A SUPERCURRENT ANOMALY IN SUPERGRAVITY

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Received 8 November 1977

It is shown that an anomaly exists for the supercurrent of the supersymmetric vector multiplet when coupled to external supergravity fields. As a result, this anomaly will also be present in at least one of the O(1), O(2), or O(3) extended supergravity models.

Recently supercurrent anomalies have been studied in globally supersymmetric models [1,2]. In this paper we extend that discussion to include aspects of the anomaly question in locally supersymmetric theories by demonstrating the existence of an anomaly in the supercurrent of the abelian vector multiplet in the presence of (external) supergravity fields. We use this result to deduce the existence of a supercurrent anomaly in at least one of the O(1) [3], O(2) [4], or O(3) [5] extended supergravity theories.

Let us consider a model consisting of an abelian vector (supersymmetry) multiplet coupled to the (external) graviton and spin 3/2 fields of ordinary supergravity. The spinor supercurrent of the vector multiplet is

$$S_\mu = 2^{-1/2} F_{\alpha\beta}\sigma_{\alpha\beta}\gamma_\mu\chi\,, \tag{1}$$

where χ is a Majorana spinor, $\sigma_{\alpha\beta} = \frac{1}{4}[\gamma_\alpha, \gamma_\beta]$, and $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$, where A_α is the abelian vector field. This current satisfies the *formal* relations,

$$\partial_\mu S^\mu = 0\,, \tag{2}$$

if one uses the equations of motion, and

$$\gamma_\mu S^\mu = 0\,. \tag{3}$$

Consider the matrix element

$$\langle p, k|\bar{S}_\mu|0\rangle = \bar{u}_\nu(p)\,\epsilon^*_{\alpha\beta}(k)\,\bar{S}_{\mu\nu\alpha\beta}\,, \tag{4}$$

between the vacuum and a one-graviton–one-spin-3/2-fermion state, with k and p the graviton and gravitino mo-

[1] Research supported in part by ERDA.
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[3] Research supported in part by ERDA under contract no. E(11-1) 3230.

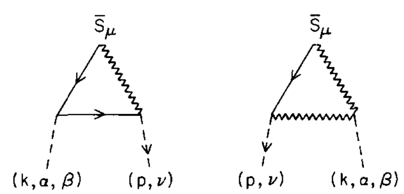

Fig. 1. Feynman graphs contributing to the amplitudes A_4 and A_5, defined in eqs. (4) and (5). A solid line with arrow denotes a spin 1/2 particle; a dashed line with arrow denotes a spin 3/2 particle; a wavy line denotes a vector boson; and a dashed line denotes a graviton.

menta respectively, $\bar{u}_\nu(p)$ the Rarita–Schwinger wave function, and $\epsilon^*_{\alpha\beta}(k)$ the graviton polarization tensor. Imposing on-shell gravitational and Rarita–Schwinger gauge invariance, we find by explicit calculation that eqs. (2) and (3) cannot be satisfied simultaneously in this sector, which is the anomaly we shall exhibit.

The most general form of $\bar{S}_{\mu\nu\alpha\beta}$ consistent with the mass-shell constraints $\bar{u}_\nu\gamma\cdot p = \bar{u}_\nu\gamma_\nu = \bar{u}_\nu p_\nu = 0$; $\epsilon_{\alpha\alpha} = k_\alpha\epsilon_{\alpha\beta} = 0$; $p^2 = k^2 = 0$, and the symmetry $(\alpha,\beta) \leftrightarrow (\beta,\alpha)$ is

$$\begin{aligned}\bar{S}_{\mu\nu\alpha\beta} = {} & A_1\gamma_\mu k_\nu p_\alpha p_\beta + A_2 p_\mu k_\nu(\gamma_\alpha p_\beta + \gamma_\beta p_\alpha) + A_3 k_\mu k_\nu(\gamma_\alpha p_\beta + \gamma_\beta p_\alpha) + A_4\gamma\cdot k p_\mu k_\nu p_\alpha p_\beta + A_5\gamma\cdot k\, k_\mu k_\nu p_\alpha p_\beta \\ & + A_6 k_\nu(g_{\mu\alpha}\gamma_\beta + g_{\mu\beta}\gamma_\alpha) + A_7\gamma\cdot k\, k_\nu(g_{\mu\alpha}p_\beta + g_{\mu\beta}p_\alpha) + A_8\gamma\cdot k g_{\mu\nu}p_\alpha p_\beta + A_9\gamma_\mu(g_{\nu\alpha}p_\beta + g_{\nu\beta}p_\alpha) \\ & + A_{10}g_{\mu\nu}(\gamma_\alpha p_\beta + \gamma_\beta p_\alpha) + A_{11}\gamma\cdot k\gamma_\mu k_\nu(\gamma_\alpha p_\beta + \gamma_\beta p_\alpha) + A_{12}\gamma\cdot k\, k_\mu(g_{\nu\alpha}p_\beta + g_{\nu\beta}p_\alpha) + A_{13}\gamma\cdot k p_\mu(g_{\nu\alpha}p_\beta + g_{\nu\beta}p_\alpha) \\ & + A_{14}k_\mu(g_{\nu\alpha}\gamma_\beta + g_{\nu\beta}\gamma_\alpha) + A_{15}p_\mu(g_{\nu\alpha}\gamma_\beta + g_{\nu\beta}\gamma_\alpha) + A_{16}\gamma\cdot k(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}) + A_{17}\gamma\cdot k\gamma_\mu(g_{\nu\alpha}\gamma_\beta + g_{\nu\beta}\gamma_\alpha),\end{aligned} \tag{5}$$

