Nuclear Physics B331 (1990) 263–276
North-Holland

WORMHOLES WITH A PAST*

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Received 28 June 1989

We discuss Minkowski space interpretations of euclidean solutions which provide semiclassical approximations to quantum processes. In particular, we consider wormhole and euclidean bounce solutions. We argue that the semiclassical description involves an instantaneous discontinuity in the Minkowski signature classical evolution and that the euclidean solutions provide final and initial value data on either side of this discontinuity. In the case of the de Sitter wormhole and bounce solutions, we resolve the problem of having a single maximal surface on the euclidean solution which must provide both past and future data across the Minkowski signature discontinuity.

1. Introduction

Although as physicists we frequently compute in euclidean space, we live in Minkowski space. Solutions of classical euclidean field equations are often assumed to represent quantum processes, but when dealing with gravity it is essential to find Minkowski signature manifolds which can be correctly linked to the euclidean solution before the physical meaning of the tunneling process can be revealed. Our purpose here is to investigate the quantum process whose amplitude is being approximated by the recently discovered euclidean wormhole solutions of the classical equations of gravity coupled to a U(1) invariant scalar field theory [1–4]. Our investigation will lead us to reconsider false vacuum decay [5] especially when it occurs in a background de Sitter space [6]. We will interpret wormhole and bounce solutions by making the appropriate connections between these euclidean solutions and the Minkowski signature manifolds in which the physical process

* Research supported by Department of Energy Contract DE-AC02-76ER03230 and DE-AC02-76ER03069.

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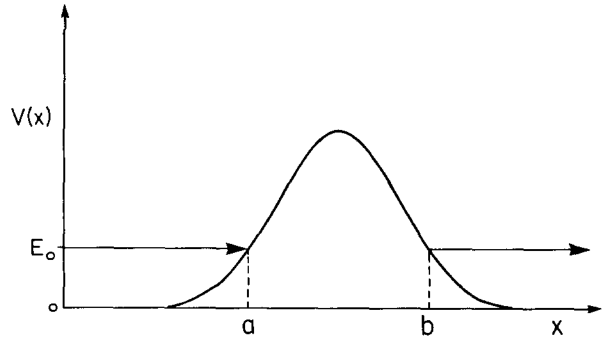

Fig. 1. A particle of energy E_0 enters from the left, tunnels through this potential barrier and exits to the right.

actually occurs. Although our results do not modify the semiclassical computations, we find that they clarify the issue of what physical process is actually being computed. In particular, we resolve a problem in the interpretation of the past evolution of de Sitter wormholes and bounce solutions.

To guide our discussions of the WKB method for field theory and for gravity it will be helpful to begin by considering a simple example from one-dimensional, non-relativistic quantum mechanics. Consider a particle of mass m moving in a potential $V(x)$ of the form shown in fig. 1. We wish to provide a semiclassical description of a particle with energy much less than the potential energy at the top of the barrier coming in from the left, hitting the barrier, tunneling through the barrier and finally moving off to the right. The wave packet describing the incoming particle in the WKB approximation is the superposition

$$\psi_{\rm in}(x,t) = \int \mathrm{d}E f(E) \exp\left(\frac{i}{\hbar}\int_{a(E)}^{x} \mathrm{d}q \sqrt{2m(E-V(q))} + \frac{i\pi}{4} - \frac{i}{\hbar}Et\right), \quad (1.1)$$

where $a(E)$ is the left-hand classical turning point, that is the left-hand-most point where $V(a)=E$. We will assume that the weight function $f(E)$ is sharply peaked at some value E_0 much less than the maximum value of V. In the semiclassical limit ($\hbar$ small) we can define the location of this wave packet by noting that the E integral is dominated by $E=E_0$ and is peaked at values of x and t which make the phase of the exponential stationary. The stationary phase condition gives

$$\int_{a(E_0)}^{x} \frac{\sqrt{m}\,\mathrm{d}q}{\sqrt{2(E_0-V(q))}} = t, \quad (1.2)$$

relating the position of the center of the wave packet x to the time. Note that as we have defined things, the packet hits the barrier ($x=a(E_0)$) at time $t=0$.

