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GRAVITATIONAL EFFECTS ON THE SU(5) BREAKING PHASE TRANSITION FOR A COLEMAN–WEINBERG POTENTIAL

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The gravitational coupling $R\phi^2$ plays a crucial role in determining the fate of the symmetric, high temperature state in a grand unified model with Coleman–Weinberg type symmetry breaking. If this term enters in the lagrangian with a negative sign, it drives the SU(5) breaking phase transition at a temperature of about 10^{10} GeV. If it enters with a positive sign, and in particular with the coefficient $\frac{1}{12}$ which is required for a conformally invariant classical theory, this term prevents the phase transition from being completed, at least until temperatures are reached for which the SU(5) coupling becomes large.

1. Introduction

In the standard big bang cosmology, the early Universe is characterized by very high temperatures at which the gauge symmetries of electroweak and grand unified (GUT) models were presumably unbroken. As the Universe cooled, phase transitions took place which brought these theories into their present spontaneously broken form. If these phase transitions were first order, extreme supercooling might have occurred with interesting cosmological consequences [1–3]. Spontaneous symmetry breaking induced by radiative corrections is known to exhibit such supercooling [2, 3]. In fact, the phase transition in Coleman–Weinberg type models [4] is so slow that it can be driven by unexpected sources. Witten [2] has shown that if the scalar potential in the standard electroweak gauge theory is of the Coleman–Weinberg form, then the phase transition which breaks $SU(2)\times U(1)$ is actually driven by QCD-induced chiral symmetry breaking. In this paper it is shown that if symmetry breaking at the GUT scale occurs in the Coleman–Weinberg mode, then the phase transition which breaks the grand unified group (taken to be SU(5) [5]) down to $SU(3)\times SU(2)\times U(1)$ is governed, i.e., either driven or suppressed, by gravitational effects.

The Coleman–Weinberg model is characterized by the absence of a mass term in the zero temperature effective potential. Finite temperature effects [6] produce an effective mass term $T^2\phi^2$ which is positive and produces a barrier (at all non-zero temperatures) between the high temperature state $\phi = 0$ and the low temperature,

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spontaneously broken vacuum state. This barrier causes the slow phase transition and extreme supercooling in this model.

Gravity plays an important role in all first-order phase transitions through cosmological effects [1–3]. While the Universe is stuck in the (high-temperature) false vacuum state, the vacuum energy density is positive. This causes the Universe to expand rapidly. The phase transition to the new vacuum states occurs through tunnelling which forms bubbles of new vacuum. If the phase transition is to be completed, these bubbles must fill space at a faster rate than space itself is expanding. Gravity (with minimal gravitational coupling) also plays a role in determining the tunnelling amplitude for the phase transition [7], but this effect is only of the order of the scalar vacuum expectation value divided by the Planck mass which is tiny (10^{-4}) even at the GUT scale.

The gravitational effect which plays such an important role for the Coleman–Weinberg type GUT model is the non-minimal coupling $R\phi^2$. This coupling is fully expected to occur and is in fact required if the scalar field theory is to be renormalizable in a classical gravitational background [8]. The Coleman–Weinberg model has a zero mass term. However, since R is non-zero in the false vacuum (due to the non-zero vacuum energy) the term $R\phi^2$ acts like a mass term while the Universe is in the false vacuum state. Although $R\phi^2$ is suppressed by inverse powers of the Planck mass, it is the leading term in the zero temperature potential near $\phi = 0$ so unlike other gravitational effects, it plays a crucial role in determining the fate of the $\phi = 0$ state. We introduce this non-minimal coupling by adding a term $\frac{1}{2}bR\phi^2$ to the scalar lagrangian. Depending on the sign of the constant b, this term either stabilizes or destabilizes the false vacuum.

Below, we will investigate the three possibilities $b = 0$, $b < 0$ and $b > 0$ for the SU(5) model [5] broken down to SU(3)×SU(2)×U(1) by a scalar field with a Coleman–Weinberg potential [4]. The results are as follows. At temperatures above about 10^{14} GeV, the Universe is in the state $\phi = 0$. Then:

(i) If $b = 0$, extreme supercooling to a temperature of about $\frac{1}{2}$ GeV takes place before the SU(5) → SU(3) × SU(2) × U(1) phase transition can occur (see refs. [9] for more detailed studies).

(ii) If $b < 0$, then the term $\frac{1}{2}bR\phi^2$ acts as a negative mass term. For temperatures below about 10^{10} GeV this term dominates over the temperature dependent mass term $T^2\phi^2$ and destabilizes the $\phi = 0$ state. Thus, the Universe will supercool only to about 10^{10} GeV at which point a rapid phase transition will occur.

