

## ARE THE WEAK INTERACTIONS STRONG?

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We consider a strong-coupling, confining version of the standard  $SU(2)_L \times U(1)$  electroweak model. We find that at low energies ( $\ll 100$  GeV) the particles and interactions given by this model are indistinguishable from those of the usual weak-coupling, spontaneously broken theory. However, at high energies ( $\gtrsim 100$  GeV) the confining model exhibits bound state quark and lepton structure, and intermediate vector bosons heavier than those of the standard model.

The low energy weak interactions are well described by the standard  $SU(2)_L \times U(1)$  model [1] of Weinberg and Salam. In this model, the mass scale of the weak interactions is determined by the vacuum expectation value of a scalar field. This vacuum expectation value also cuts off the infra-red growth of the  $SU(2)_L$  coupling constant resulting in a weak-coupling theory.

In this paper, we consider a very different approach to the standard  $SU(2)_L \times U(1)$  lagrangian. We assume that the  $SU(2)_L$  coupling constant becomes large at a mass  $\Lambda$  which sets the scale for the weak interactions ( $\Lambda$  roughly of order  $G_F^{-1/2}$ ) and that no appreciable scalar vacuum expectation value exists. Thus, we have a strong-coupling, confining  $SU(2)_L$  theory. The lagrangian we are considering is just the conventional  $SU(2)_L \times U(1)$  theory with the usual particle content and quantum number assignments. Only the values of the coupling constants and scalar vacuum expectation value changed from those of the standard model. It has been shown [2] that the particle spectrum of the standard, spontaneously broken  $SU(2)_L \times U(1)$  model is matched perfectly by the low-lying particle spectrum of this confining theory. Here, we show that, in addition, the strong-coupling, confining  $SU(2)_L \times U(1)$  model can produce low energy charged and neutral current weak interactions which are experimentally indistinguishable from those of the standard, weak-coupling model. Thus,

it may be a viable alternative. At energies near or above the weak interaction mass scale, the two models are clearly distinguishable with the strong-coupling model exhibiting bound state quark and lepton structure, heavier intermediate vector bosons, and eventually a whole  $SU(2)_L$  strongly interacting sector.

To begin, we review how the particle spectrum of the standard  $SU(2)_L \times U(1)$  model is reproduced by the strong-coupling, confining theory [2]. Since we assume that  $SU(2)_L$  is confining, all physical states must be  $SU(2)_L$  singlets. This condition is satisfied by the right-handed Fermi fields, but the left-handed fermions,  $\psi_L^i$  (where  $i = 1, 2$ ) are  $SU(2)_L$  doublets. To form left-handed fermion singlets we must bind these to the scalar doublet to form the states

$$U_L = \phi_i^* \psi_L^i, \quad D_L = \epsilon_{ij} \phi^i \psi_L^j. \quad (1)$$

(We use U and D to denote both quark and lepton fields.)

Just as in the standard model, the masses for these fermions are provided by the Yukawa coupling terms

$$\lambda_1 \bar{U}_R \phi_i^* \psi_L^i, \quad \lambda_2 \bar{D}_R \epsilon_{ij} \phi^i \psi_L^j, \quad (2)$$

which by eq. (1) correspond to the left-right transitions of the physical fermions. You would expect any fermion which has a bound state structure at the scale  $\Lambda$  to have a mass near this scale. However, in the absence of the Yukawa couplings of eq. (2) the underlying theory is chirally invariant and the mass of any physical fermion is zero to all orders. In the standard model we

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know that the coefficients of the Yukawa couplings are very small. For example,  $\lambda$  for the electron is  $\approx 10^{-6}$ . In the presence of Yukawa couplings we expect the physical fermion masses to be of the order of  $\lambda\Lambda$  which is the correct value.

Since there is no symmetry breaking in our model, the unbroken  $U(1)$  of  $SU(2)_L \times U(1)$  is identified with the electromagnetic interactions. Thus, we set the  $U(1)$  coupling constant equal to  $e$  (whereas in the standard model it is  $g_1 = e/\cos\theta$ ). Then, the  $U(1)$  hypercharges of our physical fermions must equal the electric charges of the quarks and leptons. This is easily seen to be the case [2]. For the right-handed Fermi fields the standard assignment sets the hypercharge equal to the electric charge. For the left-handed bound states of eq. (1), the hypercharges are obtained by summing the hypercharges of the constituents. The hypercharge of  $\psi_L$  is  $Y$  and the scalar field has hypercharge  $-1/2$ . Thus, the states in eq. (1) have charges  $Y \pm 1/2$  in exact agreement with the result,  $Q = Y + T_3$ , of the standard model.