There is of course a reflected wave from the barrier but this will not concern us. Using the WKB approximation, the outgoing wave on the right side of the barrier corresponding to this incoming packet is

$$\psi_{\text{out}}(x,t) = \int \mathrm{d}E f(E) T(E) \exp\left(\frac{i}{\hbar} \int_{b(E)}^{x} \mathrm{d}q \sqrt{2m(E - V(q))} + \frac{i\pi}{4} - \frac{i}{\hbar} Et \right), \tag{1.3}$$

where $T(E)$ is the barrier transmission coefficient and b is the right-hand classical turning point. The location of the center of this packet is given similarly by

$$\int_{b(E_0)}^{x} \frac{\sqrt{m}\,\mathrm{d}q}{\sqrt{2(E_0 - V(q))}} = t. \tag{1.4}$$

An essential feature of our discussion is that $T(E)$ is real. Note that the packet emerges from the barrier ($x = b(E_0)$) at time $t = 0$, the same time as it entered the classically forbidden region.

We have assumed that the initial and final states are described by narrow wave packets with the same peak energy E_0. There exist a set of conditions on $f(E)$ and the potential V which assure that this is the case, but they are stronger than those needed to assure the validity of the WKB approximation.

Eqs. (1.2) and (1.4) are the usual classical expressions relating position to time through the velocity. Therefore, the above analysis gives the following semiclassical picture of barrier penetration. A particle enters from the left and moves according to the equations of classical mechanics until at time $t = 0$ it hits the barrier at the left-hand classical turning point. At the same instant it emerges from the right-hand turning point and continues along its classical motion. The quantum process therefore appears in the semiclassical description as an instantaneous jump in the position of the particle.

Following this example we will assume that in field theory a quantum tunneling event, described semiclassically, appears as an instantaneous discontinuity in the classical evolution of the relevant set of fields. The discontinuity will be along a spacelike surface in the Minkowski space-time. The euclidean solution acts as an interpolation across this discontinuity. For a euclidean solution to represent a WKB approximation of a tunneling between two classically allowed regions, the euclidean manifold must be cut along maximal surfaces. A maximal surface is a generalized turning point, a surface on which the normal derivatives of all fields including the metric vanish. The three-geometries on either side of the discontinuity in the Minkowski manifold must be maximal with respect to the Minkowski timelike normal and must match the three-geometries of the euclidean cuts. The maximal condition on all corresponding three-geometries assures that we do not have to match non-zero Minkowski and euclidean signature time derivatives.

Field values along the euclidean cuts provide final value data on the past side and initial value data on the future side of the discontinuity in the Minkowski signature space. Initial and final value data consist of both the values of fields and their derivatives normal to the surface. Since we have chosen maximal surfaces to provide us with initial and final data, the normal derivatives of the fields will be zero and the data on either side of the discontinuity in Minkowski space are provided by the field values of the euclidean solution along the appropriate maximal cut with zero normal derivatives. In this semiclassical picture, the failure of classical physics to account for the discontinuous evolution is completely compensated by the initial and final value data across the discontinuity which are provided by the values of the euclidean solution along maximal surfaces.

As a concrete example of the above construction, consider the standard problem [5] of the decay of a false vacuum state, $\phi = \phi_{\mathrm{f}}$, to a true vacuum state, $\phi = \phi_{\mathrm{t}}$, through the formation of a single bubble appearing at the point $x = 0$ at time $t = 0$. For this example we consider a flat Minkowski space-time and gravity is ignored. The euclidean bounce solution ϕ_{B} depends only on the four-dimensional euclidean length ξ where $\xi^2 = |X|^2 + T^2$ and X and T are four cartesian, euclidean coordinates. As shown in fig. 2, the solution ϕ_{B} approaches ϕ_{f} as $\xi \to \infty$ and $\phi_{\mathrm{B}}(0) \approx \phi_{\mathrm{t}}$. According to our discussion, evolution of the field ϕ in Minkowski space is described classically, except for a discontinuity at $t = 0$. The value of ϕ before and after this discontinuity (i.e. at times $t = -\epsilon$ and $t = +\epsilon$) is given by its value along maximal cuts of the euclidean bounce solution. The value of the field at the Minkowski time $t = -\epsilon$ is given by that on a maximal slice of the euclidean bounce at euclidean time $T = -\infty$ where $\phi_{\mathrm{B}} = \phi_f$ and $\mathrm{d}\phi_{\mathrm{B}}/\mathrm{d}T = 0$. Thus, up until the time $t = 0$ the Minkowski space has $\phi = \phi_{\mathrm{f}}$. The initial-value data at time $t = +\epsilon$ which lead to a classical description of the positive-time behavior in Minkowski space are given by the value of ϕ_{B} along another maximal surface, that at euclidean time $T = 0$ (see fig. 2). Here ϕ_{B} takes the value ϕ_{f} for large $|X|^2$ but varies away from this value at a distance associated with the size of the bubble and ultimately takes a value near ϕ_{t} for $X = 0$. These initial value data characterize the bubble at its moment of birth and subsequent classical Minkowski evolution describes its expansion. Note that this picture has both a past, that is the description of the field before tunneling, and a future, a description of the field after tunneling.