(iii) If $b > 0$, the term $\frac{1}{2}bR\phi^2$ acts as a positive mass term creating a barrier even at zero temperature. This barrier prevents the SU(5) → SU(3) × SU(2) × U(1) phase transition from ever being completed.

Strictly speaking, results (i) and (iii) are statements about the Coleman–Weinberg potential and not about the SU(5) model in general. This is because at temperatures as low as those found in (i) and (iii) the SU(5) model becomes strong coupling. Then, fermion condensates may form breaking the SU(5) symmetry dynamically [10]. A consideration of this mechanism is outside the scope of this paper. The results (i)–(iii) are sufficient to demonstrate the importance of gravitational effects on the Coleman–Weinberg model at the GUT scale.

If the Coleman–Weinberg mode of symmetry breaking actually occurs, this is probably due to some sort of conformal invariance acting at the classical level. If this is the case, then the value $b=\frac{1}{6}$ is favoured. This is because the choice of $b=\frac{1}{6}$ makes the massless scalar lagrangian conformally invariant [11]. However, result (iii) indicates that for this conformally invariant theory, the GUT phase transition can never be completed.

2. Results

Consider the SU(5) model [5] broken down to SU(3)×SU(2)×U(1) in the Coleman–Weinberg mode. The effective potential which determines the scalar vacuum expectation value is [4]

$$V(\phi)=B\phi^4\left(\ln\frac{\phi^2}{\sigma^2}-\frac{1}{2}\right)+\tfrac{1}{2}B\sigma^4\,, \tag{1}$$

where (we take $\alpha_{GUT}=\frac{1}{45}$)

$$\sigma=10^{15}\,\mathrm{GeV}\,, \tag{2}$$

$$B=8\times10^{-4}\,. \tag{3}$$

At finite temperature a term is added to eq. (1) which is a complicated function [6] of ϕ and the temperature T. However, for $\phi \ll T \ll \sigma$ this term is well approximated by an effective mass term

$$V_T=\tfrac{5}{8}g^2T^2\phi^2 \tag{4}$$

where g is the SU(5) coupling constant. This term stabilizes the false vacuum, $\phi=0$, far below the critical temperature at which the true vacuum $\phi=\sigma$ becomes the lowest energy state.

Note that the term $\frac{1}{2}B\sigma^4$ has been added to eq. (1) so that the true vacuum state, $\phi=\sigma$, has zero energy*. Then, the false vacuum, $\phi=0$, has the positive energy density $\frac{1}{2}B\sigma^4$. This energy density creates a space–time curvature determined by the Einstein equation

$$R=-32\pi G(\tfrac{1}{2}B\sigma^4)\,. \tag{5}$$

This means that while the Universe is supercooling in the false vacuum state, space is expanding. The time scale for this expansion is

$$[\tfrac{8}{3}\pi G\,(\tfrac{1}{2}B\sigma^4)]^{-1/2}\,. \tag{6}$$

* Why the present vacuum should have an energy density of magnitude less than $(0.003\ \mathrm{eV})^4$ is a great mystery.

The rate per unit volume for tunnelling from the false vacuum to the true vacuum is approximately [12]*

$$\sigma^4 \, e^{-A} , \tag{7}$$

where A is the euclidean tunnelling action calculated by the techniques of Callan and Coleman [12]. In order for tunnelling to be significant, the tunnelling probability in a space–time volume determined by the time scale of eq. (6) must be of order one. Thus, the condition for the phase transition to occur is [1–3]

$$[\tfrac{8}{3}\pi G(\tfrac{1}{2}B\sigma^4)]^{-2}\sigma^4 \, e^{-A} \approx 1 . \tag{8}$$

Putting in the various numbers, we find that (8) is satisfied if

$$A \approx 50 . \tag{9}$$

If A is larger than this value, the phase transition will not be completed.

We will now compute the temperature at which the SU(5) phase transition occurs for the case of minimal gravitational coupling [9], $b = 0$. The potential for this case is just the sum of eqs. (1) and (4),

$$V_{\rm TOT} = \tfrac{5}{8} g^2 T^2 \phi^2 + B\phi^4 \left(\ln \frac{\sigma^2}{\sigma^2} - \frac{1}{2} \right) + \tfrac{1}{2} B\sigma^4 . \tag{10}$$

Witten [2] has devised a clever trick for estimating the tunnelling amplitude for this potential. The potential (10) at low temperatures has a narrow barrier near $\phi = 0$ followed by a long drop-off to the $\phi = \sigma$ state. Thus, for the purposes of tunnelling it can be well approximated by a wrong sign ϕ^4 potential.