where the A_i are functions of $(p\cdot k)$. In constructing eq. (5) we have made use of the fact that $\bar{S}_{\mu\nu\alpha\beta}$ must contain an odd number of γ-matrices, as follows from the form of the supercurrent and the Feynman rules of the theory.

At the one-loop level $\bar{S}_{\mu\nu\alpha\beta}$ is primitively cubically divergent, and thus plagued by ambiguities which arise from different possible routings of momenta through the Feynman diagrams which contribute to eq. (5). However the invariants A_4 and A_5 multiply fifth-rank tensors in the external momenta, and hence are finite, unambiguous, and uniquely calculable from the two diagrams of fig. 1. (Although other diagrams contribute to eq. (5), those of fig. 1 are the only ones which contribute to A_4 and A_5.) Gravitational and Rarita–Schwinger gauge invariance require

$$k_\alpha\bar{S}_{\mu\nu\alpha\beta} = 0, \qquad \text{and} \qquad p_\nu\bar{S}_{\mu\nu\alpha\beta} = 0, \tag{6}$$

respectively, which then determines most of the invariants in terms of A_4 and A_5.

To complete the calculation we take the point of view that a regulator scheme exists which preserves the constraint $\partial_\mu S^\mu = 0$ at the one-loop level. Thus, we may impose the constraint

$$(p+k)_\mu\bar{S}_{\mu\nu\alpha\beta} = 0, \tag{7}$$

and find as a consequence of eqs. (5)–(7) that

$$\bar{S}_{\mu\nu\alpha\beta}\gamma_\mu = -\tfrac{1}{2}p\cdot k(2A_4 + A_5)[\gamma\cdot k k_\nu(\gamma_\alpha p_\beta + \gamma_\beta p_\alpha) - p\cdot k\gamma\cdot k(g_{\nu\alpha}\gamma_\beta + g_{\nu\beta}\gamma_\alpha)], \tag{8}$$

which expresses a possible anomaly for eq. (3) in terms of finite, unambiguous invariant amplitudes. (Alternatively, if we had imposed $\bar{S}_{\mu\nu\alpha\beta}\gamma_\mu = 0$, we would have found that $\partial_\mu\langle p, k|\bar{S}_\mu|0\rangle$ was proportional to $(2A_4 + A_5)$).

The Feynman rules for supergravity interacting with the vector multiplet [5,6] enable us to calculate the amplitudes A_4 and A_5 from the diagrams of fig. 1. We find that

$$A_4 = A_5 = -\mathrm{i}\sqrt{2}/12p\cdot k(32\pi^2), \tag{9}$$

so that the right-hand side of eq. (8) is non-vanishing, demonstrating the existence of the anomaly in question. This anomaly can be reexpressed in operator form as

$$\gamma_\mu S_\mu = [1/\sqrt{2}(128\pi^2)] R_{\alpha\nu\beta\lambda}\sigma_{\beta\lambda}D_\alpha\psi_\nu \,, \tag{10}$$

which is the covariant extension of our result (modulo terms which vanish on the mass shell). (In eq. (10), $R_{\alpha\nu\beta\lambda}$ is the curvature tensor, and D_α is the gravitational covariant derivative for the spin 3/2 field.)