We would like to interpret the euclidean wormhole solutions in a similar way. The flat-space wormhole is shown in fig. 3. The euclidean wormhole should provide initial and final value data for a discontinuity in the "large" space-time across which charge is not conserved, and in addition provide initial value data for a baby universe. (To avoid confusion we will consider throughout the birth of a baby universe. Of course the wormhole can just as well be used to describe the death of a baby universe in which case the appropriate euclidean cut would provide final

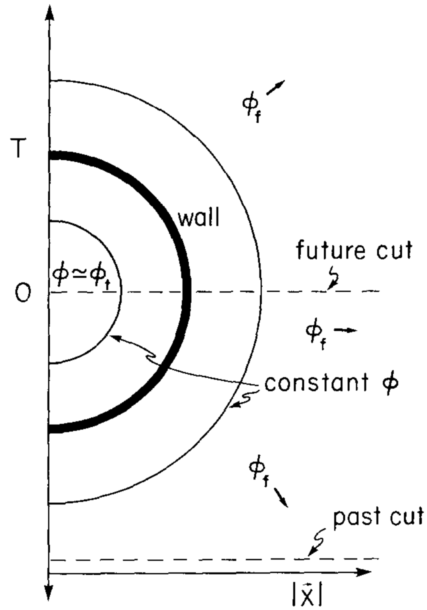

Fig. 2. The flat-space euclidean bounce solution. Lines of constant ϕ are shown as solid lines, the thick line is the bubble wall, and the dashed lines show maximal cuts used to construct the Minkowski interpretation.

rather than initial value data for the baby universe.) The usual wormhole solution is accepted as representing quantum tunneling between spaces with different topology because it can be cut along three maximal surfaces which can provide initial and final value data for the two sides of the discontinuity in the large space and initial value data for the baby universe. Two of these cuts are flat euclidean three-spaces and presumably describe the final initial surfaces of an instantaneous discontinuity extending across flat Minkowski space. The third euclidean cut is a three-sphere

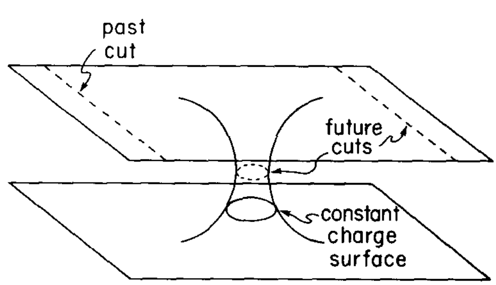

Fig. 3. The flat-space wormhole solution. Dashed lines show the maximal cuts needed to construct a Minkowski interpretation.

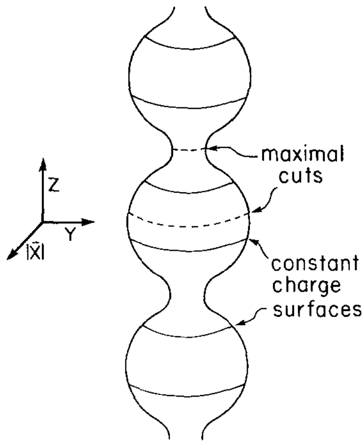

Fig. 4. The de Sitter wormhole solution. The chain of beads continues forever. Maximal cuts are shown as dashed lines and surfaces of constant charge density as solid lines.

which can provide initial-value data for a closed, Minkowski signature, Robertson–Walker universe.

