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WORMHOLES AND GLOBAL SYMMETRIES*

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We examine the effects that topological fluctuations in the structure of space-time have on the global U(1) symmetry of a scalar field theory. When the symmetry is not spontaneously broken we find wormhole solutions which break the U(1) symmetry despite the fact that they have infinite euclidean action. In the case where the symmetry is spontaneously broken we find wormhole solutions of finite action.

1. Introduction

It is possible (although far from clear) that fluctuations which change the topology of space-time have a significant impact on physics near the Planck scale. Can they also affect physics at experimentally accessible energies? Recent investigations into wormhole configurations [1–5] suggest that the answer to this question may be yes. Wormholes seem to violate spontaneously broken (i.e. nonlinearly realized) global symmetries [1–4], they may be important for understanding the vanishing of the cosmological constant [5] and they may determine the values of other parameters as well [5,6]. In this paper we will make a detailed study of the existence and effects of wormholes in a theory consisting of a complex scalar field with U(1) symmetry minimally coupled to gravity.

Let us write the complex scalar field as

$$\Phi = \frac{f}{\sqrt{2}} e^{i\theta} . \tag{1.1}$$

Then the U(1) invariant potential which characterizes our theory is solely a

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function of f,

$$V = \pm\tfrac{1}{2}m^2 f^2 + \tfrac{1}{4}\lambda f^4 + V_0\,, \tag{1.2}$$

where V_0 is a constant added to insure that V vanishes at its minimum when we have a negative mass term. This model was considered in refs. [2,7,8] with a negative mass term so that the U(1) symmetry is spontaneously broken. (In refs. [2,8] the Goldstone boson field was represented as a three-index antisymmetric tensor. In ref. [8] solutions with a non-zero cosmological constant and in dimensions other than four were also considered.) However, in this early work a simplifying assumption was made which, as we will see, is not always justified. Therefore we will reconsider this case. However, our more interesting and novel results arise when the mass term is positive. Here we will find wormhole configurations, but they have infinite euclidean action. This might suggest that such configurations have no physical relevance, but we will show that despite their infinite action, wormholes in the unbroken theory produce effects which can be reproduced in a low-energy effective field theory by including local U(1) non-invariant terms with non-zero coefficients in the effective lagrangian and we will show how these coefficients can be calculated. Even in the spontaneously broken case we will see that when the mass m is much less than the Planck mass, the full wormhole action is not what controls the strength of the explicit U(1) violating terms in the low-energy effective theory. Our work will make use of a formalism developed by Coleman and Lee [9] who have in parallel with our work looked at wormhole solutions and their implications for the low-energy effective field theory.

As in previous studies of wormholes we assume that topologically non-trivial space-time fluctuations can be described using the euclidean path integral for gravity, at least in the semiclassical approximation. Thus, we will use the euclidean Einstein equations and the scalar field equations to evaluate the gravitational action. We are interested in gravitational field configurations consisting of a wormhole in an asymptotically flat spacetime. The amplitude for a wormhole to appear and produce a baby universe carrying off U(1) charge can be written as

$$Z = \int d^4y \frac{1}{\sqrt{D}\,N_0^2}\, e^{-S}\,, \tag{1.3}$$

where S is the euclidean action of the wormhole solution, D is the determinant arising from integration over the non-zero-mode fluctuations about the wormhole solution, the integral over y reflects the integration over zero modes with normalization factor N_0. In this expression we have ignored an overall phase factor involving the phase of the Φ field since it can be eliminated by a suitable choice of boundary conditions and because it will play no role in our discussion. The U(1) symmetry associated with the Φ field results in the conserved current ($\partial_\mu J^\mu = 0$)

$$J^\mu = f^2 \partial^\mu \theta \tag{1.4}$$

and an associated conserved charge. We wish to consider solutions which transfer n units of charge through a wormhole. We assume that the wormhole solution has spherical symmetry so we can write the metric as $ds^2 = dr^2 + a^2(r) d\Omega^2$ and $f = f(r)$. Then the field equations which determine f and a are [7]

$$\ddot{f} + \frac{3\dot{a}}{a}\dot{f} = V'(f) - \frac{n^2}{4\pi^4 f^3 a^6}, \tag{1.5}$$

$$\dot{a}^2 = 1 - \frac{8\pi}{3m_{\mathrm{P}}^2} a^2 \left[V(f) + \frac{n^2}{8\pi^4 f^2 a^6} - \tfrac{1}{2}\dot{f}^2 \right], \tag{1.6}$$

where a dot denotes a derivative with respect to r.

In order for the effects we are investigating to be relevant for low-energy physics we must have $m \ll m_{\mathrm{P}}$ where m_{P} is the Planck mass. In addition, we will assume that λ is small so that we can do perturbative calculations. However, for much of our work we will assume that λ is large enough so that the effects of the scalar self-coupling dominate over the effects of the scalar mass term near the wormhole. For extremely small scalar self-couplings or for very large values of the wormhole charge this assumption is not justified. We will consider in a separate section the case where the mass term dominates near the wormhole.

