Nuclear Physics B203 (1982) 493-500

CONFINING MODELS OF THE WEAK INTERACTIONS IN TECHNICOLOR

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Received 7 January 1982
(Revised 27 April 1982)

We construct confining models of the weak interactions without fundamental scalar fields using technicolor, and show how to generate fermion masses in these models with extended technicolor. In addition, we discuss the influence of strong $SU(2)_L$ on the spectrum of several theories.

1. Introduction

It is possible to construct weak interaction models based on confining gauge theories [1,2]. In ref. [1], a model of this type, involving fundamental scalar fields, was exhibited. There are reasons to believe, however, that fundamental scalars cannot be light enough (relative to the unification scale) to play any significant role at the mass scale of the weak interactions. In this paper we will address the problem of constructing confining models of the weak interactions without fundamental scalar fields. Our approach is based on the ideas of technicolor [3] and extended technicolor [4].

In the standard model [5] the scalar sector is introduced primarily to break the weak interaction gauge symmetries. The scalar potential is arranged to have a

[1] Research supported by the US Department of Energy under contract DE-AC03-76ER03230-A005.
[2] This work is supported in part through funds provided by the US Department of Energy (DOE) under contract DE-AC02-76ER03069.
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ground state which is invariant under a smaller symmetry group than the lagrangian. This produces Goldstone bosons which through the Higgs mechanism become the longitudinal components of the W and Z. In the confining version of this model, the scalar sector plays a different role. The scalar fields are confined because they transform non-trivially under the strong unbroken $SU(2)_L$ gauge group. They bind together with fermions to produce gauge singlet composite states which are the observed left-handed fermions. The spontaneously broken and confining theories are experimentally indistinguishable at low energies primarily because their global symmetry properties are the same. In sect. 2 we will discuss the connection between these theories as you adjust the parameters which control the spontaneous symmetry breakdown.

In technicolored versions of the standard model [3], the scalar sector is replaced by strongly interacting fermions. The technicolor force causes a spontaneous breakdown of the chiral symmetry associated with the technifermions. Since the weak interactions gauge a subgroup of the chiral symmetry group, those gauge bosons corresponding to broken generators become massive. The primary role of technicolor is to break the weak interaction gauge group.

In confining models of the weak interactions without scalars, the technicolor force will serve a different function. The techniforce in this case does not dynamically break the chiral symmetry group, but rather produces spinless bound states which are not Goldstone bosons. These two fermion massive scalar states feel the strong $SU(2)_L$ force and they combine with $SU(2)_L$ non-singlet fermions to produce the observed left-handed fermions. We will show that it is consistent to view these three fermion bound states as massless because the 't Hooft anomaly conditions [6] are satisfied. We will also show that this technicolored version of the model has the same global symmetry properties as the scalar model, so it too can reproduce the observed low-energy effective four-Fermi weak interactions.

The standard mass generating mechanism in technicolored models is extended technicolor [4]. We will show that unifying technicolor and $SU(2)_L$ into a larger group can also give rise to fermion masses in confining models of the weak interactions. Our example has the additional interesting feature that the massive states produced are linear combinations of bound states with different numbers and types of constituents.

2. Global symmetries of confining gauge theories

The confining model of the weak interactions of ref. [1] and the standard model are based on the same lagrangian. The differences between these two models arise from differences in values of the coupling constants. The theory has N left-handed $SU(2)_L$ doublet fermions ψ^a_{Li} ($a = 1, N$; $i = 1,2$) and a complex scalar doublet ϕ_i. The interactions are the $SU(2)_L$ gauge interactions and the scalar self-couplings. For the moment we will ignore Yukawa couplings and other gauge couplings since at the weak interaction scale these are much weaker than the strong $SU(2)_L$ force. In addition, right-handed fermions will not be considered since they do not feel the $SU(2)_L$ interaction. There is a global $SU(N)$ symmetry associated with the N fermion doublets. The scalar potential has an $SU(2) \times SU(2)$ symmetry but one of these SU(2)'s is gauged so the total global symmetry group is $SU(N) \times SU(2)$. The fermion number current is not conserved due to anomalies.

