A MECHANISM FOR REDUCING THE VALUE OF THE COSMOLOGICAL CONSTANT

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A mechanism is presented for relaxing an initially large, positive cosmological constant to a value near zero. This is done by introducing a scalar field whose vacuum energy compensates for the initial cosmological constant. The compensating sector involves small mass scales but no unnatural fine-tuning of parameters. It is not clear how to incorporate this mechanism into a realistic cosmology.

The extremely small observational limits on the value of the cosmological constant indicate that the vacuum energy density in our universe has magnitude less than $(0.003\ \mathrm{eV})^4$. The vacuum energy density receives contributions proportional to the fourth power of virtually every mass scale in particle physics. Since each of these terms individually is many orders of magnitude larger than $(0.003\ \mathrm{eV})^4$, mysterious and unnatural cancellations must occur in order for their sum to produce a sufficiently small total energy density. This situation is very different from that of a naturally small mass parameter like the electron mass. The mass of the electron is also small compared to most other scales in particle physics but, because of a chiral symmetry, corrections to m_e are always proportional to m_e itself and thus are small for any reasonable cut-off value. Although we cannot claim to know why the electron is so light, the fact that we have a sensible low-energy effective theory in which the value of m_e does not require miraculous cancellations suggests that there may be hope of achieving a better understanding in the future using a more complete theory. In the case of the cosmological constant there is little reason for similar optimism as long as the low-energy theory requires unnatural cancellations.

The fact that the cosmological constant requires cancellations at the level of thousandths of an electron volt suggests that modifications must be made in particle physics at very low energies. An attractive possibility is the existence of a compensating field whose vacuum energy dynamically adjusts itself to cancel the large contribution coming from conventional particle physics. Any model of this type is likely to involve small mass parameters associated with the compensating field theory sector and we must require of any sensible model that these parameters be naturally small. Otherwise we are just replacing one unnaturally small mass parameter, the cosmological constant, with another. In addition, if this idea is to work it seems that the compensating sector must have a stable or metastable state at virtually every value of its own vacuum energy density in order that an arbitrary particle physics contribution can be cancelled. Also, a mechanism must exist for insuring that the compensating sector will evolve to a state with an acceptably small value of the total energy density. Finally, it must be possible to incorporate such a mechanism into a realistic cosmology.

In this note, I present a model constructed along these lines. A compensating sector is introduced which can dynamically reduce any initially large, positive cosmological constant to a value arbitrarily close to zero. The model is a low-energy effective field theory. No attempt is made to incorporate it into a complete high-energy theory. The compensating sector has very small mass parameters associated with it, but these are protected by symmetries from getting large radiative

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corrections and therefore they are natural. No fine-tuning is required to keep them small. The model provides evidence that this type of approach can work. However, it is not clear how the present mechanism can be incorporated into a realistic cosmology.

The model being discussed consists of a scalar field, B, coupled to a gauge theory. The entire compensating sector is only coupled to ordinary particles through gravity and therefore is only detectable through its gravitational effects. For example, all ordinary particles are singlets under the gauge group of the gauge theory in the compensating sector. For this reason I call it a phantom gauge theory. The lambda parameter which characterizes the coupling strength of the phantom gauge theory, Λ_{ph} is extremely small – less than 10^{-34} eV. This may seem like an extraordinarily small value, but actually it is quite natural for an isolated gauge theory to have a lambda parameter which is vastly different from Λ_{QCD} which sets the scale for hadronic masses in our world. If we characterize a gauge theory by the value of its coupling constant at the Planck mass for example, then $\alpha_{QCD}(m_{Pl}) \approx 0.02$, corresponding to $\Lambda_{QCD} \approx 100$ MeV. If our phantom gauge theory is SU(3) with phantom quarks like QCD then by requiring that $\alpha_{ph}(m_{Pl}) \leqslant 0.006$ we find that $\Lambda_{ph} \leqslant 10^{-34}$ eV. If the phantom theory is SU(2) with six flavors of phantom quarks then $\alpha_{ph}(m_{Pl}) \leqslant 0.01$ assures that $\Lambda_{ph} \leqslant 10^{-34}$ eV. Thus, with quite conventional values of the coupling constant we find extremely small values of the lambda parameter for the phantom gauge theory. A small Λ_{ph} is crucial for achieving a sufficiently small final cosmological constant in this model.

The couplings of the scalar field, B, are restricted by the symmetry

$$B \to B + \text{constant}. \tag{1}$$

This symmetry may suggest that B is a Goldstone boson. However, I will assume that the range of B goes from minus infinity to plus infinity and that the lagrangian does not have to be periodic in B. Thus, B is not a Goldstone boson associated with a compact symmetry group like U(1). It could conceivably be a dilaton, or a field associated with one of the flat directions of the potential in a supersymmetric theory. For the purposes of the present discussion it does not matter where B comes from, as long as it possesses the symmetry $B \to B + \text{constant}$.