In analyzing the field equations we will divide the euclidean space into three regions. First near the wormhole, $0 < r < r_n$ with r_n some measure of the wormhole size. In this region gravity plays an important role and both its dynamics and that of the scalar field are relevant. However, in this region the effects of the scalar mass term are negligible. Next, there is a region $r_n < r < 1/m$ where gravity plays a subdominant role and can be ignored and also where the mass term in the scalar field equation is unimportant. Thus, the dynamics relevant to this region is that of a massless scalar field in flat euclidean space. Finally for $r > 1/m$ we may ignore gravity but now the mass term becomes important. As we will see, it is only the contribution to the action of the wormhole solution coming from the region $0 < r < r_n$ which controls the strength of the explicit U(1) violating interactions in the low-energy effective field theory.

Our strategy is the following. We will construct wormhole solutions by solving the appropriate field equations in the three regions we have discussed. In the case of spontaneous symmetry breaking our results will expand on those of refs. [2, 7]. In the unbroken theory the infinite action which we find will require us to make a detailed comparison between the effects of wormholes in the full theory with quantum gravity and the effects of a local U(1) non-invariant term in the effective lagrangian of a low-energy effective field theory without the topological fluctuations of quantum gravity. By doing this comparison we will determine the coefficient of the local U(1) breaking term in the low-energy effective field theory. Our analysis will focus on the exponential factors in this coefficient and we will not explicitly compute functional determinants.

2. Near the wormhole

Near the wormhole we must consider the combined scalar and gravitational field equations but because the scalar field will be large in this region we can ignore the scalar mass term. The rescalings

$$r = \rho\sqrt{\frac{8\pi}{3\lambda m_P^2}}, \quad a = A\sqrt{\frac{8\pi}{3\lambda m_P^2}} \quad \text{and} \quad f = F\sqrt{\frac{3m_P^2}{8\pi}} \qquad (2.1),(2.2),(2.3)$$

give the field equations (ignoring the mass term)

$$F'' + \frac{3A'}{A}F' = F^3 - \frac{2Q}{F^3A^6}, \tag{2.4}$$

$$(A')^2 = 1 - A^2\left[\tfrac{1}{4}F^4 + \frac{Q}{F^2A^6} - \tfrac{1}{2}(F')^2\right] \tag{2.5}$$

with a prime denoting a ρ derivative and

$$Q = n^2\lambda^2/8\pi^4. \tag{2.6}$$

Note that as a result of the rescaling the equations only depend on n, and λ through the combination Q. We have solved these equations numerically. The most interesting results are those for $F(0)$ and $A(0)$ especially the later since this is related to the size of the wormhole neck. At the neck of the wormhole we must have $F(0) = A'(0) = 0$, and in order for our solution to match on to the solution obtained below in the far field region we must have in addition $F''(0) < 0$. These conditions give

$$F(0) < (2Q)^{1/6}/A(0) \tag{2.7}$$

and

$$\tfrac{1}{4}F^4(0)A^2(0) + \frac{Q}{F^2(0)A^4(0)} = 1. \tag{2.8}$$

From these we can obtain bounds which for convenience we re-express in terms of

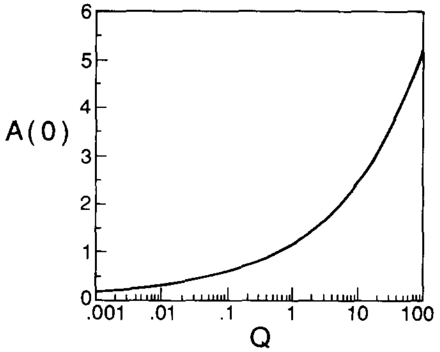

Fig. 1. $A(0)$ as a function of Q assuming that the scalar self-coupling term dominates over the mass term near the wormhole.

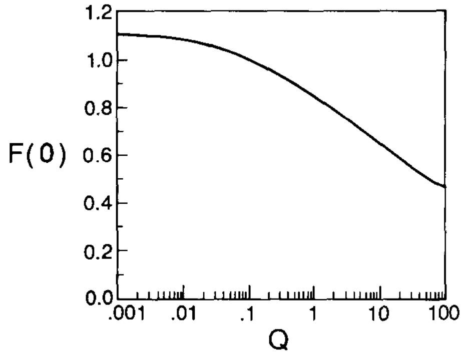

Fig. 2. $F(0)$ as a function of Q assuming that the scalar self-coupling term dominates over the mass term near the wormhole.

the original fields a and f,

$$a(0) > \frac{\lambda^{1/6} n^{2/3}}{2^{1/6} \pi^{5/6} m_{\mathrm{P}}}, \qquad f(0) < \frac{\pi^{1/6} m_{\mathrm{P}}}{2^{1/6} (n\lambda)^{1/3}}. \qquad (2.9),(2.10)$$

Note that for large values of the charge n, $a(0)$ is large and $f(0)$ small in comparison with the Planck scale so a semiclassical analysis of quantum gravity may be justified. Numerical results for $A(0)$ and $F(0)$ as functions of Q are shown in figs. 1 and 2. These results indicate that for large values of Q the above bounds are saturated, while for small Q, $F(0)$ approaches a constant value near one, independent of Q. This means that for large Q we can take

$$a(0) = \frac{\lambda^{1/6} n^{2/3}}{2^{1/6} \pi^{5/6} m_{\mathrm{P}}}, \qquad f(0) = \frac{\pi^{1/6} m_{\mathrm{P}}}{2^{1/6} (n\lambda)^{1/3}} \qquad (2.11),(2.12)$$

while for small Q we have

$$f(0) \approx \sqrt{\frac{3 m_{\mathrm{P}}^2}{8\pi}}, \qquad a(0) \approx \frac{8^{1/4} n^{1/2}}{\sqrt{3\pi}\, m_{\mathrm{P}}}. \qquad (2.13),(2.14)$$

Note that for small Q, $a(0)$ and $f(0)$ are independent of λ.