This result has implications for extended supergravity, since it permits us to conclude that a supercurrent anomaly exists in at least one of the models O(1), O(2), or O(3) of extended supergravity. Observe that the O(1), O(2), or O(3) theories can be viewed as theories of a (2, 3/2) multiplet, interacting (2, 3/2) + (3/2, 1) multiplets, or interacting (2, 3/2) + 2(3/2, 1) + (1, 1/2) multiplets, respectively. For any of these theories we can compute the matrix element $\langle p, k|S_\mu|0\rangle$ of suitably defined currents which satisfy $\partial_\mu S^\mu = 0$, and

$$\gamma_\mu S^\mu = S \,, \tag{11}$$

as *formal* relations. In contrast to the case of the vector multiplet, S is not necessarily zero, but has a specific operator form determined by the structure of the theory. (The fact that $S \neq 0$ at the tree level expresses the fact that these supergravity models are not super-conformally invariant.) We now argue that in at least one of the O(1), O(2), or O(3) supergravity theories, eq. (11) must be violated at the one-loop level. The mass-shell transition $S_\mu \to$ graviton + spin 3/2 can be considered for each of these models, and described by eqs. (4)–(8). The point is that for the calculation of the amplitudes A_4 and A_5, only triangle diagrams analogous to fig. 1 are needed, and they involve only those parts of the current quadratic in the fields, which therefore involve the supercurrents of the separate multiplets. That is, for the purpose of the calculation, for

$$\mathrm{O}(1){:}\ S_\mu = S_\mu^{(2,3/2)}, \quad S = S^{(2,3/2)} \,, \tag{12}$$

$$\mathrm{O}(2){:}\ S_\mu = S_\mu^{(2,3/2)} + S_\mu^{(3/2,1)}, \quad S = S^{(2,3/2)} + S^{(3/2,1)} \,, \tag{13}$$

$$\mathrm{O}(3){:}\ S_\mu = S_\mu^{(2,3/2)} + 2S_\mu^{(3/2,1)} + S_\mu^{(1,1/2)}, \quad S = S^{(2,3/2)} + 2S^{(3/2,1)} \,. \tag{14}$$

(Note that $S^{(1,1/2)} = 0$.) Furthermore due to the uniqueness of supergravity interactions [3–5], the current of a particular multiplet makes the *same* contribution to A_4 and A_5 no matter which of the three supergravity models we consider. This enables us to argue as follows. In O(1) either eq. (11) is violated, or

$$\gamma^\mu\langle p, k|S_\mu^{(2,3/2)}|0\rangle = \langle p, k|S^{(2,3/2)}|0\rangle \,. \tag{15}$$

If eq. (15) is valid, then in the O(2) theory, either eq. (11) is violated, or

$$\gamma^\mu[\langle S_\mu^{(2,3/2)}\rangle + \langle S_\mu^{(3/2,1)}\rangle] = \langle S^{(2,3/2)}\rangle + \langle S^{(3/2,1)}\rangle \,, \tag{16}$$

is valid, i.e.

$$\gamma^\mu\langle S_\mu^{(3/2,1)}\rangle = \langle S^{(3/2,1)}\rangle \,. \tag{17}$$

But then in O(3), either eq. (11) is violated, or eqs. (14)–(16) would imply

$$\gamma^\mu\langle S_\mu^{(1,1/2)}\rangle = 0 \,, \tag{18}$$

which is false. Therefore, at least one of the O(1), O(2), or O(3) models has a supercurrent anomaly, since it is not possible for eq. (11) to be satisfied in all three theories. In all likelihood all three have anomalies, although it is conceivable that one particular O(n) supergravity model ($n = 1, ..., 8$) is anomaly free, as is the case for a particular global supersymmetry theory [7]. To verify this possibility one must make explicit computations for all the multiplets. We hope to report such a computation in the future.

We conclude with some comments. In supergravity theories the (2, 3/2) and (3/2, 1) parts of the supercurrent are not gauge invariant, and the matrix elements of S_μ and S may depend on the choice of gauge for the internal spin 3/2 fields. However the (1, 1/2) part of the supercurrent is independent of this gauge choice so that in *any* gauge we will find an anomaly in one of the theories in question. The crucial question is whether these supercurrent anomalies could affect the hoped for renormalizability of these theories, particularly since the current S_μ couples to gauge particles of the theory, the spin 3/2 gravitinos. The issue first becomes relevant at the two-loop level where, unlike the one-loop level for which the anomaly could play no role [8], finiteness arguments rely on non-violations of global supersymmetry [9]. It would appear that as long as the anomalies can be confined to $\gamma_\mu S^\mu$, while preserving $\partial_\mu S^\mu = 0$ through a suitable regularization scheme, no problem should arise. Such a regularization scheme may indeed exist for the (1, 1/2) calculation presented here, but it is not obvious that *any* regularization scheme exists which preserves $\partial_\mu S^\mu = 0$ in the analogous (2, 3/2) or (3/2, 1) calculations. If not, this issue would have to be resolved if further progress is to be made in the search for a finite supergravity.

Much of this work was done at the Aspen Center for Physics during the summer of 1977. The authors wish to thank the Center for its hospitality. One of us (M.T.G.) also thanks B. deWit for valuable discussions.

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