Since the effect of wormholes on the value of the cosmological constant [7] is of great interest, it it essential to consider wormhole solutions with a non-zero cosmological constant as well, especially a positive one. The wormhole solution with a positive cosmological constant is shown in fig. 4. The addition of even a tiny cosmological constant causes the flat regions in fig. 3 to curl up. Because of this a novel and disturbing problem arises. The distinction between the past and the future is obscured and the de Sitter wormhole has only two, not three, maximal surfaces. How then can we interpret the de Sitter wormhole as corresponding to a change of topology?

The de Sitter wormhole has been considered before and an alternate interpretation of its role has been given [8]. In this alternate interpretation, the de Sitter wormhole corresponds to tunneling between a de Sitter universe and a closed Robertson–Walker universe and has nothing to do with topology change. However, it is clear that analyses of the effects of wormholes [7] assume that wormholes lead to changes of topology whether or not the cosmological constant is zero and, in fact, in discussions of the effect of wormholes on the cosmological constant it is always assumed that a zero cosmological constant is approached as the limit of a small positive value. Therefore our goal here is to reconstruct an interpretation for the de Sitter wormhole which clearly shows that it corresponds to a quantum-mechanically induced change of topology.

A completely analogous problem exists for vacuum decay in de Sitter space, and we will consider this problem first before tackling the wormhole. The topology of the euclidean bounce solution (see fig. 5) with a positive cosmological constant is S_4

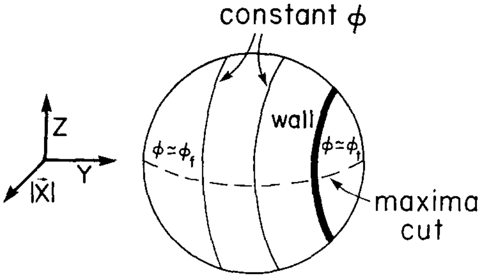

Fig. 5. The de Sitter bounce solution. Lines of constant ϕ are shown as solid lines, the thick line is the bubble wall, and the dashed lines show the maximal cuts.

rather than R_4. When we cut the euclidean manifold maximally we produce only a single surface. How can this single surface produce both final- and initial-value data for the discontinuity in the Minkowski signature description? A resolution of this difficulty and subsequent interpretation of the de Sitter bounce and the de Sitter wormhole are the main results of this paper.

2. De Sitter tunneling

We begin by considering the decay of a false vacuum state in a fixed background de Sitter space. In other words, we include the gravitational effects of a non-zero cosmological constant, but ignore the effect that the small difference in energy density between the false and the true vacuum states has on the space-time geometry. We will find it convenient to use the coordinates of a five-dimensional imbedding space to describe both euclidean and Minkowski signature manifolds. Throughout we use the convention that capital letters denote euclidean signature coordinates and lower case letters Minkowski signature coordinates.

For fixed positive cosmological constant Λ (defined as $8\pi G/3$ times the vacuum energy density) the euclidean bounce solution is defined on a sphere of radius L where $L^2 = 1/\Lambda$. We will use the coordinates of a five-dimensional flat euclidean space (X, Y, Z) in which this sphere is defined by $X^2 + Y^2 + Z^2 = L^2$. The euclidean solution ϕ_B, shown in fig. 5, depends only on the coordinate Y. The solution ϕ_B varies from a value near but not equal to ϕ_f at $Y = -L$ to a value near but not equal to ϕ_t at $Y = +L$. The value of Y where ϕ_B changes most rapidly ultimately determines the radius of the nucleated bubble. To define a suitable maximal surface, this spherical bounce solution is cut along the plane $Z = 0$. This leaves a three-sphere of radius L over which $d\phi_B/dZ = 0$. Field values on this surface can be used to define discontinuity data for the Minkowski signature space. However, as mentioned in sect. 1 this cut provides just a single surface from which we must define both initial and final data on either side of the Minkowski discontinuity.