The dynamics of this lagrangian are characterized by two dimensionful numbers. The first is Λ_2, the parameter of $SU(2)_L$ indicating where the gauge interactions become strong (or would become strong if the gauge group were not broken). The second is m^2, the mass parameter of the scalar field. If $m^2 \ll -\Lambda_2^2$ the theory is spontaneously broken and ϕ has a vacuum expectation value. This vacuum still respects a global SU(2) symmetry so the global group remains $SU(2) \times SU(N)$. It is convenient to do small coupling perturbation theory around the non-zero minimum. This is the standard model.

We now adjust the parameters until $m^2 \approx 0$. (Actually [7], $-\Lambda_2^2 \lesssim m^2 \lesssim \Lambda_2^2$). The scale of the strong interactions is larger than the symmetry breaking scale so the theory is confining. Physical states are gauge singlets and we construct composite left-handed fermions $\phi^{*i}\psi^a_{Li}$ and $\varepsilon^{ij}\phi_i\psi^a_{Lj}$ which form an $(N,2)$ under the global group $SU(N) \times SU(2)$. These states can be used to compute triangle anomalies involving three currents of the global group. In ref. [1] it was shown that these states produce the same anomalies as the gauge non-singlet fields which appear in the lagrangian*. Thus, it is consistent [6] to view these states as physical massless bound states and to assume that the chiral symmetry group $SU(N) \times SU(2)$ is unbroken. This is the basis of the confining model of the weak interactions.

If $m^2 \gg \Lambda_2^2$ the scalar field effectively drops out of the theory and we are left with only the N fermion doublets and their gauge interaction. The global symmetry of this theory is $SU(N)$. The 't Hooft anomaly conditions cannot be satisfied so the $SU(N)$ symmetry must be dynamically broken. It is most likely broken by the condensate $A^{ab} = \varepsilon^{ij}\psi^a_{Li}\psi^b_{Lj}$. If A^{ab}, which is antisymmetric in a and b, is proportional to the simplectic form then $SU(N)$ breaks down to $SP(N)$ for N even and $SP(N-1) \times U(1)$ for N odd. The phase structure outlined here is displayed in fig. 1. Note that the presence of the scalar field with $m^2 \lesssim \Lambda_2^2$ is crucial for the global symmetry group to remain unbroken.

The confining model of the weak interactions requires that the $SU(2)_L$ theory be confining and that the global symmetry group, $SU(N) \times SU(2)$, be unbroken. Thus, m^2 must not be much less than about $-\Lambda_2^2$ or too much greater than Λ_2^2. The above analysis shows that the scalar mass parameter m^2 is a relevant mass scale in this model. The model therefore depends on two mass parameters, m^2 and Λ_2^2 (unlike QCD, for example, which only depends on one scale Λ). This may be relevant to

* The fact that 't Hooft anomaly conditions can be satisfied in models with scalar and fermion constituents was also noted in ref. [9].

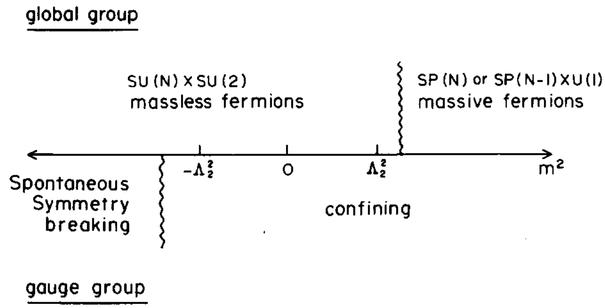

Fig. 1.

explain some discrepancies between the mass scales appearing in various form factors of the model as discussed in ref. [1]. It is also important to note that when the $SU(2)_L$ interaction is strong the scalar self-coupling gets strong too. Thus, the scalar sector is important in the interaction dynamics of the model as well as in setting mass scales. If the scalar self-coupling is strong enough, for example, this could explain why scalar exchange dominates the low-energy effective four-Fermi interaction as discussed in ref. [1].

