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Supercurrent anomaly in a supersymmetric gauge theory

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It is demonstrated that an anomaly exists for the supercurrent in a supersymmetric non-Abelian gauge theory. Since no gauge particle is coupled to the current, the anomaly does not alter the renormalizability of the theory. Some consequences of the anomaly are discussed.

I. INTRODUCTION

Since the initial work of Wess and Zumino,[1] one has witnessed an extensive development of the framework of supersymmetric theories.[2] The subject has taken on additional interest with the enlargement of these ideas to those of a local supersymmetry, which then leads to models of supergravity[3] and extended supergravity.[4] These theories seem to offer a possible link of quantum gravity to elementary-particle physics, as well as having some promise of being renormalizable theories of quantum gravity.[5]

However, the novel feature of supersymmetry, its boson-fermion symmetry, also poses one of the important problems of the subject. Since Bose-Fermi symmetry is not observed in nature, the symmetry must be broken if supersymmetry is to make contact with the physical world. One possibility is that spontaneous symmetry breaking occurs in the tree approximation, induced by appropriate interactions of scalar multiplets. In particular this leads to zero-mass Goldstone fermions,[6] which, however, cannot be identified with the neutrino.[7] The implementation of these ideas involves the Ward identities of the theory,[8] which connect the Green's functions of the supercurrent to other matrix elements. In order for these formal Ward identities to be valid, one must be assured that no anomalies of the supercurrent exist.

For the specific supersymmetric[8,9] model involving a zero-mass Yang-Mills multiplet interacting with a single massless Majorana spin-$\frac{1}{2}$ field, transforming as the adjoint representation of the internal-symmetry group, there is a connection to the issue of chiral anomalies. In this model, the axial-vector current, the supercurrent, and the stress-energy tensor are members of a supersymmetric representation, i.e., they transform among themselves under constant supersymmetry transformations. It is known that the axial-vector current has an anomaly, which suggests that the supercurrent is also anomalous. (We will show that one *cannot* derive the anomaly of the supercurrent from a supersymmetry transformation of the axial-vector current anomaly.)

The purpose of this paper is to present a diagrammatic evaluation of the anomaly of the supercurrent in this model. For completeness we present the model in Sec. II, with the anomaly evaluated in Sec. III. Some consequences of this result are considered in Sec. IV, and the conclusions are presented in Sec. V.

II. THE MODEL

Consider the SU(2) gauge-invariant Lagrangian density[9]

$$\mathcal{L} = -\tfrac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \tfrac{1}{2} i \bar{\psi}^a \gamma^\mu (D_\mu \psi)^a , \quad (2.1)$$

where the Yang-Mills field strength

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon_{abc} A^b_\mu A^c_\nu , \quad (2.2)$$

with $a = 1, 2, 3$, and ψ^a an $I = 1$ Majorana spinor. The covariant derivative is

$$(D_\mu \psi)^a = \partial_\mu \psi^a + g\epsilon_{abc} A^b_\mu \psi^c . \quad (2.3)$$

The action of this theory is invariant under the supersymmetry transformation

$$\delta A^a_\mu = i\bar{\epsilon}\gamma_\mu \psi^a \quad (2.4a)$$

and

$$\delta\psi^a = \sigma^{\mu\nu} \epsilon F^a_{\mu\nu} , \quad (2.4b)$$

in that the Lagrangian density (2.1) changes by a total derivative. Throughout this work, ϵ is a space-time-independent anticommuting Majorana spinor and $\sigma_{\mu\nu} = \frac{1}{4}[\gamma_\mu, \gamma_\nu]$. The Noether current[8,9] which generates the transformation (2.4) is $\mathcal{S}_\mu$, with

$$\begin{aligned} \bar{\mathcal{S}}_\mu \epsilon = \bar{\epsilon}\mathcal{S}_\mu &= -i(\bar{\psi}_a \gamma_\mu \sigma^{\alpha\beta}\epsilon) F^a_{\alpha\beta} \\ &= -i(\bar{\epsilon}\sigma^{\alpha\beta}\gamma_\mu \psi_a) F^a_{\alpha\beta} , \end{aligned} \quad (2.5)$$

where the two forms of (2.5) are equivalent by the Majorana condition for ϵ and $\psi(x)$. [In principle, one should add a scalar field D^a to (2.1), but it plays no role in our work.]