The space-time in which the actual physics is taking place is, of course, Minkowski signature de Sitter space. We can define this as the surface $-z^2+y^2+|x|^2=L^2$ imbedded in a five-dimensional flat Minkowski space with metric $ds^2=-dz^2+dy^2+|dx|^2$. We will choose to describe a bubble which forms at "time" $z=0$ centered at the point $y=+L$. Bubbles forming at other times and other points can be incorporated by appropriate de Sitter transformations. In the usual static coordinates,

$$r=|x|,\qquad y=\sqrt{L^2-r^2}\cosh(t/L),\qquad z=\sqrt{L^2-r^2}\sinh(t/L) \tag{2.1}$$

for which

$$ds^2=-\left[1-(r/L)^2\right]dt^2+\left[1-(r/L)^2\right]^{-1}dr^2+r^2d\Omega_2^2, \tag{2.2}$$

the bubble appears at time $t=0$ centered at the point $r=0$.

For our discussions it is important to understand the casual structure of de Sitter space which can best be illustrated using Gibbons–Hawking coordinates u and v in place of r and t where

$$ds^2=(L+r)^2(-dv^2+du^2)+r^2d\Omega_2^2, \tag{2.3}$$

$$\frac{L-r}{L+r}=u^2-v^2, \tag{2.4}$$

and surfaces of constant t are given by u equals a constant times v. The de Sitter manifold is shown using these coordinates in fig. 6 (for the moment ignore the discontinuity shown in fig. 6). Light rays travel along straight lines with slope plus or minus one in this figure so we can see that the regions labeled I and III are completely causally disconnected. This double casual structure leads us to the following interpretation of the de Sitter bounce solution.

As before, the tunneling event appears in the Minkowski signature de Sitter space as an instantaneous discontinuity. However, the cut corresponding to this discontinuity does not extend across the full de Sitter space but rather only across one causally connected half. In terms of the u and v coordinates the discontinuity occurs along $v=0$ but only extends from $u=0$ out to positive u values. Expressed using the embedding coordinates (fig. 7), the discontinuity is at $z=0$ and extends from $y=0$ to $y=+L$. We can now use the value of ϕ_B along the maximal euclidean surface $Z=0$ (shown in fig. 5) as both initial and final data for this half cut in the Minkowski signature space. The value of ϕ for $z=-\epsilon$ along the surface $0\leqslant y\leqslant L$ is given by that on the euclidean manifold at $Z=0$ in the range $-L\leqslant Y\leqslant 0$. In this range ϕ_B is near its false vacuum value ϕ_f and we match the euclidean point $Y=-L$ with the Minkowski point $y=+L$ and similarly $Y=0$

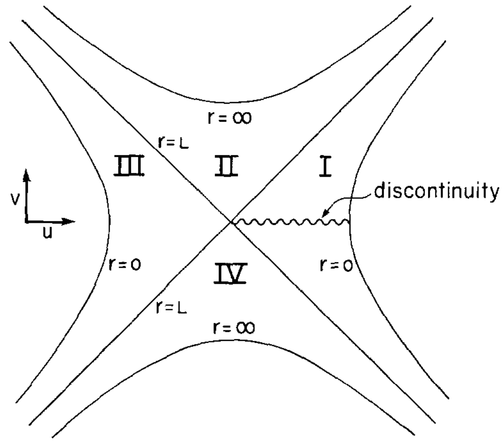

Fig. 6. De Sitter space in Gibbons–Hawking coordinates. In this diagram light travels along straight lines with slope ± 1. Regions I and III are causally disconnected but share past and future regions IV and II. The discontinuity corresponding to a wormhole or bubble at $r = 0$ is shown as a wiggly line.

with $y = 0$. Thus the euclidean point $Y = -L$ gives the field value on the past side of the Minkowski discontinuity at the location where the center of the bubble is about to appear. The initial value for future Minkowski evolution defined along the surface $z = +\epsilon$ and $0 \leqslant y \leqslant L$ is given by the value of ϕ_B along the rest of the maximal euclidean surface at $Z = 0$ extending over the range $0 \leqslant Y \leqslant L$. For example, on the future side of the discontinuity the field value at the center of the bubble is given by that at $Y = +L$ on the maximal cut of the euclidean solution. Explicitly we take ϕ as a function of y and z in the range $0 \leqslant y \leqslant L$ on either side of the discontinuity at $z = 0$ to be

$$\phi(y, -\epsilon) = \phi_B(-y), \qquad \phi(y, +\epsilon) = \phi_B(y). \tag{2.5}$$

