CANCELLATION OF THE SUPERCURRENT ANOMALY IN A SUPERSYMMETRIC GAUGE THEORY

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It is shown that the anomalous divergence of the supercurrent in a supersymmetric non-Abelian gauge theory may be removed at the one-loop level by coupling three scalar multiplets in the adjoint representation to the supersymmetric Yang-Mills multiplet.

We recently demonstrated that an anomaly exists [1] for the divergence of the supercurrent in a supersymmetric non-Abelian gauge theory. The specific model studied [2] consisted of a zero-mass Yang-Mills multiplet interacting with a single massless Majorana spin 1/2 field transforming as the adjoint representation of the internal symmetry group, taken as SU(2) for simplicity. In this case the supersymmetry Noether current $\mathcal{S}_\mu$ is only formally conserved. An anomaly exists, since the matrix element of $\mathcal{S}_\mu$ connecting the vacuum to the one boson-one fermion state is not conserved when computed in lowest non-trivial order of perturbation theory. Our calculation [1] demonstrated a fundamental clash between (Yang-Mills) gauge invariance, the conservation of the supercurrent between physical states $[\partial^\mu \langle \text{phys} | \mathcal{S}_\mu | \text{phys} \rangle = 0]$, and the spin 3/2 character of the supercurrent $[\gamma_\mu \mathcal{S}^\mu = 0]$.

In this note we extend the results of the previous calculation so as to include interactions of the Yang-Mills meson and Majorana spinor with the non-interacting scalar multiplet of Wess and Zumino [3], in order to demonstrate the existence of an anomaly killing mechanism at the one-loop level. This is possible since the added particles lead to additional contributions to both the supercurrent $\mathcal{S}_\mu$ and the one-loop diagrams contributing to the anomaly. We will show that, to leading order, the anomalous divergence $\partial_\mu \mathcal{S}^\mu$ is proportional to the Callan-Symanzik β-function of this extended gauge theory. This is related to a more restrictive result of Curtright [4], who showed (by formal arguments) that for the *pure* Yang-Mills supersymmetric theory, one may construct a *conserved*, gauge-invariant supercurrent $\hat{\mathcal{S}}_\mu$ with the properties $\partial^\mu \hat{\mathcal{S}}_\mu = 0$ and $\gamma_\mu \hat{\mathcal{S}}^\mu \sim \beta_\mathrm{V}(g)$, where $\beta_\mathrm{V}(g)$ is the β-function of the Yang-Mills multiplet alone. The discussion of ref. [1] allows us to reinterpret this in terms of the supercurrent $\mathcal{S}_\mu$, where $\gamma_\mu \mathcal{S}^\mu = 0$, and $\partial_\mu \mathcal{S}^\mu \sim \beta_\mathrm{V}(g)$ for the pure gauge theory. Here we show explicitly that $\partial_\mu \mathcal{S}^\mu \sim \beta(g)$, where now $\beta(g)$ also includes the contribution of the scalar multiplets: If the original Yang-Mills multiplet interacts with three scalar multiplets transforming as the adjoint representation, $\beta(g)$ vanishes to lowest order and the supercurrent anomaly is removed.

Consider a model consisting of a Yang-Mills vector meson v_μ^a, a scalar A^a, a pseudoscalar B^a, and Majorana spinors ψ^a and χ^a; all massless and in the adjoint representation. The Lagrangian describing this theory in the Wess and Zumino gauge [6] is

$$\mathcal{L} = -\tfrac{1}{4}(v_{\mu\nu})^2 + \frac{i}{2}\bar{\psi}^a \gamma_\mu (D^\mu \psi)_a + \tfrac{1}{2}(D_\mu A)^2 + \tfrac{1}{2}(D_\mu A)^2$$
$$+ \frac{i}{2}\bar{\chi}^a \gamma_\mu (D^\mu \chi)_a - ig\epsilon_{abc}\bar{\psi}^a (A^b + \gamma_5 B^b)\chi^c$$
$$- \tfrac{1}{2} g^2 [A^2 B^2 - (A_a B^a)^2] \,, \tag{1}$$

where $v_{\mu\nu}$ is the Yang-Mills field strength, D_μ is the covariant derivative, and $a = 1, 2, 3$. The supercurrent [2], which generates supersymmetry transformations, is

$$\mathcal{S}_\mu = \mathcal{S}_\mu{}^\mathrm{V} + \mathcal{S}_\mu{}^\mathrm{S} \,, \tag{2}$$