In addition to these fermions, there are  $SU(2)_L$  singlet, bound state, spin one bosons of the form [2]

$$\phi^* D_\mu \epsilon \phi^*, \quad \phi \epsilon D_\mu \phi, \quad \phi^* \vec{D}_\mu \phi, \quad (3)$$

where  $D_\mu$  is the  $SU(2)_L \times U(1)$  covariant derivative. Note that these have charges (hypercharges)  $+1$ ,  $-1$  and  $0$ . We identify them with the  $W^+$ ,  $W^-$  and  $W^0$  vector bosons. Finally, there is a spin 0,  $SU(2)_L$  singlet bound state  $\phi_i^* \phi^i$  corresponding to the neutral Higgs boson of the standard model.

We expect the  $W$  bosons of eq. (3) to have a mass of order  $\Lambda$ , the scale set by the  $SU(2)_L$  coupling constant. A crucial feature of this model is that up to corrections due to electromagnetic interactions and mass splittings within fermion doublets, the masses of the three  $W$ 's are identical. This is due to a global  $SU(2)$  symmetry of the standard  $SU(2)_L \times U(1)$  lagrangian. This symmetry can most easily be seen by writing the scalar fields as a matrix

$$\Omega = \begin{pmatrix} \phi_1 & -\phi_2^* \\ \phi_2 & \phi_1^* \end{pmatrix} \quad (4)$$

and likewise the  $SU(2)_L$  gauge fields as

$$A_\mu = A_\mu^a \tau^a. \quad (5)$$

Then, for  $e = 0$  and  $\lambda_1 = \lambda_2$  in eq. (2), the  $SU(2)_L \times U(1)$  lagrangian is invariant under the global transformation

$$\begin{aligned} \Omega &\rightarrow \mathcal{U} \Omega \mathcal{U}^+, \\ A_\mu &\rightarrow \mathcal{U} A_\mu \mathcal{U}^+, \\ \psi_L &\rightarrow \mathcal{U} \psi_L, \\ \begin{pmatrix} U_R \\ D_R \end{pmatrix} &\rightarrow \mathcal{U} \begin{pmatrix} U_R \\ D_R \end{pmatrix}, \end{aligned} \quad (6)$$

where  $\mathcal{U}$  is a  $2 \times 2$  unitary matrix. Note that this  $SU(2)$  symmetry is different from the local  $SU(2)_L$  symmetry. The  $W$  bosons of eq. (3) are a triplet under this global  $SU(2)$  symmetry and thus equal in mass (up to the corrections mentioned above). This is the same symmetry which assures  $m_W/m_Z = \cos\theta$  in the standard model. The  $U$  and  $D$  fermions form a global  $SU(2)$  doublet. Thus, by this  $SU(2)$  symmetry, the effective fermion-fermion-vector interaction must be of the form

$$\frac{1}{2} g (\bar{U}_L, \bar{D}_L) W_\mu^a \tau^a \gamma^\mu \begin{pmatrix} U_L \\ D_L \end{pmatrix}. \quad (7)$$

The coupling is only to the left-handed fermions because only these feel the strong  $SU(2)_L$  force. The effective coupling constant  $g$  cannot at present be calculated in terms of the  $SU(2)_L$  coupling constant, but the relation is presumably similar to that between the  $\rho$ -nucleon-nucleon coupling constant and the QCD coupling.

The interaction of eq. (7) leads through single charged  $W$  exchange to a low energy interaction of charged currents,

$$(4g^2/8M^2) J_{\mu L}^+ J_{\mu L}^-, \quad (8)$$

where  $M$  is the  $W$  boson mass and  $J_{\mu L}^\pm$  are the conventional charged currents. This gives the standard charged current interaction if

$$g^2/8M^2 = G_F/\sqrt{2}. \quad (9)$$

Since in the strong coupling theory we expect  $g$  to be of order 1 or somewhat larger,  $M$  will be about 125 GeV or greater. Thus, our bound state intermediate vector mesons are likely to be considerably heavier than those of the standard model.

