HOMOGENEOUS TRANSITIONS IN AN INFLATING UNIVERSE ☆

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We consider homogeneous field transitions in a background de Sitter space by reducing the field theory to an equivalent quantum-mechanics problem. These transitions prevent spontaneous symmetry breaking from occurring in de Sitter space. We apply our formalism to the Hawking–Moss transition in the new inflationary cosmology.

1. Introduction. In the inflationary cosmology [1], phase transitions occurring in an approximately de Sitter nature of the universe during the inflationary period can have an important impact on these transitions. For example, transitions can occur which are homogeneous across an entire spacelike surface as first discussed by Hawking and Moss [2]. In this paper, we study homogeneous transitions in a background de Sitter space. We show that they can be analyzed by reducing the four-dimensional field theory to an equivalent one-dimensional quantum-mechanics problem. This problem can be solved by WKB methods or other techniques. One interesting consequence of homogeneous transitions is the absence of spontaneous symmetry breaking in de Sitter space. This is discussed in section 2. In section 3 we apply our methods to the Hawking–Moss transition. The nature of this transition is particularly clear in our formulation. Finally, we analyze an issue of relevance to the new inflationary scenerio [3].

2. Formalism and absence of spontaneous symmetry breaking. Throughout this paper we will work in a de Sitter space with cosmological constant Λ. A convenient coordinate system is one in which the metric takes the form

$$d\tau^2 = dt^2 - L^2\cosh^2(t/L) \times [d\alpha^2 + \sin^2\alpha(d\theta^2 + \sin^2\theta\, d\phi^2)]\,, \tag{2.1}$$

where

$$L = (3/\Lambda)^{1/2}\,. \tag{2.2}$$

We begin by considering a scalar field with the action

$$S = \int d^4x\,\sqrt{-g}\,[\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)]\,. \tag{2.3}$$

To evaluate homogeneous transitions we consider field configurations which are functions of time only

$$\phi = q(t)\,. \tag{2.4}$$

The action for such configurations is

$$S = 2\pi^2L^3\int dt\,\cosh^3(t/L)[\tfrac{1}{2}\dot{q}^2 - V(q)] \tag{2.5}$$

This is just the action for a one-dimensional quantum-mechanical system. Corresponding to the action (2.5) is the Schrödinger equation ‡1

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‡1 We can generalize (2.6) to include gravity as follows. To the lagrangian in (2.3) we add the scalar curvature term, $-(16\pi G)^{-1}R$. Since the transitions are homogeneous [in the coordinates (2.1)] we can take R to be a function of t only. We represent this by generalizing (2.1) to

$$d\tau^2 = dt^2 - \rho^2(t)\,d\Omega^2\,.$$

Then the Schrödinger equation (2.6) is replaced by a similar one, but with two independent variables, q and ρ.

$$(\mathrm{i}/2\pi^2L^3)\partial\psi/\partial t = \{[-1/2(2\pi^2L^3)^2\cosh^3(t/L)]\partial^2/\partial q^2 + \cosh^3(t/L)V(q)\}\psi\,. \quad (2.6)$$

Note that in (2.5) and (2.6) the role of $\hbar$ is played by the quantity $1/2\pi^2L^3$. In the flat-space limit L goes to infinity so the effective $\hbar$ goes to zero. This means that in flat space q behaves classically and, for example, symmetries of $V(q)$ can be spontaneously broken. However, in de Sitter space with L finite, the effective $\hbar$ is not zero and as in any one-dimensional quantum system the ground state is unique and spontaneous symmetry breaking is impossible.

Consider, for example, a potential $V(q)$ with degenerate minima at $q = \pm a$ and a discrete $q \to -q$ symmetry. Classically the symmetry is broken and this result also holds for the quantum theory in flat space. However, in de Sitter space the symmetry is restored by homogeneous transitions. There are two ways that the transition from $\phi = -a$ to $\phi = +a$ can occur. First, because the hamiltonian in eq. (2.6) is time-dependent energy is not conserved and the time dependence can induce hopping over the top of the barrier separating the $q = \pm a$ states. This is the Hawking–Moss type of transition. It will be evaluated for a general potential in the next section. Second, a tunnelling may occur between the $q = \pm a$ states. We will assume that the tunnelling process occurs quickly on the scale of L. Note that the barrier height for the potential in eq. (2.6) goes like $\cosh^3(t/L)$. It is therefore smallest at $t = 0$ and so the tunnelling amplitude is dominated by transitions occurring at $t = 0$. This does not imply any breakdown of the de Sitter symmetries. We have chosen a particular coordinate system and in this system homogeneous transitions occur on the hypersurface $t = 0$. Of course transitions along any other hypersurface which can be obtained from this one by a de Sitter transformation are equally likely.

