PARTICLE PRODUCTION IN THE NEW INFLATIONARY COSMOLOGY

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Techniques are developed for computing particle production due to the time dependence of a scalar field expectation value during a phase transition. We review the new version of the inflationary universe and discuss baryon production in this model.

1. Introduction. In the big bang cosmology, the universe may undergo a series of phase transitions as it cools. Under such circumstances it is likely that the universe (or some part of it) was temporarily in an unstable or metastable field configuration and then underwent a transition to the stable vacuum state. The purpose of this paper is to show how particle production occurs during this transition and how the final vacuum state is reached. Techniques for analysing such a situation are developed in section 2.

Recently, a new variant of Guth's inflationary cosmology [1] has appeared [2–4]. We review this scenario in section 3 and point out that the presence of Hawking radiation may limit the amount of exponential growth. In addition we discuss the baryon asymmetry produced in the new inflationary model.

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2. Particle production. We wish to compute the density of particles produced by a scalar field when it changes from some initial value $\phi = A$, oscillates and then settles into a new state $\phi = 0$. Such a situation could occur in the early universe due to thermal effects [5]. Here in a simple model, we will simulate these thermal effects by introducing a source J which holds the system in the $\phi = A$ state until at some time J is turned off and the system subsequently evolves to the $\phi = 0$ state. For simplicity, we consider a scalar field of mass m which is free except for Yukawa couplings to fermions [‡1]. The various species of fermions to which the scalar field is coupled will be labelled by an index i.

Classically, the evolution of the ϕ field in this model is determined by the field equation

$$\delta S/\delta\phi = (\Box^2 + m^2)\phi = -J\,, \tag{2.1}$$

where S is the classical action. The source J is chosen to be

[‡1] We require that the fermion mass be less than half the scalar mass.

$$J = -m^2 A\theta(-t)\,, \tag{2.2}$$

so that the conditions discussed above are met for the ϕ field. The solution of eq. (2.2) satisfying the boundary conditions appropriate to this case is

$$\begin{aligned} \phi &= A \qquad t < 0\,, \\ \phi &= A\cos mt \quad t > 0\,. \end{aligned} \tag{2.3}$$

Thus, classically the ϕ field oscillates indefinitely after it is released from the $\phi = A$ configuration.

The oscillations of the ϕ field result in the production of fermion–antifermion pairs through the Yukawa interactions $\Sigma_i g_i \phi \bar{\psi}_i \psi_i$. To calculate the total number of fermions of type i which are produced (to lowest order in g_i), we must compute the amplitude

$$\mathcal{A} = {}_+\langle \mathrm{F}_i, \bar{\mathrm{F}}_i | \int \mathrm{d}^4x\,(-\mathrm{i}g_i\phi\bar{\psi}_i\psi_i)\,|0\rangle_-\,. \tag{2.4}$$

In the expression (2.4) the fields $\bar{\psi}_i$ and ψ_i will create the fermions F_i and $\bar{\mathrm{F}}_i$ leaving the matrix element ${}_+\langle 0|\phi|0\rangle_-$. The field given by (2.3) is the expectation value ${}_-\langle 0|\phi|0\rangle_-$. The expectation value we want, ${}_+\langle 0|\phi|0\rangle_-$, is also a solution of the field equation (2.1) but with Feynman rather than retarded boundary conditions. This solution is

$$\begin{aligned} {}_+\langle 0|\phi|0\rangle_- &= A\,[1 - \tfrac{1}{2}\exp(\mathrm{i}mt)] \qquad t < 0\,, \\ {}_+\langle 0|\phi|0\rangle_- &= \tfrac{1}{2}A\exp(-\mathrm{i}mt) \qquad t > 0\,. \end{aligned} \tag{2.5}$$

If the field (2.5) is inserted into the amplitude (2.4) we find that an infinite density of fermions will be produced. This is because the field (2.5) [like (2.3)] oscillates indefinitely and thus keeps producing particles forever. However, in reality particle production damps out these oscillations and a finite fermion density results. This can be seen by considering quantum corrections to the field equations. The full quantum-mechanical field equation is

$$\delta\mathbf{\Gamma}/\delta\phi = -J\,, \tag{2.6}$$

where $\mathbf{\Gamma}$ is the effective action. To leading order in the g_i the quantum corrections to the effective action are given by fig. 1 and, to this order,

$$\mathbf{\Gamma} = -\tfrac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^4y\,\phi(x)\mathbf{\Gamma}^{(2)}(x,y)\phi(y)\,, \tag{2.7}$$

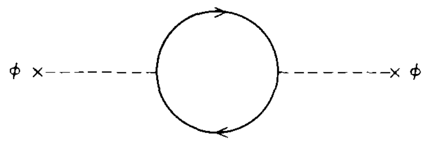

Fig. 1. One-loop contribution to the effective action.

