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CONSTRAINTS ON THE ELECTRON-NEUTRINO MASS FROM THE SUPERNOVA DATA
A systematic analysis

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The energy versus time of arrival pattern of neutrinos from SN1987A is sensitive to a neutrino mass, m_ν, of order a few eV. To disentangle constraints on m_ν from the data, a theory of supernova emission is necessary. We recall the present status of this theory and approximate its predictions in two diffusion models: one designed to reflect the present supernova lore, the other devised to pessimize, within reason, the consequent upper limits on m_ν. We discuss the model dependence and statistical significance of our results, as well as the experimental uncertainties and caveats to which they are subject. We address the question, do the supernova results supercede the present laboratory limits on m_ν?

1. Introduction

Some amount of instant attention [1] has been given to the extraction of limits on (or even measurements of) neutrino masses from the underground data [2–6] on the supernova SN1987A. In this post-gold-rush paper, we analyze neutrino mass limits in detail, paying careful attention to the questions of model-dependence, fit quality, statistical significance, and the effect of measurement errors and other experimental uncertainties on the results. We do not ignore the fact that upper limits, to deserve that qualification, must be pessimized relative to the unknown parameters of the underlying theory.

The supernova neutrino data for each experiment consist of a handful of events, all or most of which can be attributed to the dominant process of $\bar{\nu}_e +$ (free)p $\to$ $e^+ + n$ scattering. Each event, i, is characterized by a time of arrival t_i and a positron energy E_i, measured to 20–30% precision. For our purposes, the millisecond errors in t_i are negligible, at least in the time differences $t_i - t_j$ within a given experiment. The directionality of the events, which can be measured to some extent in water-Cherenkov counters, only plays an incidental role in our analysis. The energies and timings of the 12 Kamiokande II(K2) events [3] and of the 8 IMB events [4] registered at ~ 7h 35min UT on Feb. 23rd 1987 are shown in table 1. The table also includes the 5 LSD events [2, 6] registered ~ 4h 43min earlier, that should not be lightly ignored [7].

Let N be the total expected number of events, $E_e(E_\nu)$ be the $e^+(\bar{\nu}_e)$ energy, σ be the $\bar{\nu}_e p \to e^+ n$ cross section, $\varepsilon(E_e)$ be the detector's positron detection and reconstruction efficiency*, and E_{TH} be the e^+ threshold detection energy, below which the events are explicitly cut off to supress the radioactive background. Let the time-integrated neutrino flux from the supernova, $d\Phi/dE_\nu$, be a Fermi-thermal spectrum characterized by a temperature T, an approximation to be presently discussed. The expected time-integrated positron energy distribution is:

$$dN/N\,dE_e = \sigma\varepsilon(d\Phi/dE_\nu) \bigg/ \left[\int_{E_{TH}}^{\infty} \sigma\varepsilon(d\Phi/dE_\nu)\, dE_e \right], \tag{1a}$$

$$d\Phi/dE_\nu = E_\nu^2/[1 + \exp(E_\nu/T)], \tag{1b}$$

$$E_\nu \cong E_e + m_n - m_p \equiv E_e + Q, \tag{1c}$$

$$\sigma = (G_F^2/\pi)\cos^2\theta_c(1 + 3g_A^2)E_e p_e \cong (2.3 \times 10^{-44}\ \text{cm}^2)(E_e p_e/m_e^2). \tag{1d}$$

Let $f(E_\nu, t_{SN})$ be the time distribution of emitted neutrinos at fixed energy E_ν, normalized to a constant integral from $t_{SN} = 0$ (the core's implosion time) to $t_{SN} = \infty$. Let r be the distance to the supernova and m the electron neutrino mass. The expected normalized distribution of events in energy and earthly time, t, is:

$$dN/N\,dE_e\,dt = (dN/N\,dE_e) f(E_\nu, t_{SN}(t)), \tag{2a}$$

$$t_{SN}(t) \simeq t - \frac{r}{c}\left(1 + \frac{1}{2}\frac{m^2}{E_\nu^2}\right) = t - 2.675\left(\frac{m}{1\,\text{eV}}\right)^2\left(\frac{1\,\text{MeV}}{E_\nu}\right)^2\left(\frac{r}{52\,\text{kpc}}\right) + \text{const}. \tag{2b}$$

In what follows we use a distance of $r = 52$ kpc: to be precise our results on m

* We have parametrized the K2 efficiency $\varepsilon(E_e)$ as a function that rises linearly from zero to unity in the interval $(E_0, 2E_0)$, with $E_0 = 5.5$ MeV; and is suppressed to a vanishing value below $E_{TH} = 7.5$ MeV. An overall multiplicative constant (relevant to the efficiency in the detector's total volume) plays no role in our neutrino-mass analysis. Some events may "leak" from below E_{TH}, a possibility that we allow by smearing their energies with the quoted [3] gaussian errors. The LSD efficiency is parametrized with the same functional form, but with parameters $E_0 = 3.5$ MeV, $E_{TH} = 4.5$ MeV. This is a very good approximation to the recently recalibrated efficiencies [6]. The IMB efficiency is parametrized by the function $\varepsilon(E_e) = 1 - \exp[-0.08(E_e - E_1)^{0.93}]$, with $E_1 = 18$ MeV. We have checked that our results on neutrino masses are not significantly affected by reasonable modifications of $\varepsilon(E_e)$. This is less so for the optimal values of T, and not at all the case for the supernova's total neutrino luminosity [7].

TABLE 1
Results from the K2, IMB and LSD experiments: times from the first event, energies, and (in the case of water detectors) angles of the Cherenkov cones relative to the direction pointing away from SN1987A

Experiment	Event	Time (s)	E_e (MeV)	θ_{LMC} (deg.)
Kamioka II	1	0	20.0 ± 2.9	18 ± 18
	2	0.107	13.5 ± 3.2	15 ± 27
	3	0.303	7.5 ± 2.0	108 ± 32
	4	0.324	9.2 ± 2.7	70 ± 30
	5	0.507	12.8 ± 2.9	135 ± 23
(K2)	(6	0.686	6.3 ± 1.7	68 ± 77)
	7	1.541	35.4 ± 8.0	32 ± 16
	8	1.728	21.0 ± 4.2	30 ± 18
	9	1.915	19.8 ± 3.2	38 ± 22
(Water)	10	9.219	8.6 ± 2.7	122 ± 30
	11	10.433	13.0 ± 2.6	49 ± 26
	12	12.439	8.9 ± 1.9	91 ± 39
	33162	0	38 ± 9.5	74 ± 15
	33164	0.42	37 ± 9.3	52 ± 15
IMB	33167	0.65	40 ± 10	56 ± 15
	33168	1.15	35 ± 8.8	63 ± 15
(Water)	33170	1.57	29 ± 7.3	40 ± 15
	33173	2.69	37 ± 9.3	52 ± 15
	33179	5.01	20 ± 5	39 ± 15
	33184	5.59	24 ± 6	102 ± 15
	994	0	6.2 ± 0.7	
LSD	995	3.86	5.8 ± 0.7	
(UNO)	996	4.22	7.8 ± 0.9	
(Scintil.)	997	5.91	7.0 ± 0.7	
	998	7.01	6.8 ± 0.8	

The LSD energies are the recently revised ones [6].

should be interpreted as constraints on $m(r/52\ \text{kpc})^{1/2}$. To extract information on m, a model of $f(E_\nu, t_{SN})$ is *necessary*. Clearly one can conceive of models in which neutrinos are emitted in sharp bursts at the same time intervals as the observed events. Such models would constrain m to vanish, with an error governed only by energy measurement uncertainties (the K2 and IMB data show evidence of periodicity, and models akin to this [8] have actually been discussed). For any given value of m one can also imagine a totally ad-hoc pattern of emission that would accommodate any given experimental results. Our intention is to investigate how m is constrained by the data on sensible general grounds. To this end we must discuss models of $f(E_\nu, t_{SN})$ incorporating only the generally accepted properties of neutrino emission by a supernova [9], and let the data itself fix the parameters of the model and ascertain the quality of the fits. This is standard procedure, followed for instance in the extraction of neutrino mass limits from β-decay experiments.