We can also determine the second derivatives of the fields at $r=0$ by performing a power series expansion. We find

$$A''(0)=\frac{1}{A(0)}\left[\frac{3Q}{F^2(0)A^4(0)}-1\right], \tag{2.15}$$

$$F''(0)=F^3(0)-\frac{2Q}{F^3(0)A^6(0)}. \tag{2.16}$$

We note in passing that the combined scalar and gravitational field equations have a static solution consisting of a "tube" with

$$a=a(0), \qquad f=\frac{n^{1/3}}{(2\pi^2)^{1/3}\lambda^{1/6}a(0)} \qquad (2.17),(2.18)$$

with $a(0)$ chosen to make $\dot{a}$ zero. However, since this is not asymptotically flat it is not relevant for our present analysis.

We will call the contribution to the action coming from the region near the wormhole $S_{\mathrm{w}}^{(n)}$. Its definition is somewhat arbitrary since it depends on the precise value of r_n which we use to separate the wormhole region from the region we consider next. However, we expect to be able to take $r_n \approx a(0)$. The total action includes contributions from both the scalar and gravitational euclidean actions,

$$S=S_\phi+S_{\mathrm{G}} \tag{2.19}$$

with

$$S_\phi=2\pi^2\int_0^\infty a^3\,\mathrm{d}r\left[\tfrac{1}{2}\dot{f}^2+\frac{n^2}{8\pi^4 f^2 a^6}+V(f)\right], \tag{2.20}$$

$$S_{\mathrm{G}}=-\frac{1}{16\pi G}\left[\int \mathrm{d}^4x\sqrt{g}\,R+2\int_{\partial V}\mathrm{d}S\sqrt{g^{(3)}}\,(\kappa-\kappa_0)\right], \tag{2.21}$$

where $g^{(3)}$ is the determinant of the three-metric on the boundary ∂V of the volume of integration and $\kappa-\kappa_0$ is the difference between the extrinsic curvature of the boundary and that of the boundary surface imbedded in flat space*. Although we

* By considering the region $0<r<\infty$ we are actually constructing one half of a wormhole solution, the second half being an identical reflection of the solution we discuss. As a result we should not include the boundary at $r=0$ in our calculation of the action, and the action we give is actually one half of the complete wormhole action.

do not explicitly calculate $S_w^{(n)}$ it is of order

$$S_w^{(n)} \approx m_P^2 a^2(0) \approx \frac{A^2(0)}{\lambda}. \tag{2.22}$$

Using the above results for large and small charge we find

$$S_w^{(n)} \approx \lambda^{1/3} n^{4/3} \tag{2.23}$$

for large Q and

$$S_w^{(n)} \approx n \tag{2.24}$$

for small Q.

3. The massless region

Far from the wormhole, $r > r_n$, but at distances $r < 1/m$ we can ignore gravity and take the euclidean spacetime to be flat $a(r) = r$ and still ignore the effects of the scalar mass term. Thus the only equation we need to consider is that for f,

$$\ddot{f} + \frac{3}{r}\dot{f} = \lambda f^3 - \frac{n^2}{4\pi^4 f^3 r^6}. \tag{3.1}$$

Since f is still large near $r = r_n$ we look for a decreasing solution in order to minimize the action. In fact, an exact solution can be obtained,

$$f = \beta/r, \tag{3.2}$$

with β determined by

$$\frac{n^2}{4\pi^4 \beta^4} - \lambda\beta^2 = 1. \tag{3.3}$$

Of course this solution must be matched on to the solution we obtained near the wormhole but this can be done by adjusting the value of $f(0)$. To see how this is done we note that the above solution is actually unstable against small perturbations. Writing

$$f = \beta/r + C(r) \tag{3.4}$$

and linearizing in C we find that

$$C(r) \propto r^{p_\pm}, \tag{3.5}$$

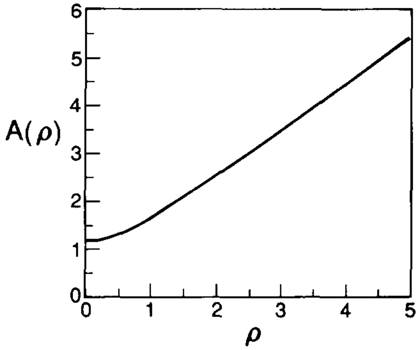

Fig. 3. $A(\rho)$ as a function of ρ for the region near the wormhole and the massless region. In this figure we have taken $Q = 1$.

