A COSMOLOGICAL BOUND ON THE INVISIBLE AXION

L.F. ABBOTT [1]
Physics Department, Brandeis University, Waltham, MA 02254, USA

and

P. SIKIVIE [2]
Particle Theory Group, University of Florida, Gainesville, FL 32611, USA

Received 14 September 1982

The production of axions in the early universe is studied. Axion models which break the $U(1)_{PQ}$ symmetry above 10^{12} GeV are found to produce an unacceptably large axion energy density.

The absence of *CP* violation in the strong interactions can be explained naturally by incorporating the $U(1)_{PQ}$ symmetry of Peccei and Quinn [1] into the standard model. A consequence of this symmetry is the axion [2], a spin 0 boson of mass

$$m_A \approx m_\pi f_\pi / v, \tag{1}$$

where v is the magnitude of the vacuum expectation value which breaks the $U(1)_{PQ}$ symmetry. An axion of mass (1) with v of order the weak interaction scale has not been observed. However, it has been pointed out [3] that v may be much larger, for example of order the grand unification mass. The axion is then extremely light and weakly coupled and is essentially invisible to laboratory experiments. It has recently been noted [4] that most axion models have a spontaneously broken $Z(N)$ symmetry and that the resulting discrete, degenerate vacua give rise to domain walls which are incompatible with standard cosmology. Several solutions to this problem have been proposed [4,5]. One approach is to adopt an inflationary cosmology [6]. This solves the domain wall problem provided that the reheating temperature, after the inflationary period, is not high enough to restore the $U(1)_{PQ}$ symmetry.

In this letter, we derive a new constraint which standard cosmology imposes on axion models. We consider the production of axions in the early universe and find that axions are copiously produced when the temperature, T, gets of order Λ, where Λ is the QCD scale parameter. At this time, QCD instanton effects align the vacuum producing a coherent state of axions at rest. After they are produced, the axions decouple. They have a large energy density which only goes down like T^3 since they are nonrelativistic. Thus, we expect a large energy density of axions in the present universe. We find that unless v is less than 10^{12} GeV, the present axion energy density will exceed the closure density of the universe by a factor of ten. This bound holds whether or not the vacuum of the axion model is unique. It is also independent of the history of the universe before the temperature reaches about 100 GeV.

To obtain our bound we must consider how the axion field behaves as the universe cools. At a high temperature of order v, a scalar field develops an expectation value

$$\langle \Phi \rangle = v e^{i\theta}, \tag{2}$$

breaking the $U(1)_{PQ}$ symmetry. The axion field is

[1] Supported in part by DOE contract DE-AC03-76ER03230-A005.
[2] Supported in part by DOE contract DSR80136je7.

0 031-9163/83/0000–0000/$ 03.00

$\phi_A \equiv \theta v$. Initially, the value of θ in eq. (2) is chosen randomly, but after the quarks develop a mass at a temperature of about 100 GeV, QCD instantons introduce a potential for the axion field with minima at values of θ for which there is no *CP* violation. This is the Peccei–Quinn mechanism [1]. We will choose our phases so that the vacuum is described by $\theta = \phi_A = 0$. The initial value of θ in eq. (2) is presumably some number of order one. Thus, initially ϕ_A is of order v and as T approaches Λ the instanton induced potential will cause the axion field to oscillate,

$$\phi_A \approx Av \cos m_A t. \tag{3}$$

We will parameterize the amplitude of these oscillations by the dimensionless variable A. Initially, $A = \theta$ and is of order one. The oscillating ϕ_A field is equivalent to a coherent state of axions at rest. Since we are dealing with large numbers of axions in a coherent state we can use classical field theory as a good approximation. The present value of A in eq. (3) must be extremely small if the axion energy density which is about $m_A^2 v^2 A^2 \approx m_\pi^2 f_\pi^2 A^2$ is to be compatible with cosmology. If we require this energy density to be less than ten times the critical energy density of the universe we find that

$$A_{\text{present}} < 10^{-21}. \tag{4}$$

We will now compute the present value of the amplitude of the axion field oscillations, A, to see if the bound of eq. (4) is satisfied. If we ignore (for the moment) axion interactions, the axion field is governed by the equation of motion

$$\mathrm{d}^2\phi_A/\mathrm{d}t^2 + 3H(t)\mathrm{d}\phi_A/\mathrm{d}t + m_A^2(T)\phi_A = 0, \tag{5}$$

where $H(t)$ is the Hubble "constant" and $m_A(T)$ is the temperature-dependent axion mass. We use the Hubble constant ($H = 1/2t$) and temperature–time relation ($T \propto t^{-1/2}$) of a spacially flat, radiation-dominated cosmology. The axion mass, $m_A(T)$, can be determined from finite-temperature instanton calculations [7]. For three quark flavors (which is the relevant number for the region of interest) we find

$$m_A(T) = 0.04\, \alpha_s^{-3}(\pi^2 T^2)(m_u m_d m_s \Lambda^9)^{1/2} v^{-1} T^{-4}. \tag{6}$$