The coupling in eq. (7) also leads through  $W^0$  exchange to a neutral current interaction

$$(4g^2/8M^2) J_{\mu L}^3 J_{\mu L}^3 = (4G_F/\sqrt{2}) J_{\mu L}^3 J_{\mu L}^3. \quad (10)$$

Note that the global  $SU(2)$  symmetry has assured that the normalization of the interaction (10) is identical to

that of the charged current interaction. The interaction of eq. (10) agrees exactly with the  $J_{\mu L}^3 J_{\mu L}^3$  term in the neutral current interaction of the standard model,

$$(4G_F/\sqrt{2})(J_{\mu L}^3 - \sin^2\theta J_{\mu}^{\text{em}})^2. \quad (11)$$

Eq. (10) does not represent the full neutral current interaction in the confining model. Since all left-handed physical fermions are bound states, they exhibit structure at a length of order  $1/\Lambda$  where  $\Lambda$  is the mass scale set by the  $SU(2)_L$  coupling constant. This bound state structure leads to a non-trivial form factor for the fermions. At low  $Q^2$  values, the electromagnetic current of a given Fermi field  $F$  has a contribution from the form factor which is

$$\kappa(Q^2/M^2)\bar{F}_L\gamma^\mu F_L. \quad (12)$$

We have used  $M$ , the mass of the W boson, to set the scale of the form factor since it is of order  $\Lambda$ .  $\kappa$  is an unknown parameter roughly of order 1. There is no magnetic form factor piece because it vanishes between left-handed fields.

It has been previously noted [3] that the neutral current interaction of eq. (10) and a photon exchange interaction produced by the form factor of eq. (12) give a neutral current interaction for neutrino scattering processes which is identical to that of the standard model. In the photon exchange process the  $1/Q^2$  from the photon propagator cancels the  $Q^2$  in eq. (12) resulting in an effective interaction of the form

$$(e^2\kappa/M^2)\bar{\nu}_L\gamma^\mu\nu_L J_{\mu}^{\text{em}}, \quad (13)$$

coupling the neutrino to the electromagnetic current of the target fermion. In the standard model the neutrino current interaction is [from eq. (11)]

$$(4G_F/\sqrt{2})\bar{\nu}_L\gamma^\mu\nu_L(J_{\mu L}^3 - \sin^2\theta J_{\mu}^{\text{em}}). \quad (14)$$

Eqs. (10) and (13) reproduce this interaction exactly if

$$\kappa e^2/M^2 = (4G_F/\sqrt{2})(-\sin^2\theta), \quad (15)$$

or by eq. (9)

$$\kappa = -(g^2/2e^2)\sin^2\theta \approx -1.25g^2. \quad (16)$$

If  $g \approx 1$  then  $\kappa$  is of order 1 as was expected. However, if  $g$  is much larger than 1,  $\kappa$  becomes unreasonably large. Therefore, we require a fairly small  $g$ . For example, the  $\rho$ -nucleon-nucleon coupling constant [4] is somewhere between 2.5 and 5.5. The  $\rho$ -nucleon-nucleon coupling is the QCD counterpart of our  $g$ . How-

ever, the correspondence is not precise because, for example, the constituents have different spins. Therefore, we consider  $g \approx 1$  quite reasonable.

The above analysis indicates that our model can naturally account for neutral current phenomenology. To be more precise we will now assume a specific model for the fermion form factors based on vector-meson dominance. In this model a photon interacts with a bound state left-handed fermion in two ways. First, there is the usual pointlike coupling proportional to the charge. Secondly, the photon can turn into a  $W^0$  and then the  $W^0$  interacts with the left-handed fermion via eq. (7). (Just as in ordinary hadronic physics, a photon can interact with a nucleon by first becoming a strongly interacting vector meson.) We define the amplitude for a photon to become a  $W^0$  as  $(2\kappa e/g)Q^2$ . This second contribution gives rise to the fermion form factor and leads to a photon exchange interaction between two arbitrary fermions  $f$  and  $f'$  of the form

$$eJ_{\mu}^{\text{em}}(f)Q^{-2}(2\kappa e/g)Q^2M^{-2}gJ_{\mu L}^3(f') + (f \leftrightarrow f') \\ = (2\kappa e^2/M^2)[J_{\mu}^{\text{em}}(f)J_{\mu L}^3(f') + J_{\mu}^{\text{em}}(f')J_{\mu L}^3(f)]. \quad (17)$$