If we set $t = 0$ in eq. (2.6) we get the simple Schrödinger equation

$$(\mathrm{i}/2\pi^2L^3)\partial\psi/\partial t = \{[-1/2(2\pi^2L^3)^2]\partial^2/\partial q^2 + V(q)\}\psi\,. \quad (2.7)$$

This can be evaluated, for example, by WKB techniques. In this approximation, tunnelling introduces a splitting between the degenerate ground states [‡2]

$$\Delta E = A\exp\left(-2\pi^2L^3\int_{-a}^{a}\mathrm{d}q[2V(q)]^{1/2}\right), \quad (2.8)$$

where A is undetermined since it depends on the effects of the other modes which are not spacially uniform. The unique ground state in this approximation is the symmetric combination $2^{-1/2}(|a\rangle + |-a\rangle)$.

Restoration of continuous symmetries also occurs in de Sitter space. Consider, for example, a complex scalar field with U(1) symmetry and a potential V with a minimum at $|\phi|^2 = a^2$. We can evaluate homogeneous transitions by considering field configurations

$$\phi = a\exp[\mathrm{i}\delta(t)/\sqrt{2}a]\,. \quad (2.9)$$

The action for such configurations is

$$S = 2\pi^2L^3\int\mathrm{d}t\,\cosh^3(t/L)\tfrac{1}{2}\dot{\delta}^2\,. \quad (2.10)$$

Once again for the flat-space case L goes to infinity and (2.10) describes a classical rotor. For finite L, quantum effects are relevant and the ground state is a symmetric S-wave state. Note that in this case there is a gap in the spectrum indicating the absence of a massless Goldstone excitation.

Thus, during the de Sitter phase of an inflationary cosmology spontaneously broken symmetries are restored by homogeneous transitions. When the de Sitter phase ends the universe is left in a symmetric superposition of states like $2^{-1/2}(|a\rangle + |-a\rangle)$ or the S-wave for δ. However, the normal effects of spontaneous symmetry breaking will reappear as soon as the transition amplitudes between classically degenerate states become negligible. For operators which are singlets under the symmetry group it is easy to see that, in the absence of transitions between different vacuum states,

[‡2] This agrees with the result of Coleman and DeLuccia (ref. [4]) in the limit $\epsilon \to 0$ if a factor of two is introduced for application of their formalism to degenerate minima.

matrix elements between a symmetric superposition of states are equal to those between any particular non-symmetric ground state. On the other hand, if a matrix element of a non-symmetric operator is measured the vacuum wavefunctional will immediately collapse into a non-superimposed state which will then be stable.

3. Hawking–Moss transitions. Up to now we have largely been ignoring the time-dependence of the hamiltonian in eq. (2.6). Transitions caused by this time dependence can change the energy (as defined below) of the quantum system and lead to transitions of the type considered by Hawking and Moss [2]. In their language the energy non-conservation of these transitions is due to thermal fluctuations caused by the non-zero Hawking temperature. To evaluate transitions for a time-dependent hamiltonian, we must imagine that the time-dependence is adiabatically turned off in the far past and future. Then the probability for a transition between two states with asymptotic energies E_0 and E_1 induced by a time-dependent hamiltonian is given in the WKB approximation by [5]

$$P = A \exp\left(-\frac{2}{\hbar} \mathrm{Im} \int_0^{t_0} (E_1 - E_0)\, \mathrm{d}t\right), \tag{3.1}$$

where t_0 is the (imaginary) time for which

$$E_1(t_0) = E_0(t_0). \tag{3.2}$$

A is an undetermined constant. Hawking and Moss considered a transition between $q = q_0$ and $q = q_1$ where q_0 is a minimum of V and q_1 the local maximum of V at the top of the barrier. For this case

$$E_0 = \cosh^3(t/L)V(q_0), \quad E_1 = \cosh^3(t/L)V(q_1), \tag{3.3}$$

and

$$\hbar = 1/2\pi^2 L^3.$$

Setting $E_0 = E_1$ we find that $t_0 = \mathrm{i}\frac{1}{2}\pi L$ and

$$P = A \exp\{-\tfrac{8}{3}\pi^2 L^4 [V(q_1) - V(q_0)]\}. \tag{3.4}$$

In performing this calculation we have considered a constant background spacetime and have ignored the effect which the difference $V(q_1) - V(q_0)$ has on the cosmological constant. Thus, our calculation is valid in the limit $8\pi G[V(q_1) - V(q_0)] \ll \Lambda$. In this limit (3.4) agrees with the result of Hawking and Moss.