Eq. (5) is the equation of a damped harmonic oscillator with time-dependent parameters. Until the age of the universe, t, is of order $1/m_A(t)$ the axion field will not have had enough time to oscillate. At a time t_1, where $t_1 = 1/m_A(t_1)$, when the temperature is T_1, the axion field begins to oscillate. The drag term in eq. (5) will cause the amplitude of these oscillations to decrease like $T^{3/2}$. The time variation of the axion mass also causes the oscillation amplitude to decrease. After the time t_1, the relative change in the axion mass is slower than the oscillation frequency, that is $(1/m_A)\mathrm{d}m_A/\mathrm{d}t < m_A$. This means that the mass variation is adiabatic and the quantity $m_A(T)A^2$ will remain a constant since it is an adiabatic invariant [8]. Thus, the amplitude is reduced by the square root of the ratio of the axion mass at temperature T_1 to the final axion mass, m_A of eq. (1). Putting all this together we find that at a temperature T (where $T < \Lambda$)

$$A = [m_A(T_1)/m_A]^{1/2}(T/T_1)^{3/2}, \tag{7}$$

assuming initially $A = \theta = 1$. T_1 and $m_A(T_1)$ can be determined from eq. (6). We find that $T_1 \approx 1$ GeV $(10^{11}\ \text{GeV}/v)^{1/6}$ and

$$A_{\text{present}} \approx 10^{-21}\ (v/10^{12}\ \text{GeV})^{7/12}. \tag{8}$$

For eq. (4) to be satisfied, we must therefore require that

$$v < 10^{12}\ \text{GeV}. \tag{9}$$

This bound is relatively insensitive to the normalization and even to the power of T used in the formula for $m_A(T)$. Uncertainties about the value of Λ and about the effects of chiral symmetry breaking introduce at most a factor of 2–3 uncertainty in the bound.

Up to now we have been ignoring axion couplings. The invisible axion, of course, decouples at temperatures much below v. However, when the axion field is oscillating, we have a huge density of axions in a coherent state and coherence can enhance the effects of axion interactions. Axion couplings to fermions do not lead to coherent effects and so are irrelevant. Likewise, axion couplings to gravitons are too weak. However, axion self-couplings and photon couplings are of interest.

Since the coherent state axions are non-relativistic, their energy density goes down like T^3 as indicated by eq. (7). However, if through self-couplings these axions could become relativistic, their energy density would start going down like T^4 and the present value of A would be much smaller than the result (8). To see if significant amounts of energy can be transferred from the initial zero momentum mode to non-zero momentum modes let us consider the effect of a self-coupling term $\lambda\phi_A^4/4!$ and write the axion field as

$$\phi_A = Av \cos m_A t + \delta A_k(t) \exp(i\boldsymbol{k}\cdot\boldsymbol{x}). \tag{10}$$

Then, for small δA_k, the non-zero momentum mode is governed by the equation (ignoring for the moment the Hubble term)

$$d^2\delta A_k/dt^2 + (k^2 + m_A^2 + \tfrac{1}{2}\lambda A^2 v^2 \cos^2 m_A t)\delta A_k = 0. \tag{11}$$

This is the Mathieu equation and it has the following properties [9]. There are bands of k values for which eq. (11) is unstable and δA_k grows exponentially. The most rapidly growing modes occur for a band of width

$$\Delta k \approx (m_A/4\sqrt{3})(\tfrac{1}{8}\lambda A^2 v^2/m_A^2)^2, \tag{12}$$

which grow at a rate

$$\Gamma \approx \tfrac{1}{16} m_A(\tfrac{1}{8}\lambda A^2 v^2/m_A^2)^2. \tag{13}$$

A given unstable mode will grow as long as it lies in the band of instability, but eventually it will be red-shifted out of this band. The time it takes to be red-shifted out of the unstable band is

$$\tau \approx \Delta k/Hm_A. \tag{14}$$

For the axion field $\lambda \approx m_A^2/v^2$, so,

$$\Gamma\tau \approx (1/64\sqrt{3})(\tfrac{1}{8}A^2)^4(m_A/H). \tag{15}$$

For $v \geqslant 10^{12}$, we find that $\Gamma\tau$ is always much less than one so no appreciable growth takes place in the non-zero momentum modes. Thus, axion self-couplings have no effect on the bound of eq. (9).