The remainder of this paper is organized in the obvious fashion, reflected in the titles of its different sections.

2. Supernova neutrinos: theory and uncertainties

In eq. (1) we have assumed that the time-integrated neutrino flux is thermal. Detailed numerical simulations of supernova core collapse do indeed yield neutrino energy spectra that are approximately thermal, except for deviations in their low and high energy tails. The experiments at hand have low-energy thresholds that lie near or above the mean energy of the spectra that best fit the data. Only "high-energy" deviations from a thermal spectrum are potentially relevant. These deviations are subject to large theoretical uncertainties: some authors [10] find that neutrino absorption depletes the high-energy tail relative to a thermal fit, others [11, 12] find a small enhancement of the high-energy neutrino flux. But for data sets as scarce as the ones at hand, it is not possible to establish a meaningful deviation from a thermal spectrum: all that a sensible amount of effects such as absorption may entail [7] is a modification of the effective temperature approximately describing the observed energy spectrum. The temperature and the neutrino mass are not strongly correlated in the determination of the time-energy pattern of neutrino arrival, and the precise shape of the energy spectrum is not crucial to constraints on the neutrino mass, as long as it fits the data well. Thus, we stick to a thermal spectrum as in eq. (1b).

Electron antineutrinos produced and trapped [13] in the collapsing core of a stellar object are expected to diffuse their way out to a "neutrino-sphere" of tens of kilometers radius R_ν, beyond which the column density is small enough for neutrinos to escape with little or no further interaction*. The characteristic diffusion times (seconds) are predicted [9] to be much longer than the time scale for core collapse (fractions of a second). The observed neutrino pulses from SN1987A last for a few seconds, indicating the existence of a time-stretching diffusion process, independently of whether the core of SN1987A collapsed into a neutron star or/and [7] a black hole. Due to its extraordinary complexity, supernova physics is still in its infancy, and even the relatively simple processes occurring within a few seconds of core collapse are not understood in utmost detail.

To extract neutrino mass limits we need an explicit expression for the function $f(E_\nu, t_{SN})$ in eq. (2a). Most of the supernova literature of the past did not concern itself with the explicit time-energy correlation function f, for which no generally

* The radius of the neutrino-sphere, from which neutrinos finally escape from the supernova, is not perfectly defined. Low energy neutrinos escape earlier and from greater depths than the more strongly interacting high energy neutrinos. For the sharply falling density distributions characteristic of the collapsed core, this effect is not very important and the neutrino sphere is rather well defined: R_ν is a slowly varying function of E_ν, except for the relatively scarce neutrinos of considerably higher or lower energy than the mean.

accepted form exists; but rather with the bulk properties of neutrino emission: luminosity, temperature, and overall duration of the pulses. There are some exceptions to this rule, in which the authors present results [12, 14] for the average energy of different neutrinos as a function of time. It is well established that the neutrino mean free paths within the collapsed core are short enough for neutrinos to be trapped and thermalized at high temperatures, that in the inner collapsed core reach 10–30 MeV. As neutrinos diffuse their way out of this trap, at least two neutrino scattering processes are important in the determination of $f(E_\nu, t_{SN})$ and of the energy spectrum $d\Phi/dE_\nu$:

(i) Neutrino elastic scattering off, not only neutrons; but also protons, α-particles and even heavy nuclei, that persist in various time-dependent fractions inside the neutrino-sphere, for seconds after core-collapse. These neutral current processes have cross sections that, in the degenerate and non-degenerate limits, are quadratic in neutrino energy, $\sigma_\nu \sim E_\nu^2$, and are relatively large: they dominate the determination of the neutrino mean free path. Elastic scattering of $E_\nu \sim O(10\ \text{MeV})$ neutrinos off heavy targets is conservative and does not affect neutrino energies or temperatures.

(ii) Neutrino-electron elastic scattering and charged current neutrino absorption and reemission by nucleons. These processes have relatively small cross sections and do not significantly contribute to the determination of the neutrino scattering length, but they tend to re-thermalize neutrinos and degrade their temperature as they diffuse their way out to the neutrino-sphere.

If the process (ii) did not play a significant role and process (i) dominated neutrino diffusion from the core, the average energy of emitted neutrinos $\langle E_\nu \rangle$ would increase with time, since in this case the cross section of neutrinos and thus the length of their random walk journey within the neutrino-sphere increases with energy. The temperatures that roughly characterize the energy spectra stemming from detailed numerical calculations are smaller than the temperature in the inner neutronized collapsed core. This means that the cooling rethermalization processes, (ii), do play some role. If process (ii) is important $\langle E_\nu \rangle$ may be constant or even decrease with time due to the heat loss implied by neutrino emission. Some explicit calculations [14] do indeed yield an average $\bar{\nu}_e$ energy that, after decreasing sharply for a brief $\sim$ 1s transient, continues to decrease (albeit very very slowly) on the few seconds time scale of $\bar{\nu}_e$-flux fall-off. Yet, at least for the first few seconds after core-collapse for which it is possible to implement the time-consuming numerical analysis, other calculations [15, 12] indicate that the average energy of electron anti-neutrinos increases with time.

The effect of a non-vanishing neutrino mass in the prediction of the pattern of neutrino arrival times is to delay low energy neutrinos relative to higher energy ones. An average energy of emitted neutrinos that increases with time works in the opposite direction. Thus, models wherein the average energy of neutrinos decreases with time give tighter upper limits on neutrino masses than models in which $\langle E_\nu \rangle$

increases. We will consider two types of models for $f(E_\nu, t_{SN})$, one in which $\langle E_\nu \rangle$ increases with time as if process (i) totally dominated the neutrino diffusion process, and another in which $\langle E_\nu \rangle$ is constant. The first model is very conservative with respect to upper limits on the neutrino mass since for it $\langle E_\nu \rangle$ increases more rapidly than in more detailed numerical simulations of supernova dynamics. The second model is a closer approximation to current supernova lore, yet it is slightly more conservative than models in which $\langle E_\nu \rangle$ decreases slowly with time.

3. Diffusion models of the electron-antineutrino flux.

Neutrino diffusion approximations closely match the results of much more complex numerical investigations that include extra realistic details of neutrino-transport, such as non-isotropic cross sections. We shall work in the diffusion approximation and exploit the brevity of the dynamical time scale of core collapse, relative to the expected and observed neutrino-transport times within the neutrinosphere, to solve the problem of diffusion in the approximation that the supernova's core temperature and inner density profile are static*. We let the data itself fix the relevant parameters of this profile.

Faced with incomplete and inconclusive detailed theoretical information on the neutrino-emission time-energy correlation function, $f(E_\nu, t_{SN})$ in eq. (2a), we choose to study our constraints on neutrino masses in two extreme variants of two extreme diffusion models. This allows us to explore and compare with the data a range of possibilities that is as large or presumably larger than that of reasonable detailed supernova models.