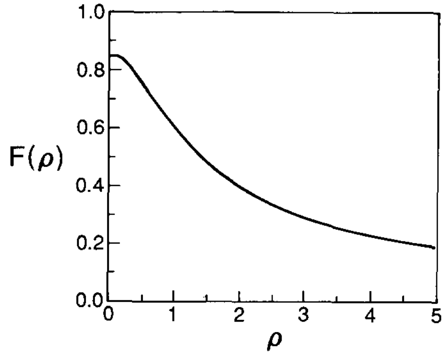

Fig. 4. $F(\rho)$ as a function of ρ for the region near the wormhole and the massless region. In this figure we have taken $Q = 1$.

where

$$p_{\pm} = -1 \pm \sqrt{4 + 6\lambda\beta^2}\,. \tag{3.6}$$

Thus, we find a growing mode $p_+ > 0$ and a dying mode $p_- < 0$. Since there is only one unstable mode we can use our freedom to adjust $f(0)$ to insure that the coefficient of this growing mode is precisely zero. This is how the solutions in the near and massless regions are matched. The resulting solutions are shown in figs. 3 and 4.

For small Q we have

$$\beta^2 = n/2\pi^2 \tag{3.7}$$

while for large Q

$$\beta^6 = n^2/4\pi^4\lambda\,. \tag{3.8}$$

Comparing this with our results for $a(0)$ and $f(0)$ suggests that we can take $r_n \approx a(0)$ in both the large and small Q limits and we have verified numerically that this is the case for other Q values. The action in this region away from the wormhole is dominated by the scalar contribution

$$S_1^{(n)} = 2\pi^2 \int_{r_n}^{1/m} r^3 \mathrm{d}r \left[\tfrac{1}{2}\dot{f}^2 + \frac{n^2}{8\pi^4 f^2 r^6} + V(f) \right]. \tag{3.9}$$

Although this result is not surprising, the sub-dominance of the gravitational contribution to the action in this region is not trivial because it involves a cancellation between the volume integral of $\sqrt{g}R$ and the contribution of the extrinsic curvature term from the boundaries at $r = r_n$ and $r = 1/m$. We can write $a(r) = r(1 + h(r))$ and far from the wormhole h is small. For small h

$$\sqrt{g}R = 6r^3\left(\ddot{h} + \frac{3}{r}\dot{h}\right) = 6\frac{\mathrm{d}}{\mathrm{d}r}(r^3\dot{h}), \tag{3.10}$$

$$\sqrt{g^{(3)}}(\kappa - \kappa_0) = -3r^3\dot{h}. \tag{3.11}$$

From this we see that the volume term and the surface term in (2.21) cancel and gravity can be ignored far from the wormhole. The contribution to the action coming from this intermediate region is then

$$S_1^{(n)} = -\gamma\ln(mr_n) + F_1^{(n)}(r_n), \tag{3.12}$$

where

$$\gamma = 2\pi^2\left(\beta^2 + \tfrac{3}{4}\lambda\beta^4\right) \tag{3.13}$$

and $F_1^{(n)}$ is a factor correcting for the approximations we have used in obtaining the first logarithmic term.

4. The far region

For $r > 1/m$ we can again ignore gravity but not the mass of the scalar field. Thus, in this region we must specify whether we are dealing with a positive or negative mass. For negative mass the value of f will decrease exponentially to the stationary value $m/\sqrt{\lambda}$ and there will be a finite contribution to the action from the far region. However, the positive mass case is very different. Here f must go to zero, obeying the scalar field equation

$$\ddot{f} + \frac{3}{r}\dot{f} = m^2 f + \lambda f^3 - \frac{n^2}{4\pi^4 f^3 r^6}. \tag{4.1}$$

For large r the first and last terms on the right side dominate and

$$f = \sqrt{\frac{n}{2\pi^2 m r^3}}\left(1 - \frac{3}{16m^2r^2} + \cdots\right). \tag{4.2}$$

Since this field goes to zero only as $r^{-3/2}$ the contribution to the total action from this region diverges. If we consider the action coming from a finite region $1/m < r < r_{\max}$ we find

$$S_2^{(n)} = nmr_{\max} + F_2^{(n)}, \tag{4.3}$$

where $F_2^{(n)}$ again corrects for our various approximations*.

5. Interpretation

The total action we have obtained is

$$S^{(n)} = S_w^{(n)} - \gamma \ln(mr_n) + F_1^{(n)}(r_n) + S_2^{(n)}. \tag{5.1}$$

For the spontaneously broken theory this is finite. However, when the theory is unbroken the result for $S_2^{(n)}$ giving

$$S^{(n)} = S_w^{(n)} - \gamma \ln(mr_n) + F_1^{(n)}(r_n) + F_2^{(n)} + nmr_{\max} \tag{5.2}$$

might suggest (since $r_{\max}$ should go to infinity) that wormhole configurations cannot break exact global symmetries. This is not correct as we will now show. Since the wormhole removes n charges it seems reasonable to assume that in a low-energy effective field theory the effects of a wormhole carrying charge n can be reproduced by adding the term

$$g_n \int d^4y\, \Phi^n(y) \tag{5.3}$$

to the effective lagrangian. The effective field theory is only valid at distances greater than some cutoff value $r > 1/\Lambda$ and if we take $1/\Lambda > r_n$ matters simplify because then we can ignore the gravitational effects of the wormhole. In the effective field theory we can express the amplitude Z for a wormhole induced U(1) violating process by inserting the above operator into the usual functional integral. Following Coleman and Lee [9], we compute the amplitude between states of definite charge by inserting a charge projecting δ-function into the functional integral. Using the hamiltonian form of the functional integral and including our