de Wit and Freedman[8] have made a detailed study of the Ward-Takahashi identities for the Green's functions of the supercurrent $\mathcal{S}_\mu(x)$. The current is a gauge-invariant operator, because the supersymmetry transformations have a gauge-covariant action on the fields of the Lagrangian. It was shown[8] that, although $\mathcal{S}_\mu(x)$ is not strictly conserved because of the technicalities of gauge-field quantization, it is *formally* conserved in matrix elements between physical states. That is, one can show by formal arguments that

$$\partial_\mu \langle \text{phys} | \mathcal{S}^\mu(x) | \text{phys} \rangle = 0 \qquad (2.6)$$

and $\gamma_\mu \mathcal{S}^\mu = 0$.

Of course these formal arguments do not rule out an anomaly for $\mathcal{S}_\mu(x)$, but (2.6) emphasizes that in order to establish the existence of an anomaly, one must compute the matrix elements of $\mathcal{S}_\mu(x)$, restricted to *physical* states.

One may also add scalar multiplets to the model,[8,9] and arrange the interactions so that supersymmetry is spontaneously broken in the tree approximation, leading to a modification of the Ward identities due to the presence of Goldstone fermions. We will omit scalar multiplets from the theory, and confine our attention to the Lagrangian (2.1).

III. THE ANOMALY

We are led to search for (mass-shell) anomalies of the supercurrent $\mathcal{S}_\mu(x)$, as specified by Eq. (2.5) and the Lagrangian (2.1). Consider the process

$$\mathcal{S}_\mu \to \psi + A \; ,$$

with both the fermion and the boson on-shell. The relevant one-loop diagrams for this transition are shown in Figs. 1 and 2, omitting the trivial mass renormalization and wave-funtion renormalizations, which play no role at the one-loop level because of our mass-shell constraint. Each of the diagrams in Figs. 1 and 2 is primitively linearly divergent, so that the existence of an anomaly is a possibility.[10]

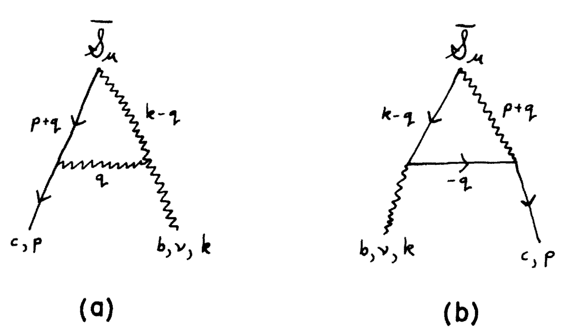

FIG. 1. Triangle graphs contributing to the process $\bar{\mathcal{S}}_\mu \to \psi + A$.

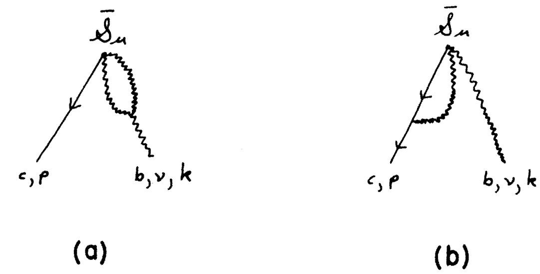

FIG. 2. Bubble graphs contributing to the process $\bar{\mathcal{S}}_\mu \to \psi + A$.

Since the diagrams of Figs. 1 and 2 are linearly divergent, the evaluation of the Feynman integrals is ambiguous due to the presence of surface terms which depend on the particular routing of momenta through the diagrams. This ambiguity may be removed by the imposition of gauge invariance on the external gauge-boson line. We cannot define these divergent integrals by means of a regulator scheme, since this may introduce explicit violations of supersymmetry not related to the anomaly. Rather, we follow a method used by Adler[10] in order to avoid similar problems in the axial-vector anomaly. Our strategy is as follows:

1. Write the matrix element for $\mathcal{S}_\mu \to \psi + A$ in terms of all possible on-shell invariant amplitudes, and relate the amplitudes by imposing gauge invariance, and $\mathcal{S}^\mu \gamma_\mu = 0$ which maintains the spin-$\frac{3}{2}$ character of the supercurrent.
2. Express $\partial_\mu \mathcal{S}^\mu \to \psi + A$ in terms of the same invariant amplitudes, with the same mass-shell constraints imposed.
3. Evaluate the *finite parts* of the four diagrams of Figs. 1 and 2 in the Feynman-'t Hooft gauge.
4. By dimensional arguments, the finite parts are both first-rank and third-rank tensors in the external momenta.
5. The finite parts which are first-rank tensors in the external momenta are ambiguous, since they have contributions from the routing-dependent surface terms. (See Appendix A.)
6. The finite parts which are third-rank tensors in the external momenta are uniquely defined by the Feynman integrals, and are independent of the routing of momenta. It is crucial to note that in the Feynman-'t Hooft gauge these terms come from the diagrams of Fig. 1 *only*. In this gauge, Fig. 2 plays no further role in our considerations.
7. Express invariants in terms of the unambiguous finite amplitudes, which are third-rank in momenta, as found from the Feynman diagrams.
8. Compute $\partial_\mu \mathcal{S}_\mu \to \psi + A$ on the mass-shell to determine the anomaly.
9. Express the result in operator form.

10. Reexpress the operator form of the anomaly as a total divergence, valid on-shell.

We have presented this detailed outline of our methodology in order to ensure that the steps leading to our result are clear.

We define the amplitude for $\mathcal{S}_\mu \to \psi + A$ as

$$\epsilon_\nu^*(k)\bar{u}(p)S_{\mu\nu}^{bc} = \epsilon_\nu^*(k)\bar{u}(p)(R_{\mu\nu}^{bc} + T_{\mu\nu}^{bc}) , \quad (3.1)$$

where p (k) is the fermion (boson) momentum, and the boson and fermion wave functions are $\epsilon_\nu(k)$ and $u(p)$. The amplitude is divided into two parts, $R_{\mu\nu}$ and $T_{\mu\nu}$, which come from Figs. 1 and 2, respectively. The mass-shell restrictions are

$$\epsilon^\nu(k)k_\nu = 0 ,$$
$$\bar{u}(p)\gamma\cdot p = 0 , \quad (3.2)$$
$$k^2 = p^2 = 0 .$$

From the Feynman rules of the theory, and the structure of the diagrams in Figs. 1 and 2, one observes that $S_{\mu\nu}$ has an odd number of γ matrices, a fact which is useful in constructing this amplitude. The form of the current also gives an explicit factor of $\gamma_\mu \sigma^{\alpha\beta}$ at the upper vertex of each diagram.

The most general form for $S_{\mu\nu}^{bc}$ on the mass-shell is therefore

$$iS_{\mu\nu}^{bc} = \delta_{bc} S_{\alpha\beta\nu}\gamma_\mu \sigma^{\alpha\beta} \quad (3.3a)$$

$$= \delta_{bc}[A_0 \gamma\cdot k(\gamma_\alpha g_{\beta\nu} - \gamma_\beta g_{\alpha\nu}) + A_1(p_\alpha g_{\beta\nu} - p_\beta g_{\alpha\nu}) + A_2\gamma_\nu(p_\alpha\gamma_\beta - p_\beta\gamma_\alpha) + A_3\gamma_\nu(k_\alpha\gamma_\beta - k_\beta\gamma_\alpha)$$
$$+ A_4\gamma\cdot k\gamma_\nu(p_\alpha k_\beta - p_\beta k_\alpha) + A_5\gamma\cdot k p_\nu(k_\alpha\gamma_\beta - k_\beta\gamma_\alpha) + A_6\gamma\cdot k p_\nu(p_\alpha\gamma_\beta - p_\beta\gamma_\alpha) + A_7 p_\nu(p_\alpha k_\beta - p_\beta k_\alpha)$$
$$+ A_8(k_\alpha g_{\beta\nu} - k_\beta g_{\alpha\nu})]\gamma_\mu\sigma^{\alpha\beta} . \quad (3.3b)$$

Gauge invariance requires

$$k^\nu S_{\alpha\beta\nu} = 0 , \quad (3.4)$$

which implies

$$A_0 - A_3 = p\cdot k A_5 , \quad (3.5a)$$

$$A_1 = -p\cdot k A_7 , \quad (3.5b)$$

$$A_2 = -p\cdot k A_6 . \quad (3.5c)$$

Notice that the constraint $\mathcal{S}^\mu\gamma_\mu = 0$ is automatically satisfied, since $\gamma_\mu\sigma^{\alpha\beta}\gamma^\mu = 0$, and provides no additional information. The invariant amplitudes A_4, A_5, A_6, and A_7 multiply third-rank tensors in momenta, and hence are *uniquely* determined by the Feynman integrals.