In our first diffusion model the neutrino source (a newly born neutron or strange star, or the material accreting onto an infant black hole) is a thermalized "hot spot" of radius R_{hs}, and the neutrinos escaping from its surface diffuse through relatively cooler material out to a surface of much larger radius R_ν, before they freely escape. The corresponding spherically symmetric solution of the diffusion equation, in the approximation $R_\nu^2 \gg R_{hs}^2$, is:

$$f_{hs}(E_\nu, t_{SN}) = \frac{2\tau_{hs}^{3/2}}{\sqrt{\pi}\, t_{SN}^{5/2}}\, e^{-(\tau_{hs}/t_{SN})}, \tag{3a}$$

$$\tau_{hs}(E_\nu) = 3R_\nu^2/[4\pi\lambda(E_\nu)c] \equiv (1\,\text{sec})\left(\frac{E_\nu}{10\,\text{MeV}}\right)^n \frac{1}{\gamma_{hs}}. \tag{3b}$$

* The cooling of the "neutrino-fireball" born at core-collapse time may not be a simple process wherein, at a fixed radial position, the composition of the star in terms of different nuclear species is static, and the temperature decreases uniformly with time. In a model of the evolution into a neutron star of an 1.4 $M_\odot$, $R \sim 10$ km collapsed core, for example, Burrows and Lattimer [14] find that the temperature rises for the first few seconds, before cooling starts from the surface inwards, with a characteristic time of ~ 10 sec. The initial entropy and lepton gradients within the forming neutron star are unstable against convection, and might even produce a "mantle overturn" whose effect on neutrino fluxes remains to be fully clarified.

Here $\lambda = (\sigma\rho)^{-1}$ is the neutrino mean free path and γ_{hs} is a diffusion parameter, defined by the second of eqs. (3b), to be fixed by the fits to the data. In writing eqs. (3) we have neglected the fact that the neutrino temperature may be expected to decrease with time as neutrinos are emitted. There are two reasons for this. First, the effect is small: in the diffusion solutions including this effect, the temperature decreases at a much slower pace than the neutrino flux does, even in the unrealistic limit where all "heat" is in neutrinos. Second, as we have emphasized, the correction works in the direction of slightly tightening the limits on m, making them less safe. We have let n vary in eq. (3b) to investigate the quality of fits with different values of n. The hot-spot model corresponds to the extreme where the energy-conserving processes, labelled (i) earlier in the text, completely determine the fate of the diffusing neutrinos. The neutrino cross sections behave as $\sigma \sim E_\nu^2$, so that $n = 2$ in eq. (3b). A time-energy correlation which is a function of t/E^2 is the crucial ingredient of any model of this type, in the approximation of a fixed radius neutrino-sphere*.

To test the model-dependence of our results within the realm of diffusion models, we introduce a "hot-ball" scenario wherein all the material within R_ν is uniformly "hot" at $t_{SN} = 0$, an initial-condition temperature distribution extremely different from that of the "hot-spot" model previously discussed. After a brief transient, the solution for $f(E_\nu, t_{SN})$ in the hot-ball model is dominated by the principal spherical mode of diffusion:

$$f_{hb}(E_\nu, t_{SN}) = \frac{1}{\tau_{hb}} e^{-(t_{SN}/\tau_{hb})}, \tag{4a}$$

$$\tau_{hb}(E_\nu) = 3R_\nu^2/[\pi^2\lambda(E_\nu)c] \equiv (1\,\text{sec})\left(\frac{E_\nu}{10\,\text{MeV}}\right)^n \frac{1}{\gamma_{hb}}. \tag{4b}$$

The natural value of n in the hot-ball model is $n = 0$. This corresponds to the assumption of a well defined neutrino-sphere, to which neutrinos arrive thermalized by whichever rethermalization process is relevant, and from which they fly off with a thermal spectrum of approximately constant temperature. This scenario is akin to the one describing photons diffusing inside the Sun and leaving from its photosphere, the only major difference is that the Sun's central power supply is steady on the photon-diffusion time scale. If neutrinos are assumed to carry all of the thermal energy of the cooling ball, the characteristic exponential time scale of temperature fall-off in the hot-ball model is three times longer than that of neutrino flux fall-off [16]. Once again, it is appropriate to neglect this effect, that slightly improves the upper limits on the neutrino mass. The $n = 0$ hot-ball model, unlike the $n = 2$ hot-spot model, has a trivial time-energy correlation in the neutrino flux: the shape

* See footnote in sect. 2.

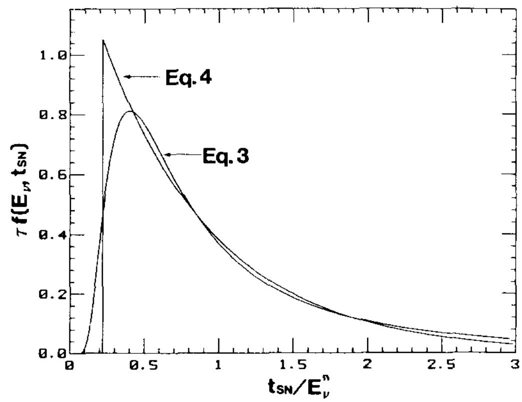

Fig. 1. The functions $\tau f(E_\nu, t_{SN})$ of eqs. (3) and (4), plotted versus $x \sim \gamma t_{SN}/E_\nu^n$, for a choice of parameters that makes them match. The actual curves are $\theta(x)x^{-5/2}e^{-1/x}$ and $1.4\theta(x-0.22)e^{-1.3x}$.

of the energy spectrum is time-independent, its overall magnitude decreases exponentially with time.

The hot-spot and the hot-ball model differ both in the value of n and in the functional dependence of the diffusion solution on the variable t/E_ν^n. However, in our fits the diffusion constant and the time delay between core collapse and the emission of the first neutrino are free parameters to be determined by the data, and given this freedom and the scarcity of the data set, we find that in practice the functional forms of eqs. (3) and (4) are indistinguishable. The reason for this is illustrated in fig. 1 where we plot $\tau f(E_\nu, t_{SN})$ as a function of t/E_ν^n using eqs. (3) and (4). The parameters have been chosen to indicate how closely the two forms can match, particularly if looked at with a "coarse-grained" eye. Because of this similarity, our results on neutrino masses are sensitive to the choice of n, but extremely insensitive to whether we choose, at fixed n, to use eqs. (3) or eqs. (4), a fact that we have repeatedly checked in our actual fits to the data. For this reason, we distinguish our models in what follows by the value of n, with $n = 2$ the hot-spot model, and $n = 0$ the hot-ball one. As we have emphasized, the hot-spot model maximizes the values of the upper limits on the neutrino mass: we shall refer to it as our "pessimized" or "conservative" extreme. All the results actually shown correspond to the $n = 2$ and $n = 0$ versions of the diffusion solution eq. (3). The main reason for this choice is economy: the singular (and unphysical) behavior at $t_{SN} = 0$ of the hot-ball model causes time-consuming difficulties in the computer analysis of the data.

We have taken for granted the general belief [9, 12] that the electron antineutrino pulse is lengthened by a diffusive process. An important question is whether or not the data indicate that this is the case. We shall investigate this question by attempting to fit the data to a model with instantaneous neutrino emission, in which the observed time spread of the data points is entirely due to the in-flight dispersive effect of a non-vanishing neutrino mass. The attempt will fail.

4. Comparison of models and data: fits and their significance

We are finally in a position to compare theory and experiment. The data points from the separate experiments are so scarce that it is not advisable to bin them in comparing them to the theoretical predictions, a likelihood method must be used instead. The likelihood function is:

$$L \propto \prod_i \frac{1}{\sqrt{2\pi}\,\sigma_i} \int \mathrm{d}E \exp\left[-\frac{1}{2}\left\{\frac{E-E_i}{\sigma_i}\right\}^2\right] \frac{\mathrm{d}N}{N\,\mathrm{d}E\,\mathrm{d}t}\bigg|_{t=t_i}, \tag{5}$$

where i runs over the data points. L is a function of the neutrino mass and temperature, of the parameters n and γ defined in eqs. (3),(4) and of t_{SN}^1: the time ellapsed between core collapse and the emergence of the first neutrino to be observed, some 170000 years later, in a particular experiment (which of the observed events this is, is a function of neutrino mass). The measurement errors on E_i have been taken into account by smearing L for each data point, i, over a distribution in E_i, that we assume to be a gaussian with the standard deviations σ_i quoted in table 1.