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where

$$\mathcal{S}_\mu{}^{\mathrm{v}} = -\mathrm{i}\sigma^{\alpha\beta}\gamma_\mu\psi_a \mathrm{v}^a_{\alpha\beta} \tag{3}$$

and

$$\mathcal{S}_\mu{}^{\mathrm{s}} = \gamma_\lambda\gamma_\mu[D_\lambda(A+\gamma_5 B)]^a\chi_a$$
$$+\tfrac{4}{3}\sigma_{\mu\lambda}\partial^\lambda[(A^a+\gamma_5 B^a)\chi_a] . \tag{4}$$

with $(\gamma_5)^2 = -1$ and $\sigma_{\alpha\beta} = \frac{1}{4}[\gamma_\alpha, \gamma_\beta]$. The first term in $\mathcal{S}_\mu{}^{\mathrm{s}}$ is the Noether current for the scalar contribution, while the second term in (4) is the improvement of Ferrara and Zumino [5] which ensures that $\gamma_\mu\mathcal{S}^\mu = 0$ when the equations of motion are employed. [Note that $\gamma^\mu\mathcal{S}_\mu{}^{\mathrm{v}} = 0$ is an algebraic result, and does not depend on the equations of motion.]

We previously have discussed [1] the process $\mathcal{S}_\mu{}^{\mathrm{v}} \to \psi + \mathrm{v}$ in one-loop approximation, for which

$$\langle p_\psi, k_{\mathrm{v}}|\partial^\mu\mathcal{S}_\mu{}^{\mathrm{v}}|0\rangle$$
$$= -\frac{3\mathrm{i}g^2}{8\pi^2}\epsilon_\nu{}^*(k)\bar{u}(p)[\gamma_\nu p\cdot k - p_\nu\gamma\cdot k] . \tag{5}$$

Now consider $\mathcal{S}_\mu{}^{\mathrm{s}} \to \psi + \mathrm{v}$, which is to be added to eq. (5) to obtain the complete amplitude for $\mathcal{S}_\mu \to \psi + \mathrm{v}$. Define the amplitude for the contribution of $\mathcal{S}_\mu{}^{\mathrm{s}}$ to this process as

$$\epsilon_\nu{}^*(k)\bar{u}(p)(S^{\,\mathrm{s}}_{\mu\nu})^{bc} = \epsilon_\nu{}^*(k)\bar{u}(p)\delta_{bc}Q_{\mu\nu} , \tag{6}$$

with $Q_{\mu\nu}$ computed from the diagrams of fig. 1, and the mass-shell restrictions

$$\epsilon^\nu(k)k_\nu = 0 , \quad \bar{u}(p)\gamma\cdot p = 0 , \quad k^2 = p^2 = 0 . \tag{7}$$

From a trivial analysis of the structure of the Feynman diagrams, one notes that only an odd number of γ-matrices are present in $Q_{\mu\nu}$. With this in mind, the most general form of $Q_{\mu\nu}$ on the mass-shell is

$$\mathrm{i}Q_{\mu\nu} = C_1\gamma\cdot k\delta_{\mu\nu} + C_2\gamma\cdot k\gamma_\nu\gamma_\mu + C_3\gamma_\nu p_\mu + C_4 p_\nu\gamma_\mu$$
$$+ C_5\gamma_\nu k_\mu + C_6\gamma\cdot k p_\nu k_\mu + C_7\gamma\cdot k p_\nu p_\mu . \tag{8}$$

Gauge invariance imposes the constraint $k^\nu S^{\,s}_{\mu\nu} = 0$, which requires

$$C_1 + C_5 + p\cdot k C_6 = 0 , \quad C_4 = 0 , \quad C_3 + p\cdot k C_7 = 0 ,$$

while $\gamma^\mu S^{\,s}_{\mu\nu} = 0$ implies (9)

$$C_1 + 4C_2 - C_5 = 0 \quad \text{and} \quad C_5 + p\cdot k C_7 = 0 . \tag{10}$$