The anomaly is obtained from

$$S_\nu = i(p+k)^\mu S_{\alpha\beta\nu}\gamma_\mu\sigma^{\alpha\beta} \quad (3.6)$$

evaluated on-shell. Combining (3.3) with (3.6), one finds

$$-iS_\nu = p\cdot k\gamma_\nu[-2(A_0 - A_3) + 2A_1 + 4A_2 + 2p\cdot k A_4]$$
$$-\gamma\cdot k p_\nu[2(A_0 - A_3) + A_1 + 2A_2 + 2p\cdot k A_4$$
$$-4p\cdot k A_5 - 2p\cdot k A_6 - p\cdot k A_7] . \quad (3.7)$$

This can be simplified by using (3.5) to obtain

$$-iS_\nu = 2p\cdot k(p\cdot k\gamma_\nu - \gamma\cdot k p_\nu)(A_4 - A_5 - 2A_6 - A_7) , \quad (3.8)$$

which makes it evident that the anomaly is *uniquely* determined by amplitudes A_4 to A_7, which are in fact unambiguously calculable from the Feynman amplitudes. Therefore, our calculation of the anomaly will be unique.

Let us proceed to the necessary Feynman integrals. It is straightforward to show that in the Feynman–'t Hooft gauge, the diagrams of Fig. 1 can be represented as

$$iR_{\mu\nu}^{bc} = -4g^2\delta_{bc}\int \frac{d^4q}{(2\pi)^4}\frac{N_{\alpha\beta\nu}\gamma_\mu\sigma^{\alpha\beta}}{(p+q)^2q^2(k-q)^2}$$
$$= -8g^2\delta_{bc}\int_0^1 dx\int_0^{1-x} dy\int \frac{d^4q}{(2\pi)^4}\frac{\bar{N}_{\alpha\beta\nu}\gamma_\mu\sigma^{\alpha\beta}}{(q^2+M^2)^3}$$
$$+ \text{surface terms} , \quad (3.9b)$$

where

$$M^2 = p^2x(1-x) + k^2y(1-y) + 2p\cdot kxy \quad (3.10a)$$

$$\xrightarrow{\text{on-shell}} 2p\cdot kxy . \quad (3.10b)$$

We have translated the origin of the momentum-space integration, hence the distinction between $N_{\alpha\beta\nu}$ and $\bar{N}_{\alpha\beta\nu}$, and the appearance of surface terms. (A discussion of the surface terms appears in Appendix A.)

The actual expressions for $N_{\alpha\beta\nu}$ and $\bar{N}_{\alpha\beta\nu}$ are somewhat lengthy. However, if we use the mass-shell restrictions (3.2), then $\bar{N}_{\alpha\beta\nu}$ simplifies considerably. At this stage, $\bar{N}_{\alpha\beta\nu}$ contains terms which contribute to both the divergent and the finite part of $R_{\mu\nu}^{bc}$. As argued above, only that part of $\bar{N}_{\alpha\beta\nu}$ which is a third-rank tensor in the external momenta contributes to the unambiguous part of the Feynman integral. Therefore, with these considerations, the use of (3.2), and some

γ algebra, we have the result

$$\bar{N}_{\alpha\beta\nu} = xy[2\gamma\cdot k\gamma_\nu(p_\alpha k_\beta - p_\beta k_\alpha) + \gamma\cdot kp_\nu(k_\alpha\gamma_\beta - k_\beta\gamma_\alpha) + \gamma\cdot kp_\nu(p_\alpha\gamma_\beta - p_\beta\gamma_\alpha) - 4p_\nu(p_\alpha k_\beta - p_\beta k_\alpha)] + \text{lower-rank tensors.} \quad (3.11)$$