To ascertain the constraints imposed by the data on a particular parameter, p, one must compute the likelihood $L(p)$ with the rest of the parameters set at their likelihood-maximizing optimal value, for that particular choice of p. The function $L(p)$ describes the relative probabilities of different values of p, and integrals over $L(p)$ can be used in the consuetudinary fashion to set confidence levels of different p-intervals. Once the optimal values of the parameters are determined we check the overall goodness of fit using the Kolmogorov-Smirnov test. The test is constructed from integrals of $\mathrm{d}N/N\mathrm{d}E\,\mathrm{d}t$ over various ranges of t and E. (In our figures these integrals are denoted KS(t) and KS(E_ν), respectively.) The integral nature of the test solves the problem faced by the customary χ^2 fits to meager data sets: undue sensitivity to the way the data are binned. The goodness or badness of fit is determined by the maximum distance between the smooth theoretical curve and a rising, staircase-like function constructed directly from the data.

5. Constraints from the K2 data: results and auto-critique

The most stringent limits on the neutrino mass come from the K2 experiment, as we now discuss in detail. Fig. 2a shows $L(m^2)$, normalized to unit integral, for all K2 events except #6 of table 1. (This event, eliminated by the Kamiokande authors themselves [3] as a likely low-energy background, will be systematically ignored.) We have not a priori constrained neutrinos not to be tachyons: m^2 is allowed to run over positive and negative values.

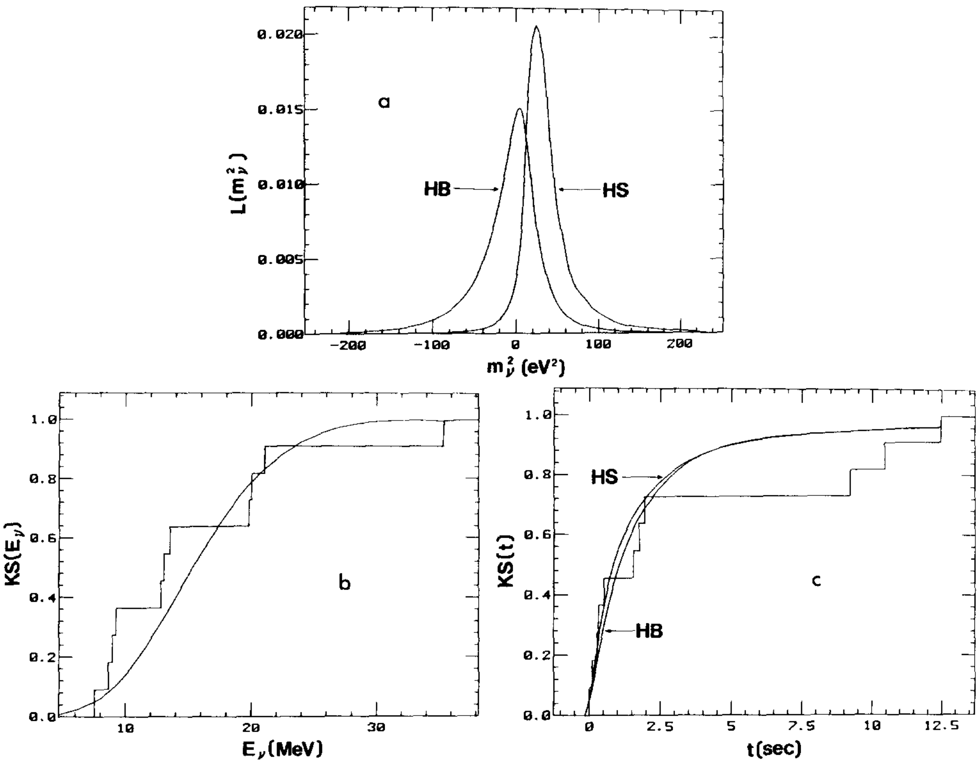

Fig. 2. K2 results with event #6 excluded. (a) Likelihood function $L(m^2)$. (b) Kolmogorov-Smirnov test as a function of neutrino energy. (c) Same test as a function ot time. Here and in other figures HS (HB) refers to the hot-spot (hot-ball) model. The two models coincide in their KS(E_ν) tests, and they are not explicitly labelled in the corresponding figures.

Integrating under the corresponding curve in fig. 2a, we find that for the hot-spot model, the 90%, 95%, and 99% pessimized upper limits on m are 9 eV, 11 eV, and 15 eV respectively. If we impose that m^2 be positive and renormalize the area under $L(m^2)$ to unity above $m^2 = 0$, the upper limits are in this particular case almost identical, indistinguishable within two significant figures from the previously stated ones. The probability that m^2 is zero or less (an indication of the likelihood of a fit to massless neutrinos) is 6%: though $L(m^2)$ peaks at $m^2 > 0$, massless neutrinos are not overwhelmingly excluded. At the maximum of $L(m^2)$ $m = 4.5$ eV, $\gamma_{hs} = 1.4$, $T = 2.40$ MeV, and $t^1_{SN} = 0.7$ s. These results are gathered, along with many others to be discussed, in table 2.

Also shown in fig. 2a is $L(m^2)$ for the hot-ball model. As anticipated, the upper limits on m, reported in table 2, are lower for the hot-ball model ($n = 0$) than for the hot-spot case ($n = 2$). Fig. 2a and table 2 show that the indication of a nonvanishing mass in the $n = 2$ case completely disappears for $n = 0$: *there are no*

TABLE 2
Upper limits on the electron antineutrino mass m (in eV) and most likely values of m and of the remaining parameters.

Exper.	Events excluded	Hot ball model [or $n = 0$ in eq. (3b)] limits on m (eV) all m^2			$m^2 \geq 0$ only			T (MeV)	γ_{hb}	t^1_{SN} (s)	m (eV)
K2	6	6	7	10	6	8	11	2.4	0.57	0.5	2
K2	3,6	14	20	25	20	23	27	2.6	0.44	0.6	2
K2	6,10,11,12	5	6	7	6	7	8	2.9	1.1	0.4	3
K2	3,6,10,11,12	5	6	7	6	7	7	3.3	1.2	0.3	3
K2	1,2,3,6	16	22	27	22	25	28	2.5	0.38	−0.7	8i
IMB	none	45	48	52	45	48	52	3.7	4.1	2.2	36
LSD	none	17	20	23	21	23	24	0.9	2.0	−9.1	11i
K2 + IMB	K2:6	6	8	9	7	8	10	2.8	0.46	*0.14	3
K2 + IMB	K2:3,6	12	17	25	16	20	27	2.6	0.54	*0.04	1
Confidence level		90	95	99	90	95	99		most likely		

Exper.	Events excluded	Hot spot model [or $n = 2$ in eq. (3b)] limits on m (eV) all m^2			$m^2 \geq 0$ only			T (MeV)	γ_{hs}	t^1_{SN} (s)	m (eV)
K2	6	9	11	15	9	11	15	2.4	1.4	0.7	5
K2	3,6	27	30	33	27	30	34	2.6	1.5	0.8	5 + 24
K2	6,10,11,12	6	7	9	6	7	9	2.9	5.1	0.3	4
K2	3,6,10,11,12	8	9	10	8	9	11	3.3	5.0	0.4	5
K2	1,2,3,6	22	26	31	26	28	32	2.5	1.1	0.2	2 + 23
IMB	none	49	53	59	49	53	59	3.7	9.5	4.5	41
LSD	none	11i	9i	20	22	23	24	0.9	3.0	−11.9	12i
K2 + IMB	K2:6	7	8	9	7	8	10	2.8	1.0	* − 1.4	5
K2 + IMB	K2:3,6	26	28	30	26	28	30	2.6	1.9	* − 0.24	5 + 24
Confidence level		90	95	99	90	95	99		most likely		

T: temperature in MeV; γ: dimensionless diffusion parameter of eqs. (3b) and (4b); t^1_{SN}: time of exist from the neutrino-sphere of the first observed neutrino, in seconds after core-collapse). Imaginary values of m reflect a negative m^2. The times marked with a star are time-differences $\Delta t = t_1(\text{IMB}) - t_1(\text{K2})$, in seconds.

serious grounds to claim that these data favour massive neutrinos. Figs. 2b, c display the Kolmogorov-Smirnov goodness of fit tests in the variables E_ν and t, respectively, with every parameter, including m, fixed at its most likely value. Both tests assign a probability to the data set better than 20%, a value above which it no longer makes much sense to compare different fit qualities. We have checked that as one moves significantly away from the best value of any parameter, the Kolmogorov-Smirnov tests properly reflect the bad quality of the corresponding fit. The probability assigned to $m = 30$ eV in the hot-spot model, for instance, is less than 1%.