If  $\kappa$  satisfies eq. (15), then the interaction of eq. (17) is identical<sup>†1</sup> to the  $J_{\mu L}^3 J_{\mu}^{\text{em}}$  cross-term in the neutral current interaction of the standard model given by eq. (11). The standard neutral current interaction of eq. (11) also has a term proportional to  $J_{\mu}^{\text{em}} J_{\mu}^{\text{em}}$ . However, this term has never been observed since it is parity conserving in electron scattering and vanishes in neutrino scattering processes. The interactions of eq. (10) plus eq. (17) are then identical to that of eq. (11) for all experimental purposes.

Thus the strong-coupling  $SU(2)_L \times U(1)$  model, supplemented by the assumption of vector-meson dominance, reproduces exactly the neutral current phenomenology of the standard model for arbitrary fermion scattering processes. Note that the strong-coupling model

<sup>†1</sup> Hung and Sakurai [5] have shown that a photon  $W^0$  mixing term similar to ours along with the interaction of eq. (10) reproduces exactly the full low energy neutral current interaction of the standard model, eq. (11). Our point of view differs from theirs because we do not expect appreciable  $\gamma$ - $W^0$  mixing at  $Q^2 \approx M^2$ . The expression  $(2\kappa e/g)Q^2$  for the  $\gamma$ - $W^0$  coupling is only a valid approximation at low  $Q^2$ . We use the vector dominance model only as a means of estimating the form factors of the fermions.

gives the correct low energy effective lagrangian characterized by three parameters  $e, \kappa/M^2$  and  $g^2/M^2$  just as the standard model is characterized by  $e, \sin^2\theta$  and  $\langle\phi\rangle$ .

If the weak interactions are due to a strong-coupling gauge theory whose scale is roughly of order  $G_F^{-1/2}$  then we expect to see physics characteristic of strong interactions near this energy. At energies around 80 GeV the bound state structure of the fermions will become apparent but the intermediate vector bosons will not appear until energies above 100 GeV. At still higher energies more strongly interacting particles should exist. For example <sup>#2</sup>, there should be excited  $W$ 's much like the  $\rho^*$ 's of ordinary hadronic physics.

Perhaps the scalar field appearing in the  $SU(2)_L \times U(1)$  lagrangian is actually itself a bound state as in technicolor models [6]. Then the left-handed ordinary fermions could be bound states of three or more fermions.

The embedding of  $SU(3) \times SU(2)_L \times U(1)$  in a grand unified model is considerably different for the confining  $SU(2)_L$  theory than for the standard model. As the energy is lowered from the grand unification scale, the  $SU(2)_L$  coupling constant must get big before the  $SU(3)$  coupling <sup>#3</sup>. Perhaps  $SU(2)_L$  is unified with another

group (possibly the technicolor group) somewhere above the weak interaction scale and below the grand unification scale. Different unification patterns must be explored.

We have shown that the weak interactions can be described by a strong-coupling non-abelian gauge theory. The underlying theory is renormalizable and is as well defined as QCD. If the intermediate vector bosons are observed to have masses above 100 GeV this possibility must be taken seriously.

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<sup>#2</sup> We have only analyzed the weak interactions arising from the exchange of the lowest mass  $W$ 's. However, the exchange of a whole sequence of intermediate vector bosons can be analyzed using the techniques given by Bjorken in ref. [3].

<sup>#3</sup> There exists the possibility that the strongly interacting  $SU(2)_L$  theory could produce a non-zero vacuum expectation value for the diquark condensate  $\epsilon_{abc} q_i^b q_j^c \epsilon^{ij}$  where  $a, b, c$  are colour indices and  $i$  and  $j$  are  $SU(2)_L$  indices. This would break the  $SU(3)$  QCD theory liberating integer charged nucleons and mesons with masses of  $\Lambda \approx G_F^{-1/2}$  and leaving only an  $SU(2)$  gauge theory of the strong interactions. We assume that this does not happen.