* The zero-mode factor in the wormhole amplitude Z given in sect. 1 includes a factor $\int d^4x\,(\partial_\mu f)^2$ which we note is finite. Only the action factor in the amplitude has an infra-red divergence. It is customary to use the identity $\int d^4x\,(\partial_\mu f)^2 = S$ valid when f solves the classical field equation to write N_0 in terms of the action S. However, the derivation of this identity involves integration by parts and it is only valid if the resulting surface terms vanish. In the unbroken case the slow decrease of the field at large r gives a non-vanishing surface term and we cannot write N_0 solely in terms of the action.

operator insertion and the charge projecting δ-function we have

$$Z = \int [\mathrm{d}f][\mathrm{d}\theta][\mathrm{d}\Pi_\mu]\, \delta\left(\int \mathrm{d}^3x\, \Pi_0 - n\right) \frac{g_n}{\sqrt{2^n}} \int \mathrm{d}^4y f^n(y) \mathrm{e}^{in\theta(y)}$$

$$\times \exp\left(-\int \mathrm{d}^4x \left[\tfrac{1}{2}(\partial_\mu f)^2 + \frac{1}{2f^2}\Pi_\mu^2 + V(f) - i\Pi_\mu \partial_\mu\theta\right]\right). \tag{5.4}$$

Performing the θ integration gives

$$Z = \int [\mathrm{d}f][\mathrm{d}\Pi_\mu]\, \delta\left(\int \mathrm{d}^3x\, \Pi_0 - n\right) \prod_x \delta\left(\partial_\mu \Pi_\mu(x) - n\delta^4(x-y)\right)$$

$$\times \frac{g_n}{\sqrt{2^n}} \int \mathrm{d}^4y f^n(y) \exp\left(-\int \mathrm{d}^4x \left[\tfrac{1}{2}(\partial_\mu f)^2 + \frac{1}{2f^2}\Pi_\mu^2 + V(f)\right]\right). \tag{5.5}$$

Here we have ignored the possible surface term arising from integrating the $\Pi_\mu \partial_\mu \theta$ term by parts. This is just the phase factor which we mentioned in sect. 1 which can be eliminated by appropriate boundary conditions and which plays no role in our analysis. Now we do the Π_μ integration following Coleman and Lee [9] by writing

$$\Pi_\mu = f^2 \partial_\mu \phi + \partial_\nu \epsilon_{\mu\nu}, \tag{5.6}$$

with

$$\partial_\mu\left(f^2 \partial_\mu \phi\right) = n\delta^4(x-y) \tag{5.7}$$

and

$$\int \mathrm{d}^3x f^2 \partial_0 \phi = n \tag{5.8}$$

and integrating over $\epsilon_{\mu\nu} = -\epsilon_{\nu\mu}$. The result is

$$Z = \int [\mathrm{d}f] \int \mathrm{d}^4y \frac{g_n f^n(y)}{\sqrt{2^n D_\Pi}} \exp\left(-\int \mathrm{d}^4x \left[\tfrac{1}{2}(\partial_\mu f)^2 + \tfrac{1}{2}f^2(\partial_\mu\phi)^2 + V(f)\right]\right) \tag{5.9}$$

with ϕ given by the above equations. D_Π is the determinant coming from the Π_μ integration. It is not hard to see that if we do this functional integral by semi-classical approximation we are repeating exactly the problem we did before. Here the operator insertion has produced a δ-function source reproducing the effects of the wormhole. If we assume spherical symmetry and solve for ϕ we find

$$\dot{\phi} = n/2\pi^2 f^2 r^3 \tag{5.10}$$

and substituting this into the above expression for Z we obtain the same action (3.9) and thus the same scalar field equation that we used before. In the massless and far field regions it makes no difference to our computation whether we have a δ-function source or a real wormhole and because of the short-distance cutoff in this low-energy effective theory there is no wormhole region to be considered here. The results of computing this last functional integral over f can then be taken from our previous results remembering the region of integration starts at the cutoff $1/\Lambda$ rather than at the wormhole radius r_n. Including in our effective field theory (with short-distance cutoff $1/\Lambda$ and long-distance cutoff $r_{\max}$) a factor D' for one-loop quantum corrections which also includes the factor D_{Π}, we find

$$Z = \int \mathrm{d}^4 y \, \frac{g_n f^n(0)}{\sqrt{2^n D'}} \, \mathrm{e}^{-S} \tag{5.11}$$

with

$$S = -\gamma \ln(m/\Lambda) + F_1^{(n)}(1/\Lambda) + F_2^{(n)} + nmr_{\max} \,. \tag{5.12}$$