where

$$\mathbf{\Gamma}^{(2)}(x,y) = \int \frac{\mathrm{d}^4p}{(2\pi)^4}\exp[\mathrm{i}p\cdot(x-y)] \times [p^2 - m_0^2 + \Sigma(p^2)]\,. \tag{2.8}$$

Here, m_0 is the bare mass and Σ the one-loop correction to the scalar propagator (fig. 1). The quantum-mechanical field equation is then

$$\int \mathrm{d}^4y\,\mathbf{\Gamma}^{(2)}(x,y)\,\phi(y) = -J(x)\,. \tag{2.9}$$

The solution of eq. (2.9) which replaces the zeroth order solution (2.5) is, to lowest order in g_i,

$$\begin{aligned} {}_+\langle 0|\phi|0\rangle_- &= A\,[1 - \tfrac{1}{2}\exp(\mathrm{i}mt)\exp(\tfrac{1}{2}\Gamma_{\mathrm{tot}}t)] \qquad t < 0, \\ {}_+\langle 0|\phi|0\rangle_- &= \tfrac{1}{2}A\exp(-\mathrm{i}mt)\exp(-\tfrac{1}{2}\Gamma_{\mathrm{tot}}t) \qquad t > 0, \end{aligned} \tag{2.10}$$

where

$$\Gamma_{\mathrm{tot}} = m^{-1}\,\mathrm{Im}\,\Sigma(m^2)\,, \tag{2.11}$$

$$m^2 = m_0^2 + \mathrm{Re}\,\Sigma(m^2)\,. \tag{2.12}$$

Eq. (2.12) gives the usual mass renormalization, while (2.11) gives the damping rate for the field oscillations. By unitarity $m^{-1}\,\mathrm{Im}\,\Sigma(m^2)$ is just the decay rate for scalar decays into fermions so the field damping rate Γ_{tot} is identical to the scalar particle decay rate. This result has a simple physical interpretation. The initial state $\phi = A$ can be thought of as a coherent state of scalar particles. This state then decays through the ordinary particle decays of these scalars.

We can now calculate the final density of fermions produced in the transition of the scalar field from $\phi = A$ to $\phi = 0$. We insert the damped solution (2.10) into the amplitude (2.4) and determine the density of fermions of type i produced as

$$n_i = mA^2B_i\,, \tag{2.13}$$

where

$B_i = \Gamma_i/\Gamma_{\mathrm{tot}}$,

is the scalar particle branching ratio into type i fermions. Since $\Sigma_i B_i = 1$, the total density of fermions produced is

$$n = mA^2 \,. \tag{2.14}$$

On the average these fermions are produced with an energy $m/2$ so the final energy density of all fermions is $1/2m^2A^2$ which is exactly the energy stored in the initial field configuration $\phi = A$.

In a realistic model the potential will also contain a $\lambda\phi^4/2$ term. Then the classical solution (with Feynman boundary conditions) is

$$\phi(t) = m/\lambda^{1/2} \sinh m(\tau + \mathrm{i}t)$$

$$= \frac{2m}{\lambda^{1/2}} \sum_{n=0}^{\infty} \exp\left[-(2n+1)m(\tau + \mathrm{i}t)\right] \,, \tag{2.15}$$

for $t > 0$. τ is an integration constant chosen to match the solution for $t < 0$. The oscillating ϕ field now produces fermion pairs with energies $E_n = (2n + 1)m$; $n = 0, 1, 2, \ldots$. The instantaneous production rate per unit volume for a fermion pair of energy E_n is

$$\Gamma^i_{\text{production}}/V \tag{2.16}$$
$$= (m^2/\lambda\pi) \exp(-2E_n\tau) g_i^2 E_n^2 (1 - 4\mu^2/E_n^2)^{3/2} ,$$

where the fermion mass $\mu < E_n/2$. If λ is very small the solution (2.16) must match the harmonic oscillator result and $\exp(-m\tau) \simeq A\lambda^{1/2}/4m$. In this case the production of fermion pairs with energy $E_0 = m$ dominates and the production of more energetic fermions is exponentially suppressed. Finally we note that scalar self couplings introduce spacial inhomogeneities which may grow more rapidly than the fermion production damps the oscillations. This is an indication that the coherent state scalars are interacting and developing non zero-momenta.

3. The new inflationary cosmology. The inflationary cosmology [1] provides a possible explanation for the extreme flatness and large scale homogeneity of the present universe. The inflationary scenario requires that the universe, as it cools, spends a significant time in a metastable or unstable field configuration with a large energy density. This energy density causes the Robertson–Walker scale factor to grow as $\exp(Ht)$ where H^2 is $8\pi G/3$ times the field energy density. If the universe remains in this configuration for a long enough time (on the scale $1/H$) then the Robertson–Walker factor can easily grow by the 28 orders of magnitude needed to explain the flatness of the present universe.