To estimate the size of this ϕ^4 coupling we write [2]

$$\ln\left(\frac{\phi}{\sigma}\right) = \ln\left(\frac{g\phi}{T}\right) - \ln\left(\frac{g\sigma}{T}\right) . \tag{11}$$

At low temperatures the second logarithm on the right-hand side of eq. (11) will dominate over the first logarithm over the range of the barrier. Thus, for calculating the tunnelling amplitude we can write

$$V_{\rm TOT} \approx \tfrac{5}{8} g^2 T^2 \phi^2 - 2B \ln\left(\frac{g\sigma}{T}\right)\phi^4 + \tfrac{1}{2} B\sigma^4 . \tag{12}$$

(At low T, the barrier is narrow enough so that the approximation $\phi \ll T$ is also valid.) The amplitude A for this potential can be determined from previous computations [13] and is

$$A = (1.5)\, 4\pi \sqrt{\frac{5}{4}} \frac{g}{8B \ln (g\sigma/T)} . \tag{13}$$

* The factor σ^4 multiplying the exponential in this equation is determined on dimensional grounds. However, the temperature T offers another possibility and in ref. [2] this factor is written as T^4. If we took this approach the tunnelling amplitude would be smaller than eq. (7) and tunnelling would be further suppressed for $b \geqslant 0$.

From this we find that eq. (9) is satisfied at a temperature

$$T = 0.4\ \mathrm{GeV}\,. \tag{14}$$

Thus, for the case $b = 0$, there is extreme supercooling in this model [9].

When $b \neq 0$ an additional term is added to the scalar potential,

$$V_G = -\tfrac{1}{2} bR\phi^2 \tag{15}$$

so that

$$V_{\mathrm{TOT}} = -\tfrac{1}{2}bR\phi^2 + \tfrac{5}{8}g^2T^2\phi^2 + B\phi^4\left(\ln\frac{\phi^2}{\sigma^2} - \frac{1}{2}\right) + \tfrac{1}{2}B\sigma^4\,. \tag{16}$$

If $b < 0$, this additonal term destabilizes the $\phi = 0$ state. To see this, consider the total potential near $\phi = 0$. Then, R is determined by eq. (5) and

$$V_{\mathrm{TOT}} = (8\pi GbB\sigma^4 + \tfrac{5}{8}g^2T^2)\phi^2 + \mathrm{O}(\phi^4) + \tfrac{1}{2}B\sigma^4\,. \tag{17}$$

At sufficiently low temperatures the term in brackets will go negative (since $b < 0$) destabilizing the $\phi = 0$ state. This will occur at

$$T = \left(\frac{64\pi G|b|B\sigma^4}{5g^2}\right)^{1/2} = 2.8\times 10^{10}\sqrt{|b|}\ \mathrm{GeV}\,. \tag{18}$$

Since $|b|$ is likely of order one, this means that at a temperature of about 10^{10} GeV the $\phi = 0$ state will decay rapidly into the true vacuum state.

We now consider the case $b > 0$. In particular, we will take $b = \frac{1}{6}$ since this gives the conformally invariant classical theory [11]. The $R\phi^2$ term in the potential of eq. (16) then provides a barrier between the $\phi = 0$ and $\phi = \sigma$ states even for $T = 0$. Our results for $b = 0$ indicate that thermal effects are insufficient to induce tunnelling through this barrier. We must, therefore, examine quantum mechanical tunnelling for the zero temperature potential

$$V_{\mathrm{TOT}} = -\tfrac{1}{12}R\phi^2 + B\phi^4\left(\ln\frac{\phi^2}{\sigma^2} - \frac{1}{2}\right) + \tfrac{1}{2}B\sigma^4\,. \tag{19}$$

For small values of ϕ^2, the term $-\frac{1}{12}R\phi^2$ acts as a right-sign mass term $\frac{1}{2}m^2\phi^2$ with m given, according to eq. (5) by

$$m = [\tfrac{16}{3}\pi G\,(\tfrac{1}{2}B\sigma^4)]^{1/2} = 6.7\times 10^9\ \mathrm{GeV}\,. \tag{20}$$

The tunnelling action for the potential of eq. (19) with R given by the Einstein equation was calculated numerically by computer. The result is

$$A = 637\,. \tag{21}$$

This is much greater than the limit, $A \approx 50$, of eq. (9). Thus, in the conformally invariant theory, the SU(5) breaking phase transition cannot be completed, at least until temperatures have been reached for which the SU(5) coupling becomes large.

3. Conclusions

We have seen that the sign of the non-minimal gravitational coupling $R\phi^2$ in the Coleman–Weinberg model operating in a grand unified theory is crucial in determining the fate of the high temperature, symmetric state. It is unusual to see gravity playing such an important role in an elementary particle field theory.

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