Comparing this with our previous result obtained in the full theory we find

$$g_n = \frac{\sqrt{2^n D'}}{f^n(0)\sqrt{D}\, N_0^2} \exp\left(-\left[S_w^{(n)} + F_1^{(n)}(r_n) - F_1^{(n)}(1/\Lambda) - \gamma \ln(r_n \Lambda)\right]\right) \tag{5.13}$$

which is finite as $r_{\max} \to \infty$, indicating that U(1) violating processes take place even though the wormhole action is infinite. Since our low-energy effective theory has a cutoff when we write $f(0)$ we really mean f averaged over a sphere around zero of radius $1/\Lambda$. As a result, "$f(0)$" is of order $\beta\Lambda$. For small Q the factor $f^n(0)$ cancels the logarithm of Λ in the last term of eq. (5.13). If we take the cutoff to be $1/\Lambda \approx r_n$ then the ratio of determinants will be a function of r_n only (locality of the effective field theory below the wormhole scale implies that any dependence of quantum corrections on long-distance physics associated with length scales greater than r_n should cancel in eq. (5.13)) and dimensional analysis suggests that

$$g_n \approx \left(\frac{r_n}{\beta}\right)^n \frac{\mathrm{e}^{-S_w^{(n)}}}{r_n^8 m_{\mathrm{P}}^4} \,. \tag{5.14}$$

We have shown that the effect of a single wormhole of charge n is equivalent in a low-energy effective field theory to an insertion of the operator

$$g_n \int \mathrm{d}^4 y \, \Phi^n(y) \,. \tag{5.15}$$

To consider the effects of multiple wormhole configurations of various charges we follow Coleman [3] and introduce creation and annihilation operators for baby universes of charge n, $a_n^\dagger$ and a_n. Then, we use for our basis of states, eigenstates

$$\left(a_n^\dagger + a_{-n}\right)|\alpha_n\rangle = \alpha_n|\alpha_n\rangle . \tag{5.16}$$

The effects of wormholes can then be included in the low-energy effective theory by adding a term

$$\sum_{n=1}^{\infty} \alpha_n g_n \int \mathrm{d}^4x\, \Phi^n(x) + \text{h.c.} \tag{5.17}$$

to the lagrangian.

6. Wormhole correlations

We have seen how wormholes break the U(1) symmetry of a scalar field theory by transferring charge between a charged state in our universe and a baby universe. We now consider the effect of wormholes on the vacuum state. Charge conservation requires that in the vacuum state wormholes appear as $\pm n$ charged pairs. Such pairs of wormholes will contribute to the vacuum energy density and thus modify the value of the cosmological constant.

To analyze the effect of wormhole pairs which are separated by distances greater than r_n we can use the effective low-energy theory rather than the full theory. For example we can represent the contribution of a charge n wormhole–antiwormhole pair to the vacuum-to-vacuum transition by

$$\langle 0| \int \mathrm{d}^4x\, \alpha_n g_n \Phi^n(x) \int \mathrm{d}^4y\, \alpha_n^* g_n (\Phi^*(y))^n |0\rangle . \tag{6.1}$$

To compute this we connect the two insertions by n scalar propagators $D_\mathrm{F}(r)$ and obtain

$$|\alpha_n|^2 g_n^2 n! \int \mathrm{d}^4x\, 2\pi^2 \int_0^\infty \mathrm{d}r\, r^3 (D_\mathrm{F}(r))^n . \tag{6.2}$$

If we sum this over multiple but uncorrelated pairs it exponentiates and summing over n we find that pairs of wormholes (to lowest order in λ) shift the bare cosmological constant Λ_0 to

$$\Lambda = \Lambda_0 - \sum_{n=1}^{\infty} |\alpha_n|^2 g_n^2 n!\, 2\pi^2 \int_0^\infty \mathrm{d}r\, r^3 (D_\mathrm{F}(r))^n . \tag{6.3}$$

At large distances $D_\mathrm{F}(r) \sim \exp[-mr]$ so the above integrals are dominated by distances $r_n < r < 1/m$. For $r < 1/m$ we have $D_\mathrm{F}(r) \sim r^{-2}$ so $r^3 D_\mathrm{F}^n \sim r^{3-2n}$. If we use this to approximate the above integral and use our estimate for g_n we conclude that

$$\Lambda \approx \Lambda_0 - |\alpha_1|^2 \frac{2\,\mathrm{e}^{-2S_w^{(1)}}}{m^2\beta^2 m_{\mathrm{P}}^8 r_1^{14}} + |\alpha_2|^2 \frac{2\,\mathrm{e}^{-2S_w^{(2)}} \ln(m r_2)}{\beta^4 m_{\mathrm{P}}^8 r_n^{12}}$$

$$- \sum_{n=3}^{\infty} |\alpha_n|^2 \frac{n!\,(2n-4)\mathrm{e}^{-2S_w^{(n)}}}{\beta^{2n} m_{\mathrm{P}}^8 r_n^{12}} . \tag{6.4}$$

Note that the $n=1$ term dominates for small m. This is because of a Kosterlitz–Thouless [10] phase transition in the vacuum distribution of wormhole pairs. Because wormholes interact by exchanging n scalar particles the quantity $-n \ln D_{\mathrm{F}}(r)$ plays the role of an interaction potential between a wormhole and an antiwormhole. At long distance the exponential behavior of D_{F} produces a linear potential which prevents wormhole pairs from separating. At shorter distances, when D_{F} goes like an inverse power of the distance, this potential becomes logarithmic which introduces the possibility of a Kosterlitz–Thouless [10] transition. For most operators the logarithmic potential confines the wormhole paris to a separation of order r_n. However, for the charge one wormhole pairs the entropy factor r^3 in the above integrals overpowers the weak logarithmic potential and $n=1$ pairs can unbind and separate out to distances of order $1/m$.