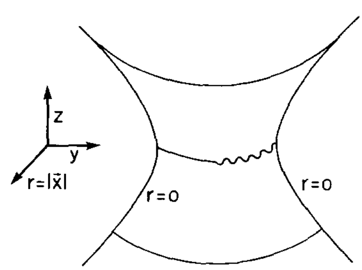

Fig. 7. De Sitter space shown as a hyperboloid. The discontinuity corresponding to a wormhole or bubble at $r = 0$ is shown as a wiggly line.

In this way the complete Minkowski signature discontinuity is specified using the single maximal euclidean surface. Note that as we approach the end of the Minkowski discontinuity at $y = 0$, the change in the value of the field across the discontinuity, $\phi_B(y) - \phi_B(-y)$, goes smoothly to zero.

The tunneling process we have described only has an impact on regions I, II and IV of the de Sitter space (see fig. 6). We believe this is just as it should be. Since region III is causally disconnected from region I where the tunneling-induced discontinuity takes place, nothing that happens in region III should have any impact on or any relevance to the tunneling process. However, in spite of this our description is complete. Given initial-value data, for example along a surface of constant negative v, we can use the classical equations of motion to predict all future behavior of the field ϕ. In region III the evolution is purely classical, while in region I we must augment the classical equations with information about the discontinuity across the cut which is just what knowledge of the euclidean bounce solution provides. Note that the behaviors in regions II and IV are partially correlated with the tunneling process since these regions are in causal contact with both regions I and III.

A novel feature of our interpretation is that it involves precisely specified final value data along the past side of the discontinuity as well as specifying initial value along the future side. We interpret this as implying that the quantum amplitude for the tunneling process to occur contains in addition to the exponential suppression factor given by the euclidean action, a prefactor proportional to the amplitude for the initial state wave function to yield field values corresponding to this final value data. Because the euclidean bounce solution de Sitter space never connects to the point $\phi = \phi_f$, we view the presence of such a factor as unavoidable.

We can also construct a similar interpretation for the euclidean bounce in the case when we allow for appreciable variation in the vacuum energy density. For example, if the false vacuum state is characterized by a positive vacuum energy density, but the true vacuum has zero cosmological constant, then the construction of the discontinuity and matching of initial and final value data proceeds as before, except that inside the bubble we match a section of flat euclidean space onto a section of flat Minkowski space along the future side of the Minkowski signature discontinuity.

As $L \to \infty$ the description we have given for the de Sitter bounce goes smoothly over to that given earlier for the flat-space bounce if we consider a region of finite size surrounding the bubble. As $L \to \infty$ the value of ϕ_B at $Y = -L$ approaches ϕ_f and the value of the field along the maximal cut in a finite region near $Y = -L$, which will provide final value data on the past side of the Minkowski discontinuity in the vicinity of the bubble, matches that along the cut at $T = -\infty$ for the flat-space bounce. Likewise, the value of ϕ_B near $Y = +L$ along the maximal cut, which will provide initial-value data for the future side of the discontinuity near the bubble, approaches ϕ_B along the cut at $T = 0$ for the flat-space bounce.

3. The de Sitter wormhole

Euclidean wormhole solutions have been found in theories of gravity with constraints [1] and gravity coupled to a three-index antisymmetric tensor field [2] and to a U(1) invariant scalar field theory with [3] or without [4] spontaneous symmetry breaking. The simplest solution is that of a scalar field with spontaneously broken U(1) symmetry and a potential constructed to insure that the magnitude of the scalar field remains constant despite all the gravitational perturbations which occur near the wormhole. Thus, we take $|\phi| = f$ where f is a constant. Since the scalar field theory has, by construction, a U(1) symmetry, there exists a charge which is conserved despite the spontaneous symmetry breaking. Solutions which have been constructed for this and other cases are spherically symmetric, so the euclidean signature metric is written in the form

$$\mathrm{d}S^2 = \mathrm{d}\xi^2 + A^2(\xi)\,\mathrm{d}\Omega_3^2, \tag{3.1}$$

where $\mathrm{d}\Omega_3$ is the line element on a unit three-sphere. These solutions have a distribution of U(1) charge with charge density depending only on ξ. The total charge on any three-sphere is conserved and is set equal to some value n.