Notice that the antisymmetry ($\alpha \leftrightarrow \beta$) is explicit. It is clear that Eq. (3.11) determines the invariant amplitudes A_4 to A_7, as required for our computation of the anomaly. That is,

$$A_i = -8g^2 \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4q}{(2\pi)^4} \frac{B_i}{(q^2+M^2)^3} \quad \text{for } i = 4 \text{ to } 7, \quad (3.12)$$

with

$$B_4 = 2xy, \quad B_5 = xy, \quad B_6 = xy, \quad B_7 = -4xy, \quad (3.13)$$

and M^2 given by (3.10). We combine (3.12) and (3.13) with (3.7) to obtain[11]

$$-iS_\nu = -8g^2 \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4q}{(2\pi)^4} \frac{[6xyp\cdot k(\gamma_\nu p\cdot k - p_\nu\gamma\cdot k)]}{[q^2+2xyp\cdot k]^3} = -\frac{3ig^2}{8\pi^2}(\gamma_\nu p\cdot k - p_\nu\gamma\cdot k), \quad (3.14)$$

which demonstrates the existence of the anomaly. Thus, we have found that

$$\langle p, c; k, b|\partial_\mu \bar{S}^\mu|0\rangle = -i\epsilon_\nu^*(k)\bar{u}(p)S_\nu\delta_{bc} \quad (3.15)$$

with S_ν given by (3.14).

IV. IMPLICATIONS

An operator form of the anomaly is

$$\partial_\mu S^\mu = -\frac{3ig^2}{8\pi^2}(\partial_\alpha A_\beta^a - \partial_\beta A_\alpha^a)(\gamma^\beta\partial_\alpha\psi_a) + \text{other terms}, \quad (4.1)$$

which reproduces (3.14) and (3.15), where the "other terms" make no contribution to the process $\partial_\mu S^\mu \to \psi A$ on-shell. [We have used the Majorana condition to write (4.1) in terms of ψ rather than $\bar{\psi}$.] Unfortunately, the additional terms in Eq. (4.1) are not uniquely specified, and are dependent on further considerations, as we shall discuss below. Notice that the supercurrent does not couple to any fields in the Lagrangian, and hence is an "alien" operator of this theory. Therefore the presence of the anomaly does not alter the Feynman rules or the renormalizability of the theory.

One may conjecture that the complete supercurrent anomaly can be written as a total derivative, in analogy with a similar result for the axial-vector anomaly.[10] To verify this conjecture would require a study of off-shell amplitudes, which then involves the complicated gauge-dependent modifications of the formal supercurrent Ward identities discussed by de Wit and Freedman.[8] Since such an exercise is outside the scope of this paper, we will proceed by accepting the conjecture and exposing its consequences.

If the supercurrent anomaly can be written as a total divergence, then it is possible to define a conserved supercurrent which satisfies both gauge invariance and the spin-$\frac{3}{2}$ constraint. As a result of the anomaly, at least one of these constraints must be abandoned. This is a situation reminiscent of the conflict between chiral conservation and gauge invariance which occurs when there is an axial-vector anomaly.[10]

We have shown that a particular matrix element of the supercurrent is not conserved. Modified supercurrents can be defined whose one-boson–one-fermion matrix element is conserved. A gauge-invariant conserved current is

$$i\hat{S}_\mu = F_{\alpha\beta}^a(\sigma^{\alpha\beta}\gamma_\mu\psi_a) - \frac{3g^2}{8\pi^2}\tilde{F}_{\mu\nu}^a(\gamma^5\gamma^\nu\psi_a), \quad (4.2)$$

with $\tilde{F}_{\mu\nu}^a = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}^a$. This current satisfies (4.1) and

$$\partial^\mu\langle\text{phys}|\hat{S}_\mu|\text{phys}\rangle = 0, \quad (4.3)$$

but

$$i\gamma^\mu\hat{S}_\mu = \frac{3g^2}{4\pi^2}\tilde{F}_{\mu\nu}^a(\gamma^5\sigma^{\mu\nu}\psi_a) \neq 0, \quad (4.4)$$

which illustrates the conflict between gauge invariance and the spin-$\frac{3}{2}$ contraint. [Note that (4.2) implies that there is a contribution of the anomaly to $S^\mu \to \psi AA$.]