Here we have used the effective field theory to compute the contribution of wormhole pairs to the vacuum energy density. We could as well have done the computation using two-wormhole solutions in the full theory (see [11]). In fact, using our single wormhole solutions cutoff at a distance equal to the pair separation as a crude approximation of a two-wormhole solution, we can deduce that at long distances there is a linear interaction potential and that at intermediate distances the interaction potential is logarithmic.

7. Small coupling or large charge

Up to now we have assumed that the coupling constant λ was large enough and the mass m small enough so that we could ignore the mass term in the region near the wormhole. In order for this assumption to be correct we must have

$$\lambda f^2(0) > m^2 . \tag{7.1}$$

If we take $f(0)$ to be of the order of our upper bound, then this requirement is equivalent to

$$\lambda > \frac{2n^2}{\pi}\left(\frac{m}{m_{\mathrm{P}}}\right)^6 \tag{7.2}$$

which seems reasonable except at very large values of n. In this case, or if λ is very small we must include the mass term in our analysis, but can ignore the self-coupling term. First [eqs. (7.6)–(7.14)] we will consider the unbroken case with positive mass term. With the rescaling

$$r=\rho/m\,,\qquad a=A/m\quad\text{and}\quad f=F\sqrt{3m_{\mathrm{P}}^2/8\pi}\qquad(7.3),(7.4),(7.5)$$

the field equations for the positive mass case become

$$F''+\frac{3A'}{A}F'=F-\frac{2Q}{F^3A^6}\qquad(7.6)$$

and

$$(A')^2=1-A^2\left[\tfrac{1}{2}F^2+\frac{Q}{F^2A^6}-\tfrac{1}{2}(F')^2\right],\qquad(7.7)$$

where

$$Q=8n^2m^4/9\pi^2m_{\mathrm{P}}^4\,.\qquad(7.8)$$

Once again all parameter dependence can be absorbed into the effective charge Q. Proceeding as before we can derive the bounds

$$a(0)>4nm/3\pi m_{\mathrm{P}}^2\,,\qquad f(0)<\frac{3^{3/2}\pi^{1/2}m_{\mathrm{P}}^3}{2^{7/2}nm^2}\,.\qquad(7.9),(7.10)$$

Numerical results for $A(0)$ and $F(0)$ as functions of Q are shown in figs. 5 and 6.

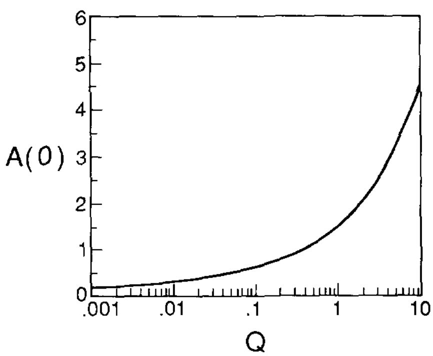

Fig. 5. $A(0)$ as a function of Q assuming that the mass term is dominant in the region near the wormhole.

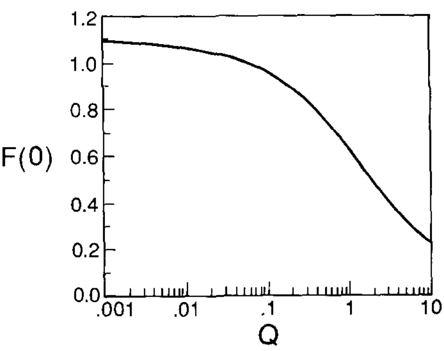

Fig. 6. $F(0)$ as a function of Q assuming that the mass term is dominant in the region near the wormhole.

For large values of Q the above bounds are saturated so

$$a(0) = 4mn/3\pi m_{\mathrm{P}}^2, \qquad f(0) = 3^{3/2}\pi^{1/2}m_{\mathrm{P}}^3/2^{7/2}nm^2, \quad (7.11),(7.12)$$

while for small Q we find

$$f(0) \approx \sqrt{3m_{\mathrm{P}}^2/8\pi}\,, \qquad a(0) \approx 8^{1/4}n^{1/2}/\sqrt{3\pi}\, m_{\mathrm{P}}. \qquad (7.13),(7.14)$$

Note that the results for small Q are not only independent of mass, but identical to those we obtained previously with an f^4 interaction and no mass. For small values of the charge the scalar potential plays no role in determining the size of the wormhole and the value of the scalar field near the wormhole.

