

Here, x_l and y_l are the x and y positions of the leader. This cost is determined for all each of the planes that are not currently in the front of the formation and chooses the plane that is the least costly to move up to the front. This plane's desired position within the formation and the leader's are then swapped and a new desired formation is generated. Based on the new desired formation a step change in the desired x,y-position of each is reassigned as necessary.

The following graph shows what happens when a switch is made mid flight. The switch is very smooth and controller remains very stable.

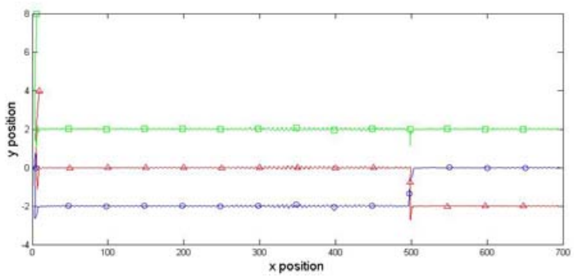

Figure 5: x-y positions with one switch

In actuality there will need to be multiple switches during formation. When we add more than one switch the switches remain smooth and the controller remains stable.

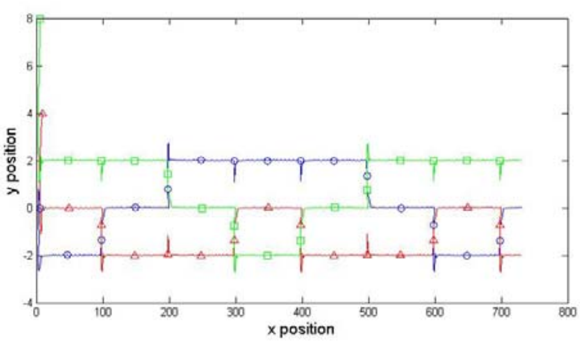

Figure 6: x-y positions with multiple switches

CONCLUSION

The paper presented an analysis of autonomous air vehicles flying in the form of a flock of birds. As shown, such a formation has implications on fuel utilization and efficiency of individual aircraft. Such a formation is shown to be mesh stable, meaning the error does not increasingly propagate downstream of the formation. Future work on the controller would include testing the theory with 6 DOF simulation software.

Comparing the simulation without switching to that with switching we can see how the changing leader accounts for greater time until one of the planes is out of fuel. Before switching was added the total distance flown before one of the planes ran out of fuel was 667 units in the x direction. After adding one switch that number went up to 697 and the simulation with multiple switches produced a total distance of 732 units in the x direction. This final number is a 9.7% increase in distance that the planes can fly. This number is right around what is expected considering the planes are getting a 15% decrease in energy expenditure two thirds of the time.

ACKNOWLEDGMENTS

The authors would like to acknowledge Columbia University Internal Funding.

REFERENCES

[1] Pant, A., Seiler, P., and J.K. Hedrick, (1999). Preliminary Investigation of Mesh Stability For Linear Systems. IMECE99/DSC-7B-1.

[2] Pant, A., Seiler, P. and J.K. Hedrick (2002). Mesh stability of look-ahead interconnected systems. IEEE Transaction on Automatic Control, 47 (2), 403-407.

[3] Weimerskirch, H., J. Martin, Y. Clerquin, P. Alexandre, S. Jiraskova. "Engergy Saving in Flight Formation." Nature, v.413 2001. p. 697-698

[4] Schkolnik, Gerard S. and Brent Cobleigh, Dryden Flight Research Center. URL: http://www.nasatech.com/Briefs/Feb05/DRC0146.html

[5] Hagenauer, Beth. "AFF: Autonomous Formation Flight is surpassing project's goals". URL: The Dryden X-press. http://www.dfrc.nasa.gov/Newsroom/XPress/stories/103101/new_aff.txt.html.

[6] Schkolnik, Gerard S. and Brent Cobleigh, Dryden Flight Research Center. URL: http://www.nasatech.com/Briefs/Feb05/DRC0146.html

[7] Iannotta, Ben. "Vortex Draws Flight Research Forward". Aerospace America Online. URL: http://www.aiaa.org/aerospace/Article.cfm?issuetocid=187&ArchiveIssueID=24.

[8] Ray, Ronald J., Brent R. Cobleigh, M. Jake Vachon, and Clinton St. John. (8/2002) Flight Test Techniques Used to Evaluate Performance Benefits During Formation Flight. NASA Dryden Flight Research Center.

[9] Ray, Ronald J., Brent R. Cobleigh, M. Jake Vachon, and Clinton St. John. (8/2002) Flight Test Techniques Used to Evaluate Performance Benefits During Formation Flight. NASA Dryden Flight Research Center. Oral Communication for Permission.

[10] Chandler, Phillip R., Meir Pachter, Dharba Swaroop, Jeffery M. Fowler, Jason K. Howlett, Steven Rasmussen, Corey Schumacher, Kendall Nygard. Complexity in UAV Cooperative Control. American Control Conference, May 2002, p. 1831-1836.

[11] Beard, R.W., Lawton, J. and F.Y. Hadaegh (2001). A coordination architecture for spacecraft formation control. IEEE Transactions on Control Systems Technology, 9 (6), 777 - 790.

[12] Hedrick, J.K. and D. Swaroop (1993). Dynamic coupling in vehicles under automatic control. In: Proc. 13th IAVSD Symposium, August 1993.

[13] Leonard, N.E. and E. Fiorelli (2001). Virtual leaders, artificial potentials and coordinated control of groups". In: Proceedings of the 40th IEEE Conference on Decision and Control, 4-7 Dec, 3, 2968 - 2973.

[14] Olfati-Saber, M. and R. Murray (2002). Graph rigidity and distributed formation stabilization of multi-vehicle systems. In: Proceedings of the 41st IEEE Conference on Decision and Control, 10-13 Dec, 3, 2965 - 2971.

[15] Sheikholeslam, S. and C.A. Desoer (1992). Control of interconnected nonlinear dynamical systems: the platoon problem. IEEE Transactions on Automatic Control, 37 (6), 806-810.

[16] S. Spry and J.K. Hedrick, "Formation Control Using Generalized Coordinates", Proceedings of the IEEE Conference on Decision and Control, CDC 2004.

[17] Swaroop, D. and J.K. Hedrick (1996). String stability of interconnected systems. IEEE Transactions on Automatic Control, 41 (3), 349-357.

[18] Tanner, H., Jadbabaie, A. and G. Pappas (2003a). Stable flocking of mobile agents, Part II: dynamic topology. In: Proceedings of the 42nd IEEE Conference on Decision and Control, Maui, Hawaii, December 9-12, 2, 2026-21.

[19] Tanner, H. Jadbabaie, A. and G. Pappas (2003b). Stable flocking of mobile agents, Part I: fixed topology. In: Proceedings of the 42nd IEEE Conference on Decision and Control, Maui, Hawaii, December 9-12, 2, 2010-15.

[20] Tillerson, M., Breger, L. and J.P. How (2003). Distributed coordination and control of formation flying spacecraft. In: Proceedings of the American Control Conference, June 4-6, 2, 1740 - 1745.

[21] Zohdi, T.I. (2003). Computational design of swarms. International Journal of Numerical Methods in Engineering, 57, 2205-2219.