One may combine eqs. (8)–(10) to write the scalar contribution to the anomaly as

$$-\mathrm{i}S^{\,s}_\nu = (p+k)^\mu Q_{\mu\nu}$$
$$= p\cdot k(\tfrac{1}{2}C_6 + C_7)(\gamma_\nu p\cdot k - \gamma\cdot k p_\nu) . \tag{11}$$

Alternatively, if we require $\partial_\mu\mathcal{S}^\mu = 0$, we find

$$\langle p, k|\gamma_\mu\mathcal{S}^\mu|0\rangle = 2\mathrm{i}p\cdot k[\tfrac{1}{2}C_6 + C_7]\delta_{bc}\epsilon^*_\nu(k)\bar{u}(p)\gamma\cdot k\gamma_\nu$$

Although the Feynman integrals defined by fig. 1 are primitively linearly divergent, and hence subject to ambiguities dependent on the routing of momenta through the diagrams, the invariants C_6 and C_7 are *not* subject to this arbitrariness since they are coefficients of terms in (8) which are *cubic* in momenta [1, appendix A]. Hence the scalar contribution to the anomaly is *uniquely* determined from figs. 1a and 1b. (Fig. 1c does not contribute to the unambiguous invariants C_6 and C_7.)

It is straightforward to show that on mass-shell

$$C_i = -2g^2\int_0^1 \mathrm{d}x\int_0^{1-x}\mathrm{d}y\int\frac{\mathrm{d}^4q}{(2\pi)^4}\,\frac{D_i}{(q^2+2xyp\cdot k)^3}$$

$$\text{for } i = 6 \text{ and } 7 , \tag{12}$$

with $D_6 = D_7 = \frac{16}{3}xy$. Therefore from (11) and (12)

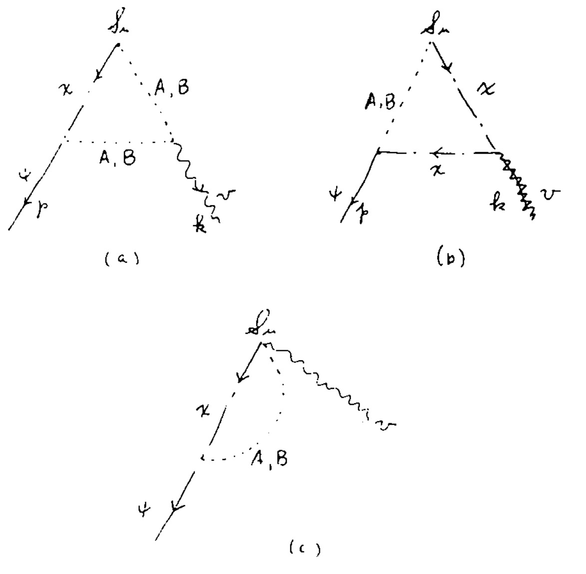

Fig. 1. Graphs contributing to the process $\mathcal{S}_\mu \to \psi + \mathrm{v}$.

$$-\mathrm{i}\, S_\nu^s = 2g^2 \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \int \frac{\mathrm{d}^4q}{(2\pi)^4}$$
$$\times \frac{8xy\, p\cdot k\,(\gamma_\nu p\cdot k - p_\nu \gamma\cdot k)}{(q^2 + 2xy\, p\cdot k)^3}$$
$$= \frac{\mathrm{i}g^2}{8\pi^2}(\gamma_\nu p\cdot k - p_\nu\gamma\cdot k)\,, \qquad (13)$$

which is −1/3 the contribution of the vector multiplet presented in ref. [1] and eq. (5). It is obvious that the *total* supercurrent anomaly for n scalar multiplets interacting with a single Yang-Mills multiplet is, to $O(g^2)$,

$$\langle p, k|\partial_\mu \mathcal{S}^\mu|0\rangle$$
$$= (n-3)\frac{g^2}{8\pi^2}\,\mathrm{i}\,\epsilon_\nu^*(k)\bar{u}(p)\delta_{bc}(\gamma_\nu p\cdot k - p_\nu\gamma\cdot k)\,. \qquad (14)$$

Since the Callan-Symanzik function is [6]

$$\beta(g) = (n-3)\frac{g^3}{16\pi^2}N + O(g^5)\,, \qquad (15)$$

for n scalar multiples in the adjoint representation interacting with the Yang-Mills multiplet of supersymmetric SU(N) theory, we have established that the anomalous divergence is proportional to the β function to this order. Clearly the anomaly is removed, to leading order, by the interaction of three scalar multiplets.