The equation governing the radial factor $A(\xi)$ is [3]

$$\dot{A}^2 = 1 - \frac{q}{A^4} - \Lambda A^2, \tag{3.2}$$

where a dot denotes a ξ-derivative and we have introduced the parameter

$$q = \frac{n^2}{3\pi^3 m_{\mathrm{P}}^2 f^2}, \tag{3.3}$$

with m_{P} the Planck mass. Λ is the cosmological constant which we will take to be positive but which may be arbitrarily small. A solution of the equation for A has been given [3] but will not be needed for our purposes. The equation for A, (3.2), is equivalent to that of a particle with total energy 1 moving in a potential $V(A) = q/A^4 + \Lambda A^2$, where A is the particle's position and ξ plays the role of time. Provided that

$$3\left(\Lambda\sqrt{q}/2\right)^{2/3} < 1, \tag{3.4}$$

there exists a solution where A oscillates indefinitely between the two points at which $V(A) = 1$. The corresponding euclidean manifold is shown in fig. 4. It looks like an infinite chain of beads linked together by narrow tubes. Each bead is approximately a euclidean de Sitter space for four-sphere while the tubes are the wormholes. In this figure we have once again used five-dimensional euclidean imbedding coordinates (X, Y, Z) in which the wormhole solution is given by the surface

$$|X|^2 + Y^2 = A^2(\xi(Z)), \tag{3.5}$$

where $\xi(Z)$ is determined from

$$Z = \int d\xi \sqrt{1 - \dot{A}^2(\xi)}\ . \tag{3.6}$$

In fig. 4 we have also drawn contours of constant charge density. The density of charge, like the metric scale factor A, depends only on the coordinate Z.

In order for us to interpret the physical meaning of this euclidean manifold we must cut it along maximal surfaces to provide data for a suitable discontinuity or initial-value surface in a Minkowski signature manifold. A maximal surface is one on which both A and the charge density are stationary with respect to translations of the surface in a direction normal to the surface. If we take a surface of constant A then, since the charge density is proportional to A^{-3}, there is only one condition, that A be stationary on the surface. Only two such surfaces are apparent, those of minimum and maximum A. One is the surface where the wormhole neck reaches its minimum radius. The other is the equator of the approximate de Sitter sphere where A takes on its maximum value as shown in fig. 4. On both of these surfaces $\dot{A} = dA/dZ = 0$ and the charge density is stationary. We will assume that the cutting which gives the dominant contribution to the wormhole amplitude is just that between the equator of an approximate de Sitter sphere and its own wormhole neck. Both maximal cuts are three-spheres, one large and one small, with uniform charge density.

To interpret the wormhole solution we must find initial and final surfaces with matching geometries in the appropriate Minkowski signature manifolds. The relevant Minkowski-signature manifolds are solutions of the Einstein equations for a universe filled uniformly with U(1) charge in the presence of a cosmological constant. We look for solutions of the form

$$ds^2 = -dt^2 + a^2(t)\, d\Omega_3^2 . \tag{3.7}$$

The equation determining a is, not surprisingly, the analytic continuation of eq. (3.2),

$$\dot{a}^2 = -1 + q/a^4 + \Lambda a^2 . \tag{3.8}$$

This is equivalent to the equation of motion for a particle with total energy -1 moving in a potential $\overline{V}(a) = -q/a^4 - \Lambda a^2$. When condition (3.4) is satisfied the allowed classical motions divide into two distinct regions. The first describes a space-time in which a is infinite at $t = -\infty$, comes in to a minimum value and then grows infinitely large again at $t = +\infty$. This is very similar to ordinary de Sitter space. For this space the effect of the charge is small, merely perturbing the exact shape of the de Sitter space but not affecting its overall structure. There is a second solution of eq. (3.8) for which the charge density plays a dominant role and the cosmological constant is subdominant. Here the motion starts at $a = 0$, progresses out to a maximum value of a and then collapses back to $a = 0$ again. This is just an ordinary expanding and collapsing Robertson–Walker universe with a small cosmological constant. The unconventional form of the energy density, proportional to a^{-6}, is due to the unusual equation of state $\rho = p$ associated with the state of uniform U(1) charge distribution in spontaneously broken theory. In the unbroken theory [4] the energy density would be the usual one for either massless or massive particles. In either case, the points $a = 0$ are space-time singularities.