Even for the hot-spot model, the very "conservative" extreme of our diffusion solutions, the upper limits on the electron anti-neutrino mass that we have just presented compete favorably with the present Tritium β-decay upper limits [17], and contradict the $m \sim 30$ eV claims of the ITEP group [18]. To ascertain the solidity of these rather strong conclusions, we proceed to analyse their dependence on three potentially weakening details of the K2 data set, and on two conceivable criticisms of the underlying theory:

(i) The distribution of the K2 events in time (see table 1) may seem a little peculiar at first sight: there are eight events concentrated in the first two seconds, followed by three events after a long ~ 9 second "silence". This is reflected in the Kolmogorov-Smirnov tests of fig. 2c: the data deviate from the expectations for the central values of t. The overall fits, with their better than 20% probabilities, are not bad enough to imply that the last three K2 events are necessarily a "hiccup" of the supernova: a deviation from the most naive diffusion picture. Yet a fit to only the first eight K2 events is even better, indeed unseemingly good, as we proceed to discuss. It is not theoretically excluded that core bounces, convection and rotation effects, mantle overturn; rarefaction waves, partially failed shock waves, or other complications of the shock propagation dynamics; and what not, might result in a supernova neutrino flux that peaks more than once as a function of time. To investigate this possibility we have somewhat artificially eliminated the last three K2 events as if they were a supernova's second thought, and redone the neutrino mass diffusion-picture analysis with the remaining eight "early" events. The results for $L(m^2)$ are shown in fig. 3b. [Fig. 3a is a repetition of fig. 2a, inserted in fig. 3 to facilitate comparisons.] The astonishingly tight limits on m that the results of fig. 3b imply are reported in table 2, along with the best values of the other parameters. The hot-spot conservative upper limits on a positive m are 6, 7 and 9 eV, to 90, 95 and 99% confidence, respectively. The indication of a nonvanishing neutrino mass in this model is very strong (98% of the area under $L(m^2)$ is above $m^2 = 0$). Once more this result evaporates in the hot-ball extreme, and cannot be taken seriously. The Kolmogorov-Smirnov tests, for both of our models, assign very high probabilities to the experiment, considerably bigger than 20%.

(ii) Event #3 in the K2 set just straddles the threshold energy cut $E_{\rm TH} \sim 7.5$ MeV and could conceivably be measured with considerable uncertainty in the efficiency, or even be part of the background. The occurrence in this experiment of

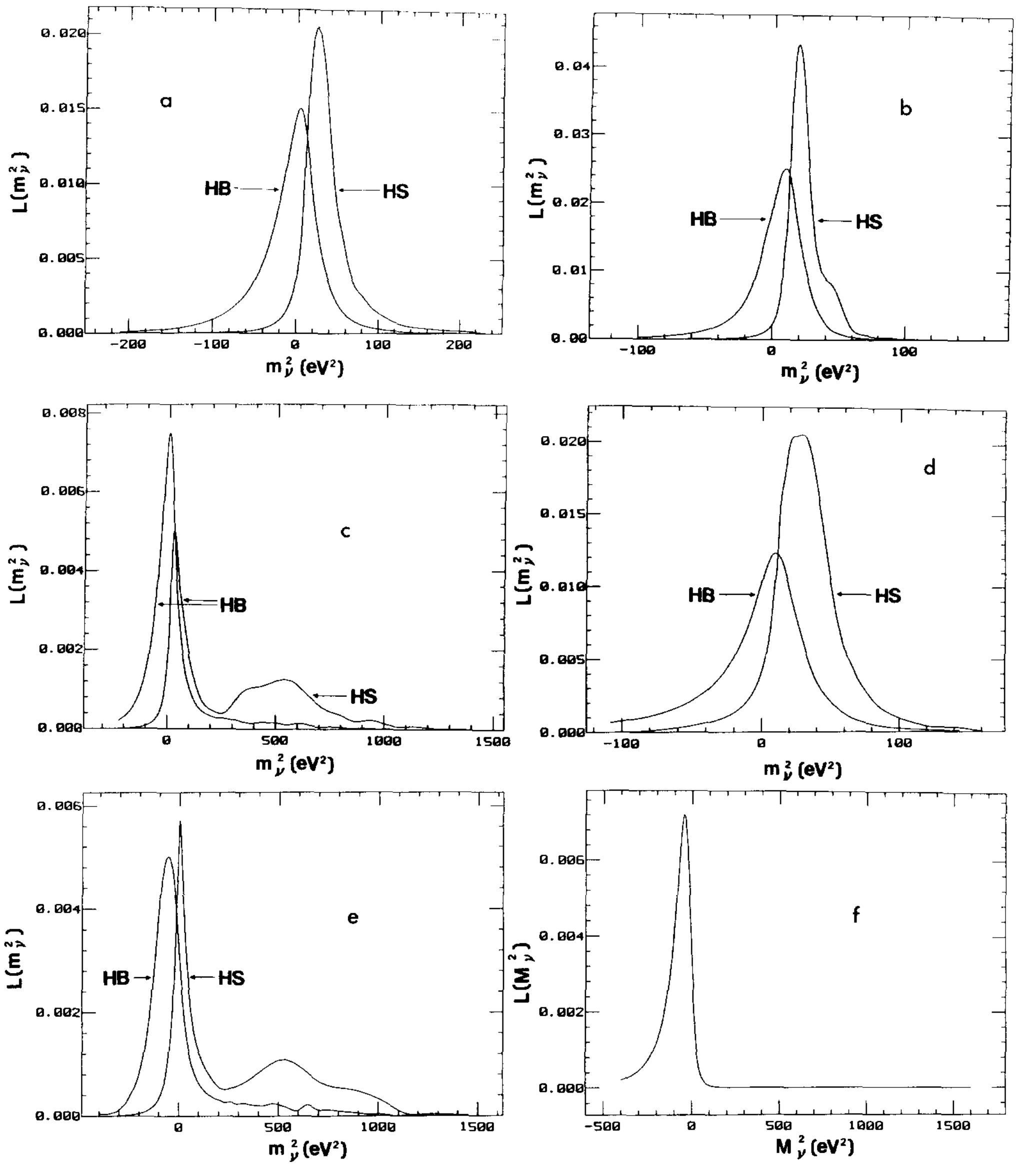

Fig. 3. Various $L(m^2)$ results based on the K2 experiment (notice that the scales are not always the same). (a) All events but #6, a repetition of fig. 2a inserted here to facilitate comparisons. (b) K2 results for $L(m^2)$ with events #6, 10, 11 and 12 excluded. (c) The same with events #6 and 3 excluded; (d) with exclusion of #3, 6, 10, 11 and 12; (e) with events #6, 1, 2 and 3 excluded. (f) Hot-ball model results for $L(M_\nu^2)$ with M_ν the neutrino mass in the "forward" events (#1 and 2), and $m = 0$ for the remaining ones.