Recently a new mechanism for holding the universe in an unstable field configuration for a significant period of time has been proposed [2,3]. It involves the phase transition from a high temperature symmetric $\phi = 0$ state to a final state $\phi = \sigma \simeq 10^{15}$ GeV in which some grand unified theory like SU(5) is broken down to SU(3) × SU(2) × U(1). In this scenario the curvature of the potential at the origin is adjusted to be small at the temperature where the $\phi = 0$ vacuum is destabilized. As a result, it takes a long time for the ϕ field to "roll down" its potential hill to get to the $\phi = \sigma$ vacuum state. A Coleman–Weinberg [6] potential has been suggested as having the desired characteristics [‡2]. During the time that the scalar field is making its slow transition, that is while it is rolling down the hill, the universe expands exponentially due to the large energy density stored in the scalar field for $\phi \approx 0$. The Robertson–Walker scale factor grows like $\exp(Ht)$ where, for a Coleman–Weinberg potential in SU(5), $H \approx 5 \times 10^9$ GeV.

Suppose that at the time when the $\phi = 0$ state becomes unstable, fluctuations are such that ϕ senses a curvature in the potential of order $-\mu^2$, where μ is much less than H. The evolution of the ϕ field is governed by the field equation in an expanding space–time [‡3]

$$\ddot{\phi} + 3H\dot{\phi} - \mu^2\phi = 0 \,. \tag{3.1}$$

Eq. (3.1) has a growing solution

$$\phi \sim \exp(\mu^2 t/3H) \,. \tag{3.2}$$

Thus, the time scale for rolling away from the $\phi = 0$ configuration is

$$\tau_{\text{rolling}} = 3H/\mu^2 \,. \tag{3.3}$$

Eq. (3.1) is for the zero momentum mode of the ϕ field. However, the other modes are rapidly red shifted towards zero momentum by the exponential expansion. Once ϕ has moved appreciably away from $\phi = 0$, the transition to the $\phi = \sigma$ state occurs rapidly. Thus, during

‡2 The potential must be unnaturally tuned against all induced mass effects including those of gravity (see ref. [7]).

‡3 See Vilenkin [4], for a similar analysis.

the transition process the Robertson–Walker factor grows to

$$R \simeq H^{-1} \exp(H\tau_{\text{rolling}}) = H^{-1} \exp(3H^2/\mu^2) . \quad (3.4)$$

It would appear that by adjusting μ in eq. (3.4) any desired degree of exponential growth can be achieved. However, the existence of a Hawking temperature [8], $T_H = H/2\pi$, in de Sitter space may in fact place a limit on how much inflation can occur [‡4]. The extreme curvature of space probably induces quantum fluctuations in the field ϕ^2 of order T_H^2. This means that even in a Coleman–Weinberg potential tuned to be flat at the origin the ϕ field senses a curvature of order T_H^2. Therefore τ_{rolling} is expected to be about $3H/T_H^2$ resulting in a scale factor

$$R \simeq H^{-1} \exp(3H^2/T_H^2)$$

$$= H^{-1} \exp(12\pi^2) \simeq H^{-1} \times 10^{51} . \quad (3.5)$$

This estimate is large enough to make the inflationary scenario work but it is very sensitive to unknowns in the exponent.

During the inflationary phase when the ϕ field is moving slowly away from $\phi = 0$, the universe expands so much that any particles present before the inflation are diluted to a negligibly small density. All of the matter in the present universe must be generated during the "Great Thaw" when the scalar field relaxes into its final state, $\phi = \sigma$, producing particles by the mechanism discussed in section 2. In particular, the baryon asymmetry of the universe must be produced in this way. In standard calculations of the baryon asymmetry [9,10] the present asymmetry may depend on the initial asymmetry, which is usually assumed to be zero. (This is because an initial baryon asymmetry might not thermalize away.) In the inflationary cosmology this assumption does not have to be made because any initial baryon asymmetry is diluted to zero.

Once the ϕ field has moved appreciably away from $\phi = 0$ the transition to $\phi = \sigma$ will occur rapidly compared with the expansion rate of the universe. Thus gravitational effects are negligible during the epoch of particle production.