The symmetry of eq. (1) is softly broken in two ways to achieve a non-trivial potential for the B field. First B, is coupled to the phantom gauge theory through the term

$$L_{int} = (\alpha_{ph}/4\pi)(B/f_B)\,\epsilon^{\mu\nu\alpha\beta}\,\mathrm{Tr}\{F_{\mu\nu}F_{\alpha\beta}\}\,, \tag{2}$$

where $F_{\mu\nu}$ is the phantom gauge field strength tensor. The parameter f_B is a large mass (perhaps of order m_{Pl}) associated with the complete high-energy theory. Since $\mathrm{Tr}\{F_{\mu\nu}F_{\alpha\beta}\}\,\epsilon^{\mu\nu\alpha\beta}$ is a total derivative this term respects the symmetry $B \to B + \text{constant}$ up to surface terms. However, instantons contribute to these surface terms and softly break the symmetry. This is exactly the type of coupling used in axion models to break the Peccei–Quinn symmetry [1]. It is well known that the coupling (2) leads to a potential for the B field of the form [1]

$$V_1(B) = -\Lambda_{ph}^4 \cos(B/f_B). \tag{3}$$

The gauge coupling of the B field breaks the symmetry $B \to B + \text{constant}$ but still preserves the symmetries

$$B \to B + 2\pi f_B \quad \text{and}\ B \to -B. \tag{4}$$

These are broken, again softly, by introducing a term

$$V_2(B) = \epsilon B/f_B \tag{5}$$

into the potential for B. The linear form in (5) is not essential but is chosen for simplicity. All that is required is a potential which has no minima over the range of B discussed below. The parameter ϵ is assumed to be less than Λ_{ph}^4 but is otherwise arbitrary. It is a naturally small parameter because its non-zero value breaks the symmetries of eq. (4). Since it breaks a symmetry all radiative corrections to the value of ϵ must be proportional to ϵ. Thus no fine-tuning is required to maintain its small value.

When this compensating sector is added to a standard particle physics model, the total vacuum energy is given by

$$V = \epsilon B/f_B - \Lambda_{ph}^4 \cos(B/f_B) + V_0, \tag{6}$$

where V_0 represents the vacuum energy density of all the fields other than B. For $\epsilon \ll \Lambda_{ph}^4$ this potential has local minima at

$$B \approx 2\pi N f_B \tag{7}$$

for integer N with energy densities

$$V_N \approx 2\pi N\epsilon - \Lambda_{\rm ph}^4 + V_0. \tag{8}$$

Since $\epsilon < (10^{-34}\ \mathrm{eV})^4$ it is always possible to find values of N for which eq. (8) gives a sufficiently small vacuum energy. In fact, for any value of V_0 there exists a state with energy density less than or equal to $2\pi\epsilon$ which can be made arbitrarily small by adjusting the value of ϵ.

Although the potential of eq. (7) always has metastable minima with acceptably small vacuum energies, to account for the small value of the cosmological constant we must explain why the universe is in one of the states with $V_N < (0.003\ \mathrm{eV})^4$ instead of being in some other metastable state. Suppose that at some initial time the universe is in a state with a large, positive cosmological constant. Thus, initially we are in a de Sitter spacetime. The local minima of the potential (6) are unstable because of the linear term. Therefore, as time passes the B field will descend down the linear slope stepping from one local minima to the next, continually decreasing the vacuum energy density. At first, when the vacuum energy density, V_N, is greater than $m_{\rm Pl}^2\Lambda_{\rm ph}^2$ the barriers separating the local minima in the potential (6) have no effect and the descent of B to lower energies is relatively rapid. There are two reasons that the barriers produced by the cosine term in the potential are irrelevant for $V_N > m_{\rm Pl}^2\Lambda_{\rm ph}^2$. First, the de Sitter space metric is time-dependent and, as a result, violations of energy conservation of order $\Delta E = (V_N/m_{\rm Pl}^2)^{1/2}$ occur due to the time dependence of the hamiltonian [2]. One can think of these violations as being caused by the non-zero Hawking temperature $T_{\rm H} = (2V_N/3\pi m_{\rm Pl}^2)^{1/2}$ in de Sitter space [3]. For $V_N > m_{\rm Pl}^2\Lambda_{\rm ph}^2$ the energy fluctuations are sufficient to make the B field hop over the barriers between successive minima of the potential. The second reason that the barriers have no effect initially is that for $T_{\rm H} > \Lambda_{\rm ph}$ instantons of the phantom gauge theory which produce the cosine term in the potential are suppressed by finite temperature effects [4]. For both reasons the B field descends down the slope unimpeded by the cosine barriers.