events of multiplicity $n < 5$ in intervals of 10 second duration, with energies above ~ 7.5 MeV (more than 20 photomultiplier hits) is well described [3] by a Poisson distribution $P_{\mu}(n)$ with a mean $\mu = 0.219$. The probability for an event such as #3 to be part of the background, in the 12.439-second interval between the first and the last events, is $\sim (12.439/10)P_{\mu}(1) \sim 21\%$. There is "only" a 4:1 chance that this event be supernova-induced, and its record low energy implies that it is the event most sensitive to a nonvanishing neutrino mass. It is therefore important to study the role that this particular event plays in our analysis (Event #4 has similar energy and timing as #3, but the probability that they both are background is only a few percent: we do not discuss the option of disposing of both of them). To study the impact of event #3 in the neutrino mass results, we have eliminated it from the set of 11 K2 events, and from the set of 8 "early" events. The corresponding results are shown in fig. 3c, d, the neutrino mass limits and other parameters are listed in table 2. Nothing very much happens, with one very notable exception, occurring only in the transition from the complete set of 11 events (all but #6) to the set of 10 (all but #3 and #6). For the hot-spot model, the 10-event likelihood function $L(m^2)$ in fig. 3c still peaks at $m \sim 5$ eV as its 11-event counterpart of fig. 3a does, but a secondary peak at $m \sim 24$ eV has developed*, to delight the ITEP team [18]. The 90, 95 and 99% confidence conservative upper limits on m move up to 27, 30 and 33 eV, respectively. These results are no longer competitive with the present generally accepted laboratory upper limits on m. The origin of the secondary peak resides in the interplay between all early events but #3 and the "late" events #10, 11, and 12. A neutrino mass of order 24 eV allows the late events, when extrapolated back to supernova time, to gather snugly with the earlier events and to provide a good overall fit to the diffusion models. Event #3 in the set of 11 events had prevented this from happening because of its record low energy: this event is even more sensitive than the relatively low energy late-comers to $m \neq 0$ and forbids a good overall fit for relatively large mass. In the hot-ball model the correlation between times and energies is weaker and no second peak arises. Instead, removal of event #3 only produces a long high mass tail in $L(m^2)$, and consequently weakens the mass bounds, see fig. 3c and table 2. *Thus the strong conclusions of the previous paragraph concerning the overall K2 data ensemble are contingent on the reality of the most suspicious event in the set*. This is not the case if the last three K2 events are attributed to a late non-diffusive supernova "hiccup" and eliminated, along with events #3 and #6, from the data set. The $L(m^2)$ results for this set of 7 events are shown in fig. 3d. They are smooth and single-peaked in both diffusion models, and they continue to provide good limits on m, that are listed in table 2. It must be admitted that for a set of only 7 events, we are getting close to the point wherein our results become suspicious on grounds of over-parametrization of a small data set.

* One of us (A. De R.) is indebted to L.B. Okun for discussions on this point.

(iii) The first two K2 events point away from the direction of the supernova location. This may be taken as a weak indication that these two events may be due to $\nu e \to \nu e$ scattering, with ν any type of neutrino or antineutrino. We have investigated the case wherein these two events are eliminated, along with #3 and #6 from the K2 set, and kept only as a constraint, on t^1_{SN}. Results for this restricted set of 8 events are shown in fig. 3e and table 2. The elimination of the forward events does not entail significant changes in $L(m^2)$: compare figs. 3e and c. Up to now we have assumed that all events are electron neutrino induced. However, if events #1 and #2 are due to $\nu e \to \nu e$ scattering then this neutrino may be of another type. To study this possibility we have investigated the case in which the electron-neutrino is massless but events #1 and #2 are induced by a different neutrino of mass M_ν. Since the hot-ball model provides a better fit to the data with a massless electron neutrino we use it for this analysis. The resulting $L(M_\nu^2)$, in the conservative approximation that the neutrino energy and the observed electron energy are equal, is reported in fig. 3f. The data show no significant evidence for $M_\nu \neq 0$ and do not support, in this sense, the anzatz that the first two forward events are any different from the others. (Except for these first two "forward" events we have not investigated the exotic possibility of ascribing various other events to neutrinos of different types with unequal masses and ad-hoc fluxes.)

(iv) The attentive reader may have noticed that, in discussing the various caveats of the K2 data, we have not shown Kolmogorov-Smirnov tests of the quality of the corresponding fits. The reason for this is that all the tests assign probabilities better than 20% to the various subsets of experimental data, and they convey very little extra information. One could be left with the unpleasant feeling that, given the poverty of the data set, we can find a good fit to any conceivable scenario. This is not correct. As a relevant illustration of this fact, we have attempted to fit the set of 11 K2 events, as well as the 8 "early" events, to a model in which electron antineutrino emission from the supernova is instantaneous (much shorter than the observed time spread of the data points). The optimal masses in this case are $m \sim 5$eV, for both sets of 11 and 8 events. The corresponding Kolmogorov-Smirnov tests are shown in fig. 4a and b. They assign probabilities to the experiment of 1% and less than 1% for the 8 and 11 data point sets, respectively. The conclusion is that the data cannot be fit with a (theoretically untenable) scenario wherein the pulse of electron antineutrinos is not lengthened by some sort of diffusive process.

(v) As a further test of our diffusion models of supernova neutrinos we study the relative likelihood function $L(n)$, for variable n in eqs. (3). A complete investigation of $L(n)$ would demand, as our previous analysis of $L(m^2)$ did, the optimization of the remaining parameters for each value of the variable under investigation. In introducing yet another parameter, we begin to incur in the danger of overparametrizing a limited data set. But in the case of the parameter n we are only interested in two related important questions. The first is whether in fixing $n = 2$ or $n = 0$ in our previous models to extract limits on m, we have not ignored a possible

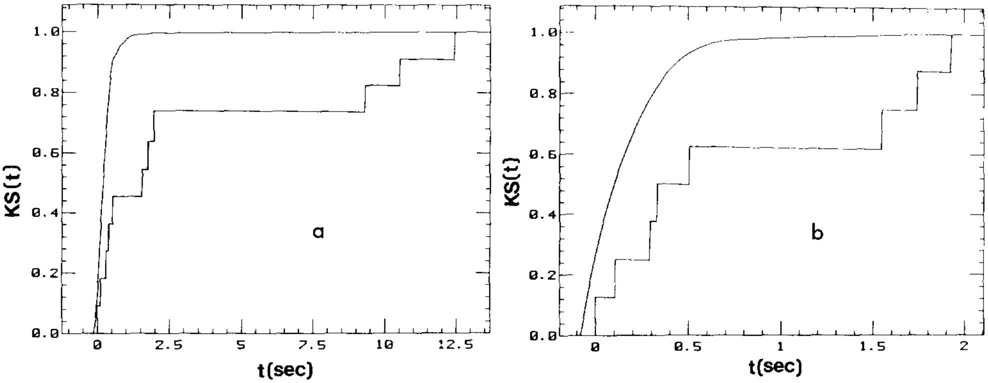

Fig. 4. Kolmogorov-Smirnov time-variable tests for a model with instantaneous neutrino emission. (a) All K2 events but #6. (b) Events #6, 10, 11 and 12 are excluded.

unduly steep dependence of our results on potential variations of n: has n being fixed to values that the fits themselves abhor? The second question is whether our conservative upper limits based on $n = 2$ do not correspond to fits that, if n where allowed to vary, would tend to favour an even larger value of n, and consequently favour larger values of m: have our upper limits been truly pessimized? To answer these two questions we compute $L(n)$ with m fixed to 5 eV (the optimum value around $n = 2$) and with m fixed to zero (an optimal value for $n \sim 0$). The remaining parameters (T, γ and $t^1_{\rm SN}$) are optimized as usual. The $L(n)$ results for the complete set of 11 K2 events are shown in fig. 5a. For $m = 0$ $L(n)$ peaks around $n = 0$: the model and the data are astonishingly consistent. For $m = 5$ eV $L(n)$ peaks around $n = 1$, and is consistent within 1σ with $n = 2$. This is also the desired result: in