Modified supercurrents can be defined whose one-boson–one-fermion matrix element is conserved, and which satisfy the spin-$\frac{3}{2}$ constraint in this sector. One such definition is

$$i\tilde{S}_\mu = F_{\alpha\beta}^a(\sigma^{\alpha\beta}\gamma_\mu\psi_a) - \frac{3g^2}{8\pi^2}[A_\lambda^a(\gamma^\mu\partial^\lambda\psi_a - \gamma^\lambda\partial^\mu\psi_a) - 3A_\mu^a(\gamma_\lambda\partial^\lambda\psi_a) + 2\epsilon_{\mu\nu\lambda\sigma}A_\nu^a(\gamma_5\gamma^\lambda\partial^\sigma\psi_a)] + \text{possible other terms.} \quad (4.5)$$

This operator obeys

$$\partial^\mu \langle 0|\tilde{\mathcal{S}}_\mu|\psi A\rangle = 0 \tag{4.6}$$

as a consequence of (3.14), (3.15), (4.1), and the Dirac equation. It is easy to verify that

$$\langle 0|\gamma_\mu \tilde{\mathcal{S}}^\mu|\psi A\rangle = 0 \tag{4.7}$$

as well. One cannot find a gauge-invariant conserved current which satisfies (4.7), since the field A_μ^a, rather than its derivatives, enters (4.5). Any possible additional terms in (4.5), which refer to other sectors of the theory, should satisfy equations analogous to (4.6) and (4.7) with the appropriate physical states appearing in the matrix elements.

The existence of a conserved operator $\tilde{\mathcal{S}}_\mu$ which satisfies (4.6) and (4.7) between any physical states would allow us to write

$$\langle \text{phys}|[H,\tilde{S}]|\text{phys}\rangle = 0\,, \tag{4.8}$$

where $\tilde{S} = \int d^3x\, \bar{\epsilon}\, \mathcal{S}_0$, $d\tilde{S}/dt = 0$ *between physical states*, and H is the Hamiltonian. Equation (4.8) will preserve all formal S-matrix rules[12] of a supersymmetric theory *if* $\tilde{S}$ generates a supersymmetry transformation. This is a reasonable conjecture, since explicit calculations in low orders of perturbation theory give no indication that global supersymmetry is broken by radiative corrections.[13] In fact, it has been conjectured[14] that if supersymmetry is not broken in the tree approximation, it is not broken in any finite order of perturbation theory. This question requires further study.

One can add scalar multiplets to the model described in Sec. II.[8,9] Explicit scalar couplings are currently being examined for a possible anomaly-eliminating mechanism. These scalar multiplets may be arranged so as to provide a spontaneous breakdown of supersymmetry in the tree approximation. Because of the anomaly, a number of the Ward identities must be modified, but one still infers the presence of a Goldstone fermion which cannot be identified with the neutrino confirming earlier analyses.[7-9] We leave the details to the interested reader.

In this model the axial-vector current j_μ^5, the supercurrent $\mathcal{S}_\mu$, and the stress-energy tensor $T_{\mu\nu}$ are related by a supersymmetry transformation. However,

$$\partial^\mu T_{\mu\nu} = 0, \tag{4.9}$$

$$\partial^\mu j_\mu^5 = \frac{g^2}{8\pi^2} F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a\,. \tag{4.10}$$

Recall that we conjectured that (4.1) can be written as either

$$\partial^\mu \tilde{\mathcal{S}}_\mu = -\frac{3ig^2}{8\pi^2}\partial_\mu [A_\lambda^a(\gamma^\mu\partial^\lambda\psi_a - \gamma^\lambda\partial^\mu\psi_a) - 3A_\mu^a(\gamma_\lambda\partial^\lambda\psi_a) + 2\epsilon_{\mu\nu\lambda\sigma}A_\nu^a(\gamma_5\gamma^\lambda\partial^\sigma\psi_a)] + \text{other terms} \tag{4.11a}$$

or

$$\partial^\mu \mathcal{S}_\mu = -\frac{3ig^2}{8\pi^2}\partial^\mu(\tilde{F}_{\mu\nu}^a\gamma^5\gamma^\nu\psi_a)\,. \tag{4.11b}$$

The two different forms of (4.11) occur because of the conflict between gauge invariance and the spin-$\frac{3}{2}$ constraint. [It is to be understood that (4.11) *is to be evaluated between physical states*. Recall that "other terms" do not contribute to $\partial_\mu \mathcal{S}^\mu \to \psi A$.] Although the *functional form* of Eq. (4.11b) can be derived from a supersymmetry transformation of the axial-vector anomaly (4.10), the numerical coefficient cannot, contrary to one's naive expectations.[11] Furthermore, the functional form of (4.11a) cannot be derived from a supersymmetry transformation, although it is Eq. (4.11a) which allows the definition of a conserved supercurrent satisfying the spin-$\frac{3}{2}$ constraint. Thus the anomalies do *not* transform irreducibly under supersymmetry transformation. The naive transformation is already suspect, since by such an argument one would incorrectly derive an anomaly for Eq. (4.9). The absence of such a relationship leads us to conjecture that there is no Adler-Bardeen theorem[10] for the supercurrent anomaly.