In this model, the supercurrent also contributes to the two-particle processes $\mathcal{S}_\mu \to \chi + A$ and $\mathcal{S}_\mu \to \chi + B$. These amplitudes must be anomaly free as well if supercurrent anomalies are to be completely absent from the model. The one-loop amplitude for $\mathcal{S}_\mu \to \chi + A$ can be written as

$$\delta_{bc}\,\bar{u}(p)\,\Sigma_\mu(p,k)\,, \qquad (16)$$

where on mass-shell

$$\mathrm{i}\Sigma_\mu(p,k) = E_1 p_\mu + E_2 k_\mu + E_3 \gamma\cdot k\gamma_\mu\,, \qquad (17)$$

since the one-loop amplitude only contains an even number of γ matrices. (For $\mathcal{S}_\mu \to \chi + B$ replace $E_i \to \gamma_5 E_i$.) There are four diagrams which contribute to (17), each primitively linearly divergent. Most importantly, *all* amplitudes in (17) are ambiguous, since each has contributions from surface terms which depend on the routing of momenta through the diagrams. The constraint $\gamma_\mu \mathcal{S}^\mu = 0$ implies that

$$E_2 + 4E_3 = 0\,, \qquad (18)$$

and an anomalous divergence for $\mathcal{S}_\mu \to \chi + A$ is to be computed from

$$\Sigma = \mathrm{i}(p+k)^\mu \Sigma_\mu$$
$$= \mathrm{i}p\cdot k\,(E_1 + E_2 + 2E_3)\,. \qquad (19)$$

Since there are four diagrams which contribute to $\mathcal{S}_\mu \to \chi + A$ (or B) to lowest order, it would appear that there is sufficient freedom to define the routing of momenta through these diagrams so that $\gamma_\mu\mathcal{S}^\mu = 0$ is compatible with $\partial_\mu \mathcal{S}^\mu = 0$. Thus one may define E_i (i = 1 to 3) so that the process $\mathcal{S}^\mu \to \chi + A$ is anomaly free.

We have shown that the anomalous divergence of the supercurrent is proportional to the Callan-Symanzik β-function in lowest order, which allows one to remove the anomaly at the one-loop level by coupling three scalar multiplets to the Yang-Mills multiplet. It is not obvious what form the anomaly will take in higher orders. If the anomaly is still proportional to the β-function, then only *two* scalar multiplets would be required to remove the anomaly at the two-loop level [6], if the coupling constant is at the fixed point of $\beta(g)$.

The existence of an anomaly for the supercurrent $\mathcal{S}_\mu$ does not affect the renormalizability of theories with global supersymmetry, since no gauge particle is coupled to this current. However, in theories of supergravity [7] and extended supergravity [8] the existence of a supercurrent anomaly would pose serious problems for the renormalization program [9], since the supercurrent couples directly to the massless spin 3/2 gauge particles of these theories. The existence of anomaly-killing mechanisms for global supersymmetry suggest a similar possibility in supergravity. We hope to report on this problem in the near future.

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Note added. The model with three scalar multiplets can be written in a particularly elegant form [10, 11], exhibiting a global SU(4) invariance, and a supersymmetric SU(4) transformation in addition to the Yang-Mills gauge invariance (see eq. (3.14) ff. of ref. [11]). The supersymmetric SU(4) transformation leads to *four* supersymmetry currents S^i_μ (i = 1 to 4). It is straightforward to show that all four supercurrents are anomaly free at the one-loop level, exactly as above. This SU(4) extended supersymmetric Yang-Mills theory is of particular interest because of its relationship to dual models. We thank J. Schwarz and J. Scherk for informing us of the relationship of our results to their work.