Eventually, by descending down the linear potential the B field will reduce the vacuum energy density to an acceptably small value,

$$V_N < m_{\rm Pl}^2\Lambda_{\rm ph}^2 \leqslant (0.001\ \mathrm{eV})^4. \tag{9}$$

At this point the cosine barriers between successive minima of the potential do become relevant. The B field cannot roll over these barriers because it is overdamped for $V_N > (m_{\rm Pl}^2/6\pi f_B^2)\Lambda_{\rm ph}^4$. It therefore must tunnel from one minimum to the next. At first, when $V_N \approx m_{\rm Pl}^2\Lambda_{\rm ph}^2$ this tunnelling is not significantly suppressed and the B field proceeds downward at essentially the same rate as it did before. However, once we get to states with $V_N \ll m_{\rm Pl}^2\Lambda_{\rm ph}^2$ the tunnelling process is highly suppressed and further downward progress is extremely slow. For $V_N \ll m_{\rm Pl}^2\Lambda_{\rm ph}^2$ the tunnelling rate per unit volume is [5–7]

$$\Gamma/V \approx \Lambda_{\rm ph}^4 \exp(-\tfrac{3}{8}m_{\rm Pl}^4/V_N). \tag{10}$$

Thus, eventually we get to states with acceptably small values of the cosmological constant [see eq. (9)] and enormously long lifetimes.

The relaxation process outlined above is quite slow. It takes at least 10^{450} y for an initial vacuum energy density of order $m_{\rm Pl}^4$ to get reduced to a value less than $(0.003\ \mathrm{eV})^4$. However, once this occurs a region the size of our observable universe will remain in a series of states with acceptably small cosmological constants for at least $10^{10^{248}}$ y. This enormously long time is due to the exponential tunnelling suppression in eq. (10). During this period tunnelling between neighboring minima of the potential occurs occasionally, but this process is extremely gentle. Converting the entire observed universe from one vacuum to the next releases only 10^{-37} eV of energy.

Unfortunately, the model discussed here arrives at states with acceptable values of the cosmological constant without ever having produced any matter with which to fill the relatively flat spacetime it has formed. Thus, as it stands, the model cannot be incorporated into a realistic cosmology. The model seems to require the spontaneous production of a non-zero matter density by some rare and presently unknown process.

The potential of eq. (6) is of course unstable and it is interesting to ask what happens after the $10^{10^{248}}$ y period during which the cosmological constant is small but positive. Clearly, the B field will eventually tunnel to a state with small negative vacuum energy. This state has two very interesting properties. First, if we could produce a state in which B was exactly at a local minimum of the potential with negative vacuum energy then this state would be stable despite the apparent instability of the potential (6). It has been shown [6, 7] that the rate for tunnelling out of such a state vanishes in an anti-de Sitter spacetime which is the spacetime for a negative value of the vacuum energy. Furthermore, a positive energy theorem can be proven for this state using the techniques of ref. [8]. However, if we produce this negative energy density state by tunnelling down from a positive energy state, which is what occurs here, then the value of B in the interior of the bubble produced during the transition is not exactly the value at the local minimum of the potential, but is slightly displaced. This situation has been analyzed in detail by Coleman and DeLuccia [6]. It seems likely that the interior of the bubble suffers a gravitational collapse in a time of order $(m_{\rm Pl}^2/2\pi\epsilon)^{1/2}$. Thus, after around $10^{10^{248}}$ y, the ultimate fate of the universe in this model is gravitational collapse.

Several issues requiring further study are raised by this work. The most important of these is to find a mechanism for introducing ordinary matter into the universe sometime during the long period during which the cosmological constant is acceptably small. Clearly this is essential if the model is to be taken seriously. It is also important to construct a more complete high-energy theory which can produce the type of non-compact Goldstone boson used here. Although it is not cosmologically realistic, the model presented here does at least demonstrate that a low-energy effective theory with a compensating sector which reduces the cosmological constant to an acceptably small value without unnatural fine-tuning of parameters can be constructed.

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References

[1] R. Peccei and H. Quinn, Phys. Rev. Lett. 38 (1977) 1440.
[2] L. Abbott and C. Burges, Phys. Lett. 131B (1983) 49.
[3] G. Gibbons and S. Hawking, Phys. Rev. D15 (1977) 2738.
[4] D. Gross, R. Pisarski and L. Yaffe, Rev. Mod. Phys. 53 (1981) 43.
[5] S. Hawking and I. Moss, Phys. Lett. 110B (1982) 35.
[6] S. Coleman and F. DeLuccia, Phys. Rev. D21 (1980) 3305.
[7] S. Parke, Phys. Lett. 121B (1983) 313;
P. Steinhardt, unpublished.
[8] W. Boucher, Nucl. Phys. B242 (1984) 282.