The axion lifetime for decays into two photons is longer than the age of the universe for an invisible axion. However, for our coherent state of axions, stimulated emission can lead to an enhancement of this decay mode. The axion photon system can be evaluated exactly like the non-zero momentum modes considered above. In this case, the oscillating axion field couples two polarization modes of the electromagnetic field and the system acts like a parametric amplifier [10]. However, the axion decay into photons is blocked until the temperature reaches about 1 MeV because the plasma frequency $[(4\pi e^2/m_e)n_e]^{1/2}$ is greater than the axion mass. After this, the axion density is too low for significant coherent effects to occur. Thus, our bound is also uneffected by photon couplings of the axion.

In conclusion, axion models which break the $U(1)_{PQ}$ symmetry above 10^{12} GeV result in a present energy density of non-relativistic axions more than ten times greater than the critical energy density. In the introduction, we suggested that the domain wall problem could be avoided in an inflationary cosmology provided that $T_{reheat} < v$. However, our bound would then require that $T_{reheat} < 10^{12}$ GeV. Since the baryon asymmetry must be generated after the inflationary period, it may be difficult to have inflation take place at such a low temperature.

Astrophysical arguments [11] place a lower bound on v of roughly 10^9 GeV [‡1]. Thus, invisible axion models with v between about 10^9 GeV and 10^{12} GeV are still acceptable. For such models, axions are a major component of the present energy density of the universe. (For example, if $v \approx 1.4 \times 10^{11}$ GeV axions would give the universe a critical energy density.) The presence of these axions would clearly have profound implications for astrophysics and cosmology.

Work similar to ours has been carried out by J. Preskill, M. Wise and F. Wilczek. We thank J. Preskill, M. Wise and S. Coleman for helpful discussions. This work was done at SLAC and by P.S. also at the Aspen Center for Physics. We thank both institutions.

Note added. We have recently learned that M. Dine and W. Fischler have also studied this problem.

‡1 This is the value from ref. [11]. Fukugita et al. [12] find a lower bound of 4×10^7 GeV.

References

[1] R.D. Peccei and H. Quinn, Phys. Rev. Lett. 38 (1977) 1440.

[2] S. Weinberg, Phys. Rev. Lett. 40 (1978) 223;
F. Wilczek, Phys. Rev. Lett. 40 (1978) 279.

[3] M. Dine, W. Fischler and M. Srednicki, Phys. Lett. 104B (1981) 199;
J. Kim, Phys. Rev. Lett. 43 (1979) 103;
M. Wise, H. Georgi and S.L. Glashow, Phys. Rev. Lett. 47 (1981) 402;
H.P. Nilles and S. Raby, SLAC-PhB-2743 (1981).

[4] P. Sikivie, Phys. Rev. Lett. 48 (1982) 1156.

[5] G. Lazarides and Q. Shafi, Phys. Lett. 115B (1982) 21;
B. Holdom, Stanford preprints ITP-712 (1982); ITP-713 (1982);
H. Georgi and M. Wise, Phys. Lett. 116B (1982) 123;
S. Dimopoulos, P. Frampton, H. Georgi and M. Wise, Phys. Lett. 117B (1982) 185;
S. Barr, D. Reiss and A. Zee, Phys. Lett. 116B (1982) 227;
S. Barr, X.C. Gao and D. Reiss, University of Washington preprint 40048-22P2 (1982);
Y. Fujimoto, K. Shigemoto and Zhao Zhi-yong, ICTP preprint IC-82-140 (1982).
[6] A. Guth, Phys. Rev. D23 (1981) 347.
[7] D.J. Gross, R.D. Pisarski and L.G. Yaffe, Rev. Mod. Phys. 53 (1981) 43.
[8] L.D. Landau and E.M. Lifshitz, Mechanics (Addison-Wesley, Reading, MA, 1960) pp. 154–157.
[9] N.W. McLachlan, Theory and applications of Mathieu functions (Oxford U.P., London, 1947).
[10] B.R. Mollow and R.J. Glauber, Phys. Rev. 160 (1967) 1076, 1097.
[11] D.A. Dicas, E.W. Kolb, V.L. Teplitz and R.V. Wagoner, Phys. Rev. D18 (1978) 1829.
[12] M. Fukugita, S. Watamura and M. Yoshimura, Phys. Rev. Lett. 48 (1982) 1522.