Now suppose you replace the scalar sector with strongly interacting fermions and a non-abelian gauge force (e.g. technicolor) whose scale is Λ_T. If technicolor is much stronger than the $SU(2)_L$ interactions, i.e. $\Lambda_T \gg \Lambda_2$, you could safely neglect $SU(2)_L$ and attempt to analyze the technicolor dynamics alone. However, if you then let Λ_2 approach Λ_T the dynamics could be radically altered. As an example suppose technicolor is SU(3) with four technifermions A, B, C, D, both left- and right-handed, transforming as triplets under SU(3). The global symmetry group is $SU(4)_L \times SU(4)_R \times U(1)$. 't Hooft has argued [6] that this theory must break chiral symmetry and there are no massless bound states. Now turn on the weak interactions by gauging the $SU(2)_L$ subgroup of $SU(4)_L$ which treats $(A,B)_L$ and $(C,D)_L$ as doublets. If $\Lambda_2 \sim \Lambda_T$, the strong group is now $SU(2)_L \times SU(3)$ and the global group is $SU(4)_R \times SU(2)$ where the SU(2) interchanges the two left-handed doublets. The only possible triangle anomaly comes from 3 $SU(4)_R$ currents. Normalize the group theory factors so that a right-handed $\underline{N}$ of $SU(N)$ produces an anomaly of 1. Then the preons produce an anomaly of 3 since there are three colors of right-handed $SU(4)_R$ $\underline{4}$'s. It is possible to construct gauge singlet bound states which produce the same anomaly. These have the form

$$Q_L^{xi} Q_L^{yj} Q_R^K \varepsilon_{ij} S^l_{xy}, \qquad (l=1,2,3), \quad (K=1,2,3,4),$$

where $Q_L^{1i} = (A,B)_L$, $Q_L^{2i} = (C,D)_L$, $Q_R^K = (A,B,C,D)_R$, SU(3) gauge indices have been suppressed and S^l_{xy} is symmetric in x and y. These states are $SU(2)_L \times SU(3)$ singlets and transform as a right-handed (4,3) of $SU(4)_R \times SU(2)$. Since these states match the anomalies of the preons it is possible that they form physical massless bound states and the theory does not break chiral symmetry. Without the $SU(2)_L$ interactions the theory has no massless states and chiral symmetry breaking occurs. Turning on the $SU(2)_L$ interactions allows the possibility of making bound states which are kept massless by unbroken chiral symmetries.

3. A technicolor model

In this section we will show how to construct confining models of the weak interactions without scalar fields. The scalar sector is replaced by a technicolored sector which does not have chiral symmetry breakdown. Suppose the technicolor group is $SU(M)$ and there is one left-handed $SU(2)_L$ doublet T^i_L ($i = 1, 2$) as well as two right-handed $SU(2)_L$ singlets A_R, B_R all of which transform as $\underline{M}$'s of $SU(M)$. This sector has a global SU(2) symmetry which acts on A_R and B_R.

In the non-technicolored sector there are N left-handed $SU(2)_L$ doublets ψ^a_{Li} ($i = 1, 2$; $a = 1, N$) with an associated global $SU(N)$ symmetry. The two sectors together admit an additional anomaly free U(1) symmetry. The particles and their transformation properties are summarized in table 1.

The confining gauge group is $SU(M) \times SU(2)_L$ and we assume it produces gauge-singlet bound states. In particular the three-fermion left-handed bound states

$$\bar{A}_R T^i_L \psi^j_{La} \varepsilon_{ij}, \qquad \bar{B}_R T^i_L \psi^j_{La} \varepsilon_{ij}$$

form a $(N, 2, -M)$ under the global group $SU(N) \times SU(2) \times U(1)$ and are just the particles required to match the anomalies of the preons of table 1. There are four non-trivial anomaly diagrams $[SU(N)]^3$, $[SU(N)]^2 \times U(1)$, $[SU(2)]^2 \times U(1)$ and $[U(1)]^3$ and the reader can check that the anomalies match in each case. Thus it is consistent to view these states as massless composites.