Both of these solutions can be embedded in five-dimensional Minkowski space $(\boldsymbol{x}, y, z)$ with

$$ds^2 = -dz^2 + |d\boldsymbol{x}|^2 + dy^2, \tag{3.9}$$

and can be expressed as the surface

$$|\boldsymbol{x}|^2 + y^2 = a(t(z))^2, \tag{3.10}$$

where $t(z)$ is determined from

$$z = \int dt \sqrt{1 + \dot{a}^2(t)}\,. \tag{3.11}$$

The de Sitter like solution is very similar to the exact space shown in fig. 7. We will call the minimum radius of the constant z three-spheres L. It obeys $\Lambda L^6 - L^4 + q = 0$ with the root chosen so that $L = \Lambda^{-1/2}$ when $q = 0$. The Robertson–Walker solution is shown in fig. 8. Here the maximum radius of the three-spheres is given by $\Lambda a_{\max}^6 - a_{\max}^4 + q = 0$ with $a_{\max} = q^{1/4}$ if $\Lambda = 0$.

The large S_3 surface of the euclidean solution provides initial and final data along a cut in the approximately de Sitter, Minkowski signature manifold exactly as it did in the case of the de Sitter bounce solution. As before, the maximal euclidean surface $Z = 0$ provides both initial and final data for a half cut in the Minkowski signature space. The field value for $z = -\epsilon$ along the surface $0 \leqslant y \leqslant L$ is given by that on the euclidean manifold at $Z = 0$ in the range $-L \leqslant y \leqslant 0$. For this surface the total charge is $n/2$ because the charge is uniformly distributed over the maximal cut surface and we have taken half of this surface. The initial-value data for future Minkowski signature evolution defined along the surface $z = +\epsilon$ and $0 \leqslant y \leqslant L$ are given by field values along the rest of the maximal surface at $Z = 0$ extending over the range $0 \leqslant Y \leqslant L$. Here the total charge has magnitude $n/2$ because we are again considering half of the total maximal cut but its sign has changed to $-n/2$ because the direction of time is now taken to be the outward normal rather than the inward normal to the euclidean hemisphere. In our interpretation the violation of charge observed in the Minkowski signature manifold is due to the rotation of the sense of time across the discontinuity as we rotate around the euclidean sphere.

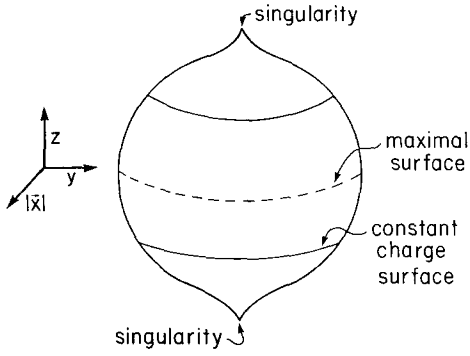

Fig. 8. A Minkowski signature Robertson–Walker universe half of which will be the baby universe.

The wormhole neck cut provides initial-value data for the birth of a closed Robertson–Walker universe at the moment of its maximum expansion. The Robertson–Walker universe is cut along the $z = 0$ plane and the space-time of the baby universe is one half of the manifold pictured in fig. 8.

The tunneling process we have described looks like an instantaneous violation of charge conservation by n units with a uniform charge distribution in region I of a de Sitter space suddenly changing to a uniform distribution with the opposite sign. In region III ordinary classical evolution takes place and no charge violation is observed. The charge which has vanished from the large space-time appears in a disconnected, closed Robertson–Walker space-time which ultimately collapses to a singularity.

We wish to thank S. Coleman, A. Guth, K. Lee, J. Preskill, A. Strominger and M. Wise for valuable discussions.

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