After the transition to the top of the barrier, $q = q_1$, the transition to the new vacuum state occurs classically. If the classical transition from the top of the barrier to the minimum of the potential is rapid, then in the coordinate system we have been using, the transition looks as in fig. 1A. That is, at a certain time (shown there as $t = 0$) a transition from old vacuum to new vacuum occurs homogeneously over a complete spacelike section. It is interesting to see what this homogeneous transition looks like in the more conventional coordinates in which

$$\mathrm{d}\tau^2 = \mathrm{d}t^2 - \mathrm{e}^{2t/L}(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2). \tag{3.5}$$

This is shown in fig. 1B. In these coordinates the transition looks like the collapse of a bubble of old vacuum. Note that the transition occurs completely inside the event horizon.

An issue of relevance to the new inflationary cosmology [3] is the velocity, $\dot{q}$, when q appears at the top of the barrier, $q = q_1$, after the

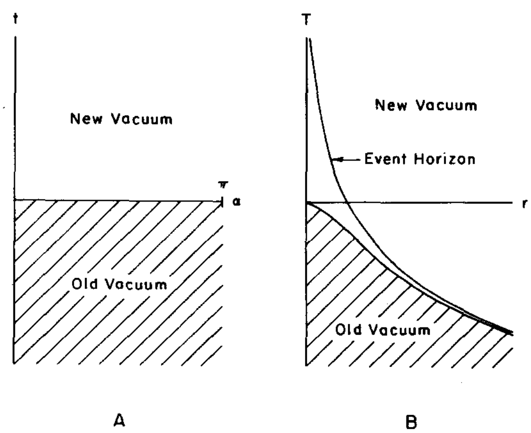

Fig. 1. Appearance of a homogeneous transition occurring at $t = 0$. Fig. 1A shows the transition in the coordinates (2.1) and fig. 1B in the coordinates (3.5). In the latter coordinates the transition looks like the collapse of a bubble of old vacuum inside the event horizon. The shaded area represents old vacuum.

Hawking–Moss transition has occurred. When $q = q_1$ and $\dot{q}$ is non-zero the energy is

$$E_q(t) = \cosh^3(t/L)[\tfrac{1}{2}\dot{q}^2 + V(q_1)] . \qquad (3.6)$$

Thus, according to our formalism, the probability for the system to appear at q_1 with velocity $\bar{q}$ is

$$P(\dot{q}) = A \exp\left(-4\pi^2 L^3 \,\mathrm{Im} \int_0^{t_0} \mathrm{d}t \cosh^3(t/L) \times [\tfrac{1}{2}\dot{q}^2 + V(q_1) - V(q_0)]\right) , \qquad (3.7)$$

where t_0 is the imaginary time for which

$$\cosh^3(t/L)[\tfrac{1}{2}\dot{q}^2 + V(q_1)] = \cosh^3(t/L)\, V(q_0) . \qquad (3.8)$$

Thus, once again, $t_0 = \mathrm{i}\tfrac{1}{2}\pi L$. The expectation value of $|\dot{q}|$ is then given by

$$\langle|\dot{q}|\rangle = \frac{\int \mathrm{d}\dot{q}\, |\dot{q}|\, P(\dot{q})}{\int \mathrm{d}\dot{q}\, P(\dot{q})} = (3/4\pi^3)^{1/2} L^{-2} . \qquad (3.9)$$

In terms of the Hawking temperature, $T_H = 1/2\pi L$, this is

$$\langle|\dot{q}|\rangle = (12\pi)^{1/2} T_H^2 . \qquad (3.10)$$

This is of the order of magnitude of what had previously been guessed on dimensional grounds [6], although we find the rather large factor $(12\pi)^{1/2}$. However, in the classical equations of motion for $q(t)$ it is the ratio of $\langle|\dot{q}|\rangle$ to $(3/L)^2$ that is relevant and this is still small. For a Coleman–Weinberg potential we find that (3.10) is small enough so that the amount of inflation is not significantly reduced by this non-zero initial velocity.

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