Now we turn our attention to the spontaneously broken theory with negative mass term. In previous analyses of this case, f was held fixed at its vacuum expectation value $f = \sigma = m/\sqrt{\lambda}$ for all values of r. This is indeed the case for very large values of the charge n. However, for smaller values of the charge the solutions depicted in figs. 1–4 apply. To see where the dividing line between these two behaviors is we note that the field f will only remain at its vacuum expectation value if the term $n^2/8\pi^4 f^3 a^6$ can be ignored in the scalar field equation relative to the mass and λf^3 terms. For $f = \sigma$

$$a^2(0) = \sqrt{\frac{1}{3\pi^3}}\left(\frac{n}{m_{\mathrm{P}}\sigma}\right). \qquad (7.15)$$

In order that we may ignore the term $n^2/4\pi^4 f^3 a^6$ in the scalar field equations near the wormhole we must then have

$$m^2 \gg m_{\mathrm{P}}^3/n\sigma\,. \qquad (7.16)$$

Using the relation $\sigma = m/\sqrt{\lambda}$ we see that this condition is essentially the inverse of eq. (7.2). It is clear that the assumption of a non-varying f field is only valid for extremely large values of the charge. Otherwise our solutions with a dynamic f field must be used. In particular it has been noted that for the solution with non-dynamic f the wormhole size diverges as $\sigma \to 0$. However, for any fixed n this limit always violates the above bound, so the divergent behavior is an artifact of the non-dynamic approximation. Instead when the above inequality is violated we have $A(0)$ given by fig. 1.

8. Conclusions

We have examined wormhole solutions in a scalar field theory with a global U(1) symmetry. We find that wormhole solutions exist both in the case where the U(1) symmetry is spontaneously broken and when it is unbroken. The complex scalar field we considered has self-interactions of the form $\lambda|\Phi|^4$ in the lagrangian density. For most of the paper we considered the scalar mass to be much smaller than the Planck mass, m_P. In the limit of large charge (more precisely large $n\lambda$) we find that at the location of the wormhole, $f=\sqrt{2}\,|\Phi|$ is much smaller than m_P and that the wormhole "size" is much greater the $1/m_P$. It follows that for large $n\lambda$ terms in the lagrangian density which we have not considered (e.g. $|\Phi|^2R$, R^2, $|\Phi|^6$, etc.) are of negligible importance and that quantum corrections can be neglected.

We have seen that integrating out wormholes introduces into the effective field theory below the wormhole scale interactions

$$L_I=\sum_{n=1}^{\infty}\alpha_n g_n \Phi^n + \text{h.c.} \tag{8.1}$$

which explicitly violate the global U(1) symmetry. Here the α_n are complex parameters which characterize the vacuum state of the theory. The coupling constants g_n, for large $n\lambda$ are of order

$$g_n \approx \left(\frac{r_n}{\beta}\right)^n \frac{e^{-S_w^{(n)}}}{r_n^8 m_P^4} \tag{8.2}$$

where r_n is approximately the wormhole size

$$r_n \approx \lambda^{1/6} n^{2/3}/m_P\,, \qquad \beta \approx n^{1/3}/\lambda^{1/6}\,, \qquad S_w^{(n)} \approx m_P^2 r_n^2\,. \quad (8.3),(8.4),(8.5)$$

In the case where the U(1) symmetry of the underlying field theory is not spontaneously broken, the action of the wormhole solutions is infinite. Physically this "infrared divergence" arises because charge must flow in from $r=\infty$ if a wormhole located at $r=0$ carries charge. The infinite action arises because in the unbroken theory all the charge carrying particles are massive. However, in the effective field theory (below the wormhole scale) the operator Φ^n acts as a point source of charge n and because the infrared divergence is reproduced exactly in the appropriate matrix element of Φ^n it does not enter into the coupling constant g_n. To compute g_n we regulate the infrared divergence by introducing a long-distance cutoff r_{max} and compare the transition amplitude between states of definite charge in the effective theory with the one-wormhole amplitude in the full theory. The value of g_n is then determined by short-distance physics on the scale of the wormhole and is independent of the infrared regulator. In fact the use of an infrared regulator could be avoided by introducing another insertion $(\Phi^*(y))^n$ since in this case the charge does not have to flow in from infinity but can be created by this insertion which acts as a source at space-time location y. In this case the effective field theory calculation would be compared to a wormhole–antiwormhole amplitude in the full theory.

Wormhole solutions have been found previously [2, 7, 8] in the case of a spontaneously broken U(1) symmetry. Our analysis of this case extends previous work by allowing for spatial variation in the field $f = \sqrt{2}\,|\Phi|$ and in particular we find the the behavior for small values of the U(1) breaking vacuum expectation value is different than in previous treatments.

Our results show that the existence of a global U(1) symmetry in the low-energy effective field theory requires that the various vacuum parameters α_n vanish so that no symmetry violating operators arise. This makes most global symmetries seem quite unnatural. However, there are exceptions. For example in the minimal standard model baryon and lepton number violating terms of dimension four or less are forbidden by gauged symmetries [12]. Since these gauged symmetries are protected from wormhole violations we would only expect baryon and lepton number violating interactions of higher dimension to be induced by wormholes. As long as the wormholes are not much bigger than the Planck size these would be suppressed by powers of m_P and hence would be acceptably small. Presumably for larger wormholes there is an even stronger exponential suppression (however, see ref. [13]).

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