PHYSICAL REVIEW D VOLUME 16, NUMBER 10 15 NOVEMBER 1977

Possible supersymmetry breaking by pseudoparticles

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(Received 3 August 1977)

The effects of pseudoparticles in a supersymmetric SU(2) gauge theory are discussed. The possibility of a nonperturbative breakdown of supersymmetry due to the pseudoparticles is suggested and explored, and the pseudoparticle contribution to the fermion 8-point function is explicitly calculated. Implications for supergravity and extended supergravity are briefly discussed.

I. INTRODUCTION

Although supersymmetry provides several desirable features when incorporated into a quantum field theory,[1–5] it is certainly not a symmetry observed in nature Thus, if supersymmetry is proposed as an underlying symmetry of field theory, a mechanism must be provided for its breakdown.[6,7] Further, since the only known massless fermions, the neutrinos, do not have couplings appropriate to a Goldstone fermion,[8] the supersymmetry-breaking mechanism should not introduce Goldstone fermions. A similar problem exists in the case of the strong interactions where chiral U(1) symmetry is apparently broken without the appearance of a corresponding Goldstone boson.

Recently, 't Hooft[9,10] has proposed a solution to this U(1) problem based on the pseudoparticle of Belavin, Polyakov, Schwartz, and Tyupkin.[11] The pseudoparticle makes nonperturbative contributions to certain Green's functions,[10,12] which break chiral U(1) invariance, apparently without introducing a Goldstone boson The pseudoparticle also leads to a fundamental change in the vacuum structure of the gauge theory, which is the origin of the chiral U(1) breaking.[13] Here we will study an analogous mechanism in the case of supersymmetry and explore the possibility that pseudoparticles induce a nonperturbative breakdown of supersymmetry.

The pseudoparticle contribution in a Minkowski-space field theory is computed by continuing the theory to Euclidean space.[10,14] In order to avoid difficulties in continuing the Majorana condition to Euclidean space, we will choose from among the various supersymmetric gauge theories[15–17] a model with one vector supermultiplet and one scalar supermultiplet For the SU(2) gauge theory the particle content is a gauge field A^a_μ, a *Dirac* spinor field Ψ_a, a real scalar field B_a, and a real pseudoscalar field C_a, all transforming as the adjoint representation of SU(2). The Lagrangian is [15–17]

$$\mathcal{L} = -\tfrac{1}{4}(F^a_{\mu\nu})^2 + i\bar\psi_a\gamma^\mu D^{ab}_\mu\psi_b + \tfrac{1}{2}(D^{ab}_\mu B_b)^2 + \tfrac{1}{2}(D^{ab}_\mu C_b)^2 - ig\epsilon_{abc}\bar\psi_a(B_b+\gamma_5 C_b)\psi_c - \tfrac{1}{2}g^2(\epsilon_{abc}B_bC_c)^2\,, \tag{1.1}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\,\epsilon_{abc}A^b_\mu A^c_\nu \tag{1.2}$$

and

$$D^{ab}_\mu = \partial_\mu\delta_{ab} + g\,\epsilon_{acb}A^c_\mu . \tag{1.3}$$

The action of this theory is invariant under the complex supersymmetry transformation[15–17]

$$\begin{aligned} \delta A^a_\mu &= i\bar\epsilon\gamma_\mu\psi_a - i\bar\psi_a\gamma_\mu\epsilon\,,\\ \delta\psi_a &= F^a_{\mu\nu}\sigma^{\mu\nu}\epsilon + iD^{ab}_\mu(B_b-\gamma_5C_b)\gamma^\mu\epsilon - i\epsilon_{abc}B_bC_c\gamma_5\epsilon\,,\\ \delta B_a &= i\bar\epsilon\,\psi_a - i\bar\psi_a\epsilon\,,\\ \delta C_a &= i\bar\epsilon\gamma_5\psi_a - i\bar\psi_a\gamma_5\epsilon\,, \end{aligned} \tag{1.4}$$

where ϵ is a *complex* anticommuting spinor.

In addition to the supersymmetry, this model has a chiral symmetry which is generated by the axial-vector current

$$J^5_\mu = i\bar\psi_a\gamma_\mu\gamma_5\psi_a + 2B_aD^{ab}_\mu C_b - 2C_aD^{ab}_\mu B_b\,. \tag{1.5}$$

The divergence of this axial-vector current is anomalous,[18] as is the divergence of the supercurrent for this model.[19] We find

$$\partial^\mu J^5_\mu = \frac{g^2}{4\pi^2}F^a_{\mu\nu}\tilde F^a_{\mu\nu}\,, \tag{1.6}$$

where

$$\tilde F^a_{\mu\nu} = \tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^a_{\alpha\beta}\,. \tag{1.7}$$

The model also possesses pseudoparticle and antipseudoparticle solutions with unit Pontryagin index,[11]

$$\frac{g^2}{32\pi^2}\int d^4x\,F^a_{\mu\nu}\tilde F^a_{\mu\nu} = \pm 1\,. \tag{1.8}$$

Thus, it would appear from integrating Eq. (1.6) over space-time and using Eq. (1.8) that the pseudoparticle or antipseudoparticle will contribute to processes for which the change of chirality satisfies

$$\Delta Q^5 = \pm 8 \,. \tag{1.9}$$

This will lead to chiral-symmetry breaking as has previously been discussed.[9,10,12] We will now investigate its effect on supersymmetry.