The technicolor force in conjunction with a strong $SU(2)_L$ produces a set of left-handed massless bound states which transform as an $(N, 2)$ under the global symmetry group $SU(N) \times SU(2)$. As was shown in ref. [1], this greatly restricts the possible residual low-energy four-Fermi interactions amongst those particles. These four-Fermi interactions include the observed weak interactions and by making certain dynamical assumptions the four-Fermi interactions can be shown to be

TABLE 1

Particle	Gauge group $SU(M) \times SU(2)_L$	Global group $SU(N) \times SU(2) \times U(1)$
T_L	$(M, 2)$	$(1, 1, N)$
A_R, B_R	$(M, 1)$	$(1, 2, N)$
ψ_L	$(1, 2)$	$(N, 1, -M)$

exactly the observed weak interactions. The phenomenology of confining models of the weak interactions, at low energies, does not depend on whether the scalar field is fundamental or composite just as in the standard weak interaction model.

4. Fermion masses

Until this point we have ignored fermion masses as well as right-handed fields. Any successful model must of course produce fermion masses, that is, transitions from left-handed to right-handed states. In confining models of the weak interactions with fundamental scalars this is done through the Yukawa couplings. For example there is a coupling $\lambda_e \bar{e}_R \phi_i \psi^e_{jL} \varepsilon^{ij}$ between the right-handed $SU(2)_L$ singlet electron the composite left-handed electron which produces an electron mass proportional to λ_e.

Without fundamental scalars it is more difficult to generate masses. In technicolored models masses can be generated through extended technicolor [4]. This approach has many difficulties in realistic models but we will show how it can be applied to confining models of the weak interactions. The basic idea is to unify* the weak interaction gauge group $SU(2)_L$ and the technicolor group into a larger group which at some high-energy scale breaks down into the two subgroups. The broken generators couple to massive gauge bosons which are responsible for fermion masses.

As an example suppose technicolor is SU(3) and there is one non-technicolored $SU(2)_L$ doublet. This is the case $M = 3$, $N = 1$ of the previous section. We begin by studying the strong $SU(3) \times SU(2)_L$ sector and later we will unify these interactions. First consider what happens when SU(3) becomes strong. The confining SU(3) produces fermionic and bosonic bound states (p, n) and π like the proton, neutron and pions of QCD. We assume that SU(3) with two flavors does not break chiral symmetries in the presence of a strong $SU(2)_L$. Thus the proton and neutron like states are massless while the pions are massive.

The SU(3) singlet state ψ^i_L must also be confined because it is not a singlet under $SU(2)_L$. It combines with the pion-like objects and forms a three-preon massless SU(2) global doublet as shown in the previous section. This state has the global U(1) quantum number -3 as can be seen from table 1. The conjugate of $(p, n)_R$ also has the charge -3. Since both these states are left-handed SU(2) global doublets they can mix and the physical massless state is really a linear combination.

In the presence of strong $SU(2)_L$ the $(p, n)_L$ doublet combines with the π's to form 5 preon, $SU(2)_L \times SU(3)$ singlet states. The $(p, n)_R$ exist as 3-preon gauge singlet states. Both of these gauge singlet states are doublets under the global SU(2) and it is possible to write down mass terms which do not violate the global chiral symmetries. Thus the strong $SU(2)_L$ can take SU(3) without chiral symmetry breakdown and eliminate the massless fermions.

* This is a slight variant of extended technicolor since extended technicolor usually unifies technicolor with the color group or with a totally broken subgroup, not with $SU(2)_L$.