As discussed in section 2, the oscillations in the ϕ field around its vacuum value are damped by particle production. If we assume that the grand unified group is SU(5) then the ϕ field is the SU(3) × SU(2) × U(1) singlet component of the 24 dimensional adjoint representation. This field has direct couplings to all particles which get mass at this stage of symmetry breaking. These include the color octet and weak triplet scalars in the adjoint, the X and Y bosons, and the 5 dimensional Higgs fields H which decompose into color triplets H_3 and weak doublets H_2. When the ϕ field relaxes these particles will be produced. The initial relative abundances (before thermalization) of these particles and of ϕ particles is difficult to determine. Regardless of the initial abundances, the interactions of these particles will eventually produce a thermal distribution with a temperature which can be determined by energy conservation. In SU(5) with a Coleman–Weinberg potential, this reheating temperature is about $T_R \simeq 5 \times 10^{13}$ GeV. Typical particle interaction rates, through gauge boson exchange, are of order $\alpha_g^2 T_R \simeq 2 \times 10^{10}$ GeV, ($\alpha_g \simeq 1/45$). This is higher than the initial expansion rate $H \simeq 5 \times 10^9$ GeV so thermal equilibrium will be reached quickly on the cosmic time scale.

It is possible that a baryon excess is produced by the ϕ field oscillations before thermal equilibrium is reached. This excess will not be thermalized away if there are no light H_3's (relative to T_R). Baryon non conserving collision rates mediated by X and Y bosons are typically $\alpha_g^2 T^5/m_X^4$ while those mediated by heavy H_3's (relative to T_R) are on the order of $\alpha_Y^2 T^5/m_H^4$ where α_Y is the associated Yukawa fine structure constant. Both of these rates are less than H so the expansion of the universe freezes in the baryon excess for all time. We can get an upper limit on this baryon excess by assuming that all of the energy stored in the potential is converted into heavy particles, P, which then decay into fermions before reaching thermal equilibrium. The initial number density n_P is determined by energy considerations

$$m_P n_P \approx \rho , \quad (3.6)$$

where ρ is the energy density stored in the ϕ field. The baryon density is then

$$n_B \approx (\rho/m_P)\Delta B , \quad (3.7)$$

where ΔB is the mean number of baryons produced in the decay of a heavy P–$\bar{\text{P}}$ pair. The ratio n_B/s is then of order $(T_R/m_P)\Delta B$. Of course this is an upper limit and would be greatly reduced if for example the ϕ

‡4 This point was stressed to us by P. Ginsparg, A. Guth and A. Strominger.

field primarily produced light Higgs doublets, H_2's. Also if light H_3's exist then this baryon asymmetry will be reduced. This is because inverse decays occur at the rate $\alpha_Y T$ which is not small compared with H for temperatures less than T_R.

After thermalization a baryon excess can result from the out of equilibrium decays of heavy bosons as the universe cools. This is similar to the standard picture of baryon production [9,10] with the important difference that the process begins at a moderate temperature $T_R \simeq 5 \times 10^{13}$ GeV with the expansion rate $H \simeq 5 \times 10^9$ GeV. For a heavy particle to decay out of equilibrium we require that there is a period of time during which its decay rate is less than the expansion rate. Since $H(t) < H_{\text{initial}}$ we require

$$\Gamma_D < H_{\text{initial}} , \tag{3.8}$$

or

$$\alpha m_P < 5 \times 10^9 \text{ GeV} ,$$

where α is the appropriate coupling constant for decays. This places an upper bound on the mass of the heavy particle. We then require the usual condition that when the particle does decay, i.e. $\Gamma_D \simeq H(t)$, the temperature of the universe is less than the mass. This gives a lower bound on m_P of

$$\alpha m_{\text{Planck}}/20 < m_P , \tag{3.9}$$

where we have used $H \sim 20T^2/m_{\text{Planck}}$. For gauge bosons $\alpha \sim 1/45$; condition (3.8) gives $m_P < 2 \times 10^{11}$ GeV while (3.9) gives $m_P > 1 \times 10^{16}$ GeV so gauge bosons can never decay out of equilibrium to produce a baryon excess.

For scalar triplets, H_3's, the appropriate coupling is the Yukawa coupling associated with the heaviest fermion. Taking $m_{\text{top}} \gtrsim 15$ GeV gives an $\alpha_Y \gtrsim 7 \times 10^{-5}$ and our two conditions become:

$$2 \times 10^{13} \text{ GeV} \lesssim m_P \lesssim 7 \times 10^{13} \text{ GeV} . \tag{3.10}$$

However, numerical studies (taken from Fry et al. [10]) have shown that violations of (3.9) result only in a power law suppression of baryon production. An acceptable baryon asymmetry can result for Higgs triplet masses between 10^{11} and 10^{14} GeV.

In conclusion, a baryon asymmetry generated when the scalar field settles into its true minimum, before thermal equilibrium is reached, would persist until today if there are no Higgs triplets with masses much below the reheating temperature. If there is insufficient baryon production from this mechanism then it is still possible to generate a baryon excess as the universe cools from the out of equilibrium decay of Higgs triplets between 10^{11} and 10^{14} GeV in mass.

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