V. CONCLUSIONS

We have demonstrated the existence of an anomaly for the supercurrent in a supersymmetric model. Since the current is an alien operator of the theory, it does not spoil the renormalizability of the theory. Further, it is conjectured that supersymmetry is preserved order by order in perturbation theory in spite of the anomaly. However, in a sequel[15] to this paper we show that pseudoparticles lead to a nonperturbative breakdown of both supersymmetry and chiral symmetry. This provides a new mechanism for supersymmetry breaking, which need not involve a Goldstone fermion.

The anomaly of the supercurrent in this model suggests that similar issues must be faced in supergravity[3] and extended supergravity.[4] If such anomalies were to exist, they would raise serious difficulties with the renormalizability of such theories, since a supercurrent $\mathcal{S}_\mu$ couples to the spin-$\frac{3}{2}$ particles of the theory, and is no longer an alien operator. This question is currently under investigation.

Note added. We have chosen to compute the supercurrent anomaly for $\partial_\mu \mathcal{S}^\mu$ subject to the constraint $\gamma_\mu \mathcal{S}^\mu = 0$, since this is an *algebraic* feature of the Noether current given by Eq. (2.5). Equivalently, we could have required $\partial_\mu \mathcal{S}^\mu = 0$ and expressed the anomaly as $\gamma_\mu \mathcal{S}^\mu \neq 0$, as is implicit in Sec. IV. This latter point of view was taken by Curtright.[16] These two forms of the anomaly are *not* incompatible since one may shift from one form of the anomaly to the other by changing the routing of momenta through the Feynman diagrams defined by Figs. 1 and 2. The important feature of the anomaly is the fundamental clash between (Yang-Mills) gauge invariance, the conservation of the supercurrent between physical states, and the spin-$\frac{3}{2}$ character of the supercurrent ($\gamma_\mu \mathcal{S}^\mu = 0$). One may satisfy any two of these constraints, but not all three, as discussed in Sec. IV.

Note added in proof. If three supersymmetric scalar multiplets are added to the model considered in this paper, the supercurrent anomaly will be canceled.[17]

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APPENDIX A

In our problem we encounter primitive linearly divergent integrals of the form

$$I = \int \frac{d^4q\, q_\mu q_\nu q_\lambda}{[(q-p)^2 + a^2]^3} \tag{A1}$$

$$= \int d^4q \frac{(q+p)_\mu (q+p)_\nu (q+p)_\lambda}{(q^2+a^2)^3} + S, \tag{A2}$$

where

$$S = -\int d^4q\,(p_\mu p_\nu p_\lambda + p_\mu q_\nu q_\lambda + p_\nu q_\mu q_\lambda + p_\lambda q_\mu q_\nu + p_\mu p_\nu q_\lambda + p_\nu p_\lambda q_\mu + p_\lambda p_\mu q_\nu)(q^2+a^2)^{-3} + \int d^4q\, q_\mu q_\nu q_\lambda \left\{ \frac{1}{[(q-p)^2+a^2]^3} - \frac{1}{(q^2+a^2)^3} \right\}. \tag{A3}$$

The surface term S is only logarithmically divergent, so that the origin of the momentum integration may be freely translated in (A3). A straightforward but lengthy computation shows that

$$S = C(p_\mu g_{\nu\lambda} + p_\nu g_{\mu\lambda} + p_\lambda g_{\mu\nu}), \tag{A4}$$

where the constant C is unnecessary for our purposes. The crucial point is that the surface term only leads to tensors which are linear in the external momenta. That is,

$$I = p_\mu p_\nu p_\lambda \int \frac{d^4q}{(q^2+a^2)^3} + D(p_\mu g_{\nu\lambda} + \text{cyclic}), \tag{A.5}$$

with the constant D dependent on the surface term. The finite term proportional to a third-rank tensor in momenta is unique. This provides the justification for the methodology of Sec. III, and Eqs. (3.9)–(3.11) in particular. The result (A5) also allows us to add Feynman diagrams for Figs. 1 and 2, in a naive fashion, if we are only interested in extracting the unambiguous tensor structures, as in Eqs. (3.1) and (3.12).

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