We now want to give the massless state a small mass. We do this by unifying $SU(2)_L$ and SU(3) into SU(5) and including a right-handed $SU(2)_L \times SU(3)$ singlet e_R with charge -3. All of the fundamental fermions we have now introduced sit nicely in the $\bar{5} + 10$ representation of SU(5). We imagine that SU(5) is broken down at a mass scale H. The resulting gauge bosons of mass H give rise to effective four-Fermi operators at low energy of the form:

$$\frac{1}{H^2} e_R A_R T_L^i T_L^j \varepsilon_{ij}, \qquad \frac{1}{H^2} e_R B_R^* T_L^i \psi_{iL}^*,$$

which are transitions between e_R and both the components of the massless left-handed state. These explicit SU(2) global violating interactions give the massless state a small mass and also allow e_R to mix with the conjugate of the 5 preon P_L. The physical electron has a small mass. Its left-handed component is mostly the fundamental leptonic preon ψ bound with the massive technipion. In addition, there is a small admixture of a 3 technifermion bound state. The right-handed electron is mostly the fundamental field e_R. Masses have been produced by unifying technicolor and SU(2) into a larger group which is explicitly breaking the chiral symmetries of the two subgroups taken alone.

It is instructive to study this $SU(3) \times SU(2)_L$ example in the limit $\Lambda_2 \gg \Lambda_3$ and for these purposes we will ignore the mass generating mechanism. If $\Lambda_2 \gg \Lambda_3$ we can ignore the SU(3) force and imagine that $SU(2)_L$ invariant condensates form. These are of two types:

$$T_L^i T_L^j \varepsilon_{ij}, \qquad T_L^i \psi_L^j \varepsilon_{ij}.$$

Since the T_L's are SU(3) triplets the first is a SU(3) $\underset{\sim}{3}^*$ with U(1) quantum number -2 while the second is a $\underset{\sim}{3}$ with the quantum number 2. We will assume that the condensates both line up in the three direction of SU(3) space so SU(3) is broken down to SU(2). The U(1) quantum number is also broken, however, there is an unbroken U(1) which can be formed [8] out of the original U(1) generator and the λ_8 SU(3) generator:

$$Q' = Q - \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

At low energies the $SU(2)_L$ force and the residual SU(2) force are both confining. The only free particles are the singlets under both the groups: A_R^3 and B_R^3. These are a doublet under SU(2) global and have $Q' = 3$. They can be used to saturate the anomaly diagrams of the unbroken chiral group and will remain massless. Thus in the $SU(3) \times SU(2)_L$ model with one set of particles having the gauge quantum numbers of an ordinary family, the existence of two left-handed massless particles with the unbroken global quantum number -3 is independent of the relative strengths of SU(3) and $SU(2)_L$. However, for models with more than one doublet this is not generally true.

5. Conclusions

Confining models of the weak interactions without scalar fields can be constructed. It is interesting to picture how such a model might be unified with SU(3) color and U(1) electromagnetism into a grand unified theory. The confining $SU(2)_L$ force must get strong at a higher mass scale than SU(3) color. Normally, this would preclude the possibility that the $SU(2)_L$ and SU(3) color coupling constants become equal at a unification mass. However, if $SU(2)_L$ is unified with a technicolor group at some intermediate scale and if the resulting group is larger than SU(3) color, the possibility of unification will exist [1]. Where will this unification point be? If the grand unified model consists solely of SU(3) color, U(1) electromagnetism and the combined $SU(2)_L$ and technicolor group and if it has the normal family structure, then the unification point will be scale where the SU(3) and the correctly normalized U(1) coupling constants meet*. This can be computed from our knowledge of α_{QCD} and α_{EM} and is 10^{19} GeV, remarkably close to the Planck mass.

This work was started at CERN and we would like to thank the CERN Theory Group for its hospitality.

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* We normalize the charge operator as in SU(5) assuming fermions come in complete sets with the gauge quantum numbers of an ordinary family.