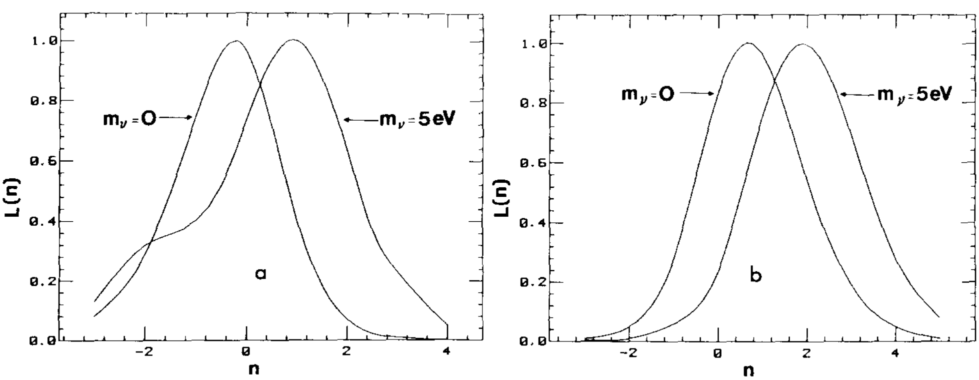

Fig. 5. K2 results for the likelihood function $L(n)$ at two fixed values of the neutrino mass. (a) All events but #6. (b) Events #6, 10, 11 and 12 are excluded.

choosing $n = 2$ as our grounds to establish neutrino mass upper limits we have, if anything, safely over-pessimized those limits. Results for the truncated set of 8 "early" K2 events are shown in figs. 5b. The $m = 0$ curve still peaks near $n = 0$, while the $m = 5$ eV curve now peaks close to $n = 2$. Both models provide excellent fits to the data.

Let us summarize the K2 results. Even in the conservative hot-spot ($n = 2$) diffusion model, our upper limits on the electron anti-neutrino mass compete and even beat the tritium β-decay ones if and only if the most suspicious event kept by the Kamiokande authors, #3, is included in the analysis, and/or if the last three events are eliminated from the diffusion picture as a second "hiccup" of the supernova, on grounds that they (not very significantly) worsen the neutrino diffusion fits. The K2 data for the hot-spot model consistently favour an electron anti-neutrino mass in the 4 to 5 eV range, excluding a null value with a statistical significance equivalent to more than two standard deviations. Though this is insufficient to jump to electrifying conclusions, it establishes a goal for β-decay experiments to match, perhaps in the near future. The hot-ball model, on the other hand, is not purposefully stretched to yield neutrino mass upper limits that are verily safe and significant, and is a good approximation of current supernova lore*. For this rather "standard" model the upper limits on m are always competitive with the current laboratory ones. It can be said in this case that, in 11 seconds of data taking, the Kamiokande collaboration equalled decades of work on the spectral endpoint of β-decay and radiative electron capture. The reason that this seemingly unjust state of affairs is possible is quite obvious [1]: the Large Magellanic cloud is at the right distance, and the $\bar{\nu}_e$ pulse is of the right duration, for the pattern of supernova data to be sensitive to a few-eV neutrino mass; the β-decay lifetimes and Q-values, contrarywise, have not been chosen by Nature with comparable generosity. There is no indication that neutrinos are massive in the hot-ball model. Given our ignorance of the intimate details of supernova physics, we feel unable to argue too strongly in favour of one or the other of our diffusion models: for one or the other set of our flabbergasting conclusions.

6. Constraints from the IMB, K2 + IMB and LSD experiments

The IMB detector has a higher energy threshold** than Kamiokande does, its data set is concentrated at relatively higher energy: IMB is less sensitive to a given neutrino mass than K2 is. The results for the 8 IMB events are displayed in fig. 6. The $L(m^2)$ results of fig. 6a show that this experiment favors nonzero neutrino masses in both of our diffusion models. The most probable masses are $m = 36$ eV

* The result of a small poll of supernova practitioners indicates that most of them (but not all of them!) would, *at this point in time*, favor $n = 0$ (the hot-ball model).

** See footnote in sect. 1.

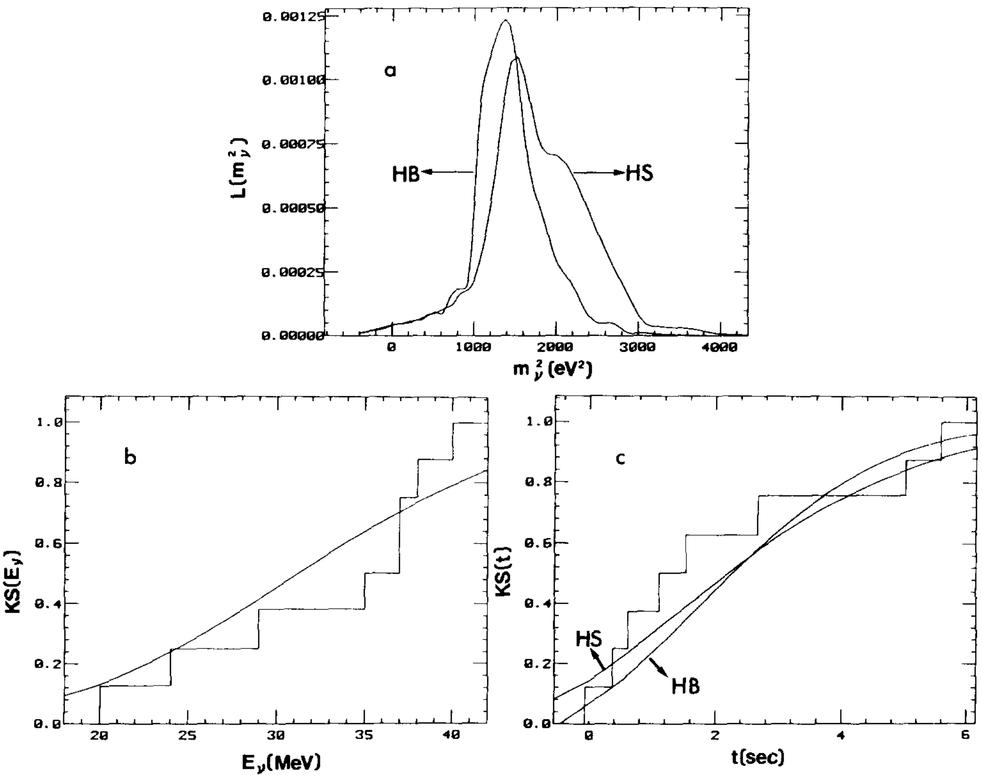

Fig. 6. IMB results. (a) $L(m^2)$. (b) KS(E_ν). (c) KS(t).

and $m = 41$ eV for the hot-ball and hot-spot cases, respectively; once again in the region that ought to delight the ITEP group. The rest of the relevant parameters are reported in table 2. The indication in favor of massy neutrinos is intriguing, but not completely overwhelming. The Kolmogorov-Smirnov tests for this data set are shown in fig. 6b, c. The goodness of the fits are of the order of 20%, totally respectable but not as good as for the K2 data. The *peak* mass values favored by the IMB data are in disagreement with mass bounds arising from K2 and with the generally accepted laboratory bounds. Unlike in the K2 case the IMB events do not suggest obvious candidates for data massage. A source of concern in the IMB data is the fact that all but one of the Cherenkov cones of the IMB events point within the hemisphere opposite to the Large Magellanic Cloud (see table 1), a fact that is not too easy to understand, except as a statistical fluctuation with a probability of $\sim 1/2^7$.

The K2 and IMB experiments clearly observed the same burst of supernova neutrinos and it is of interest to try to combine the data sets. (For discussions of the compatibility of these experiments see refs. [7, 19].) The IMB data were recorded

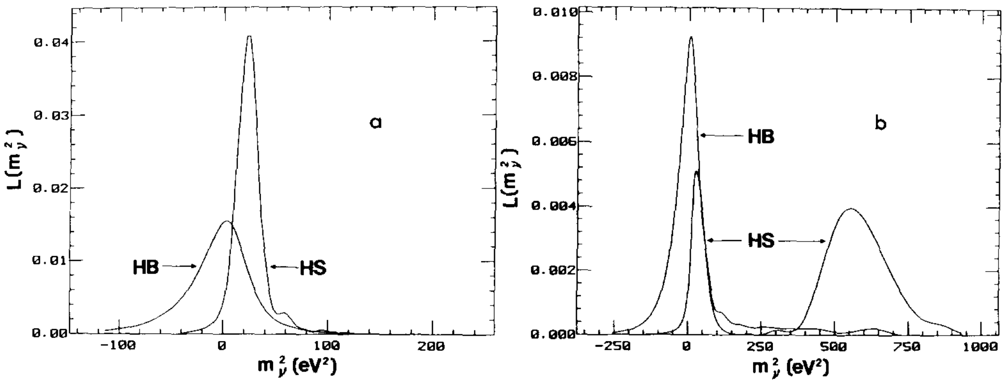

Fig. 7. Results for $L(m^2)$ in a K2 + IMB combined analysis. (a) All events but #6 of K2. (b) The K2 events #3 and 6 are excluded.

with an accuracy of a few milliseconds in their absolute universal time. The K2 data have better than millisecond accuracy in the relative timing of events, but are reported with a one-minute overall error in absolute universal time. This curious fact is said to be due to a failure in the power supply of Japan's electrical company. Data taking was apparently interrupted before the customary comparison, at the end of the tape, between the on-line computer's inner time and an external signal carrying the officially exact universal time. Within the stated one-minute error, IMB and K2 are simultaneous, but it is impossible to superimpose the data directly with the precision in relative timing that is necessary to extract information on the neutrino mass. In an attempt to skirt this unfortunate state of affairs, we have analysed the combined K2 and IMB data by introducing* as a parameter into our fits the true local time difference between the first IMB event and the first K2 event: $\Delta t = t_1(\mathrm{IMB}) - t_1(\mathrm{K2})$. Naturally, the results of the K2 + IMB analysis turn out to be closer to the individual results from K2 than they are to those from the less numerous IMB data. Fig. 7a shows the combined results on $L(m^2)$ for a set that includes the 11 K2 events and all of IMB's events. For each of the diffusion models, this figure resembles the corresponding K2 results of fig. 2a. Upper limits on m and the optimal values of the other parameters are listed in table 2. The combined K2 + IMB results after elimination of event #3 of K2 from the previous set are shown in fig. 7b. Our combined K2 + IMB fit to a common overall diffusion picture also yields an optimal value for the previously unknown time difference Δt. This value is given for various cases in table 2. Given sufficient motivation, it is presumably possible to determine, to better than one-minute accuracy, when the Japanese power failure of February 23rd 1987 actually took place. From that datum, table 2, and a little extra information from the Kamiokande authors on that

* We are indebted to S.L. Glashow for his insistence that we do the combined analysis.

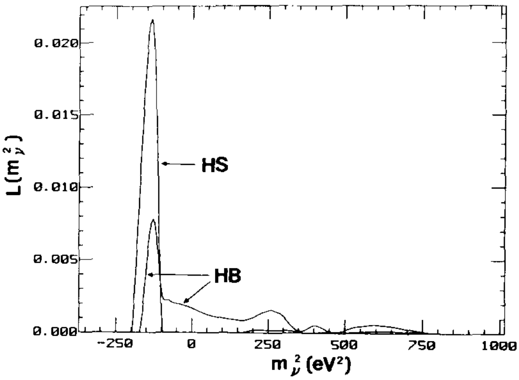

Fig. 8. $L(m^2)$ for the revised data [6] of the LSD experiment. Notice a "tachyonic" inkling of the results, worrisome only in the HS model, and not very significant statistically in either case (see text).

day's data-taking, we could determine which scenario of the table is closest to the truth. Vice versa, given one's favourite scenario, one can predict with precision when the power supply in Japan did fail. These questions of precise timing may be important in assessing the consistency [7] of the LSD data, recorded 4h 43min earlier, with the K2 results at that time.

The LSD data are often ignored, an attitude that is not the only possible one, Shelton's supernova may have banged twice [7]. The LSD events cannot be directly compared with IMB and K2 since they occurred 4h 43min earlier. We have extracted the constraints on the neutrino mass arising in our diffusion models from the LSD data. The results are reported in fig. 8 and table 2. The hot-ball model gives reasonable mass limits but the hot-spot model strongly favours tachyonic neutrinos. We must warn the worried reader that in the case of LSD we are fitting five data points with four-parameter models so that the statistical significance of these results is highly questionable. In this analysis we have used the recently revised [6] energy values and energy errors for the LSD experiment. Use of the originally published figures leads to results significantly different from those shown in fig. 8. This extreme sensitivity is also a reflection of the low ratio of the number of events to the number of parameters.

We do not have at the moment sufficient information on the characteristics of the Baskan detector [5] to attempt to analyze its data; a welcome excuse, since we would not know how to deal with the fact that the events are precisely monitored in universal time, and appear to have a ~ 30-second delay relative to IMB's data.

7. Conclusions

Our strongest electron antineutrino mass bounds stem from the Kamiokande experiment, that has a relatively low low-energy threshold and the richest data set.

The IMB data gathered at higher energies are obviously less sensitive to a given neutrino mass. The analysis of IMB by itself yields a neutrino mass likelihood function $L(m^2)$ resembling a photocopy of the ITEP results, see fig. 6a, but if the IMB data are combined with the K2 results they turn out not to modify our less controversial conclusions from the individual analysis of the K2 events. The scarce LSD data give extremely model-dependent results of very feeble statistical significance.

The belief that a diffusive process lengthens a supernova's $\bar{\nu}_e$ flux preceded the recent discoveries by decades, and is strongly supported by the data. Our diffusion models provide excellent fits to the total ensemble of K2 data points. The fits are even better if the last three K2 events are a supernova hiccup, but the difference is not sufficiently significant to draw strong conclusions. We find no evidence that the early two forward events in the K2 data set are due to massive non-electron neutrinos.

Our very standard hot-ball model very satisfactorily fits the 11 K2 events with a vanishing neutrino mass. The corresponding mass-likelihood function $L(m^2)$, shown in fig. 2a, is sufficiently close to an asymmetric gaussian for us to quote the result in the conclusions in the usual language of standard deviations:

$$m^2 = \left\{4^{+28}_{-63}\right\} \text{eV}^2, \tag{6}$$

This result favourably competes with the $\bar{\nu}_e$ limits from tritium β-decay (stronger laboratory limits depend on mixing angles and/or a hypothetical Majorana nature of neutrinos).

The statements of the last paragraph are very strong. Our knowledge of the neutrino spectrum is inferior for supernovas than it is for β-decay. For these two reasons, we have investigated in fastidious detail several theoretical alternatives and/or experimental caveats that might weaken our conclusions. We have considered a hot-spot model in which the energy-conserving neutrino-diffusion processes are overemphasized, to produce a time-energy correlation in the supernova's neutrino flux that purposefully weakens the upper limits on the neutrino mass. We have also studied the elimination of various K2 events on several disputable grounds, and found only one important instance that definitely deserves mention in the conclusions: the role played by the least convincing, lowest-energy K2 event, #3 in table 1, is crucial. The elimination of this event, with its only 4 to 1 odds of being "real," considerably weakens the upper limits on the electron antineutrino mass.

SN1987A is dim in electromagnetic radiation, in comparison with the modern sample of Type II supernovas from more distant galaxies. Supernovas akin to Shelton's may occur in the Milky Way more often than previously estimated on the basis of observations biased towards brighter objects. The next supernova in our galaxy, even if it occurs in its invisible domain, may send a manna of neutrinos sufficiently intense to constrain the neutrino mass with even greater precision. Ainsi soit-il.

We are indebted to S.L. Glashow for discussions and to S. Gutmann for instruction on the statistical significance of small data samples.

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