Our study of pseudoparticle effects will be centered around the fermion 8-point function,

$$G_8 = \langle 0 \,|\, T[\bar\psi(x_1)\bar\psi(x_2)\bar\psi(x_3)\bar\psi(x_4)\,\psi(x_5)\psi(x_6)\psi(x_7)\psi(x_8)] \,|\, 0\rangle \,. \tag{1.10}$$

In agreement with Eq. (1.9) this 8-point function receives contributions from the pseudoparticle and antipseudoparticle. In the following two sections, we will explicitly calculate the pseudoparticle and antipseudoparticle contributions to the fermion generating functional and to this 8-point Green's function. Our results can be summarized as follows:

(1) The pseudoparticle and antipseudoparticle *do not* contribute to any Green's function involving less than four ψ's and/or four $\bar\psi$'s.

(2) The pseudoparticle and antipseudoparticle *do* make a nonvanishing contribution to G_8.

In order to study the possibility of supersymmetry breaking of pseudoparticles we can investigate the validity of various Ward identities implied by supersymmetry. The Ward identities obtained from the divergence of the supercurrent are unfortunately complicated by the fact that the customary gauge-fixing term added to the Lagrangian (1.1) is not supersymmetric. This leads to additional ghost terms in the Ward identities which would not naively be expected.[17] A consequence of the Ward identities (see Ref. 17) is the schematic relation

$$G_8 + G_{\text{ghost}} = \text{terms involving six fermion fields and two boson fields,} \tag{1.11}$$

where G_{ghost} stands for a Green's function involving four ψ's, four $\bar\psi$'s, a boson field, and two ghost fields. On the mass shell of the physical states, the terms in Eq. (1.11) involving ghost fields will drop out, and one finds supersymmetry relations between an 8-fermion S-matrix element and a 6-fermion, 2-boson S-matrix element.[20] Equation (1.11) must be satisfied by the zero pseudoparticle sector and by the pseudoparticle contributions separately since the latter are nonperturbative. As remarked above, the Green's functions involving six fermion fields and two boson fields receive no contribution from the pseudoparticle or antipseudoparticle solutions. Thus, if supersymmetry is unbroken, Eq. (1.11) implies in the presence of pseudoparticles that

$$(G_8 + G_{\text{ghost}})_{\text{pseudoparticle + antipseudoparticle}} = 0 \,, \tag{1.12}$$

which is to be compared with (1.11). As emphasized above, the pseudoparticle and antipseudoparticle do make a contribution to G_8. We have no reason to believe that the pseudoparticle contributions to G_{ghost} vanish; however, we do have reason to believe that Eq. (1.12) *cannot* be satisfied. This is because, in principle, G_8 contributes to a physical amplitude whereas G_{ghost} does not. Although the usual infrared problems in the pseudoparticle sector associated with asymptotically free theories prevent us from making an actual on-shell evaluation of (1.12), we expect that in the neighborhood of a physical point G_8 and G_{ghost} will have different external-line pole structures and Eq. (1.12) will not be satisfied, signaling the breakdown of supersymmetry. If an actual S-matrix element could be extracted from G_8, the equality between the fermion 8-particle amplitude and a 6-fermion, 2-boson amplitude would certainly be destroyed by pseudoparticle effects and we would see a breakdown of supersymmetry in the S matrix. Unfortunately the infrared divergences prevent us from going to the mass shell.

The mechanism for possible supersymmetry breaking of pseudoparticles can be summarized as follows. In a purely boson theory, without fermions, the pseudoparticles will contribute to vacuum tuneling[13] and to various boson processes.[21] However, when fermions are introduced, there is a complete suppression of pseudoparticle effects in the purely bosonic Green's functions. Nevertheless, there is a change in the vacuum structure due to requirements of cluster decomposition.[13] Pseudoparticle contributions only survive for certain fermion Green's functions. This highly unsymmetric situation seems to lead to a violation of the relations between fermion and boson Green's functions implied by supersymmetry and hence to a breakdown of supersymmetry itself.

In Sec. II we will compute the one-pseudoparticle contributions to the fermion generating functional and derive the above results. Multipseudoparticle solutions[22] will also contribute to the fermion generating functional, but only to Green's functions involving more than eight Fermi fields. Thus, they will not affect our results and will be ignored. Likewise, the contributions of ordinary perturbation theory need not be considered since they will not produce supersymmetry-violating effects. (We assume the absence of spontaneous supersymmetry breaking.) Along with the possible breakdown of supersymmetry, the calculation exhibits a complete cancellation of pseudoparticle contributions to vacuum bubbles. This cancellation is known to occur for ordinary perturbation-theory contributions in supersymmetric models.[23] In Sec. III we discuss the fermion 8-point function in more detail and finally in Sec. IV we briefly discuss implications for supergravity and extended supergravity.

II. PSEUDOPARTICLE CONTRIBUTION TO THE FERMION GENERATING FUNCTIONAL

In order to define the fermion generating functional we add a source term

$$\mathcal{L}_{\text{source}} = \bar{K}_a \psi_a + \bar{\psi}_a K_a \tag{2.1}$$

to the Lagrangian (1.1). The pseudoparticle contribution is evaluated by continuing to Euclidean space and evaluating quadratic functional integrals after expanding around the classical solution,

$$\begin{aligned} &\psi_a = B_a = C_a = 0 , \\ &A_\mu^a = A_\mu^{a(\text{cl})} , \end{aligned} \tag{2.2}$$

where $A_\mu^{a(\text{cl})}$ is the pseudoparticle solution. We begin by computing the pseudoparticle contribution and then consider the antipseudoparticle. Since detailed calculations appear elsewhere[10,14] we will not go into excessive detail here.

The integration over gauge and ghost fields has been performed by 't Hooft[10] in the background field gauge

$$\partial_\mu A_\mu^a + g\epsilon_{abc} A_\mu^{b(\text{cl})} A_\mu^c = 0 \tag{2.3}$$

resulting in a multiplicative factor in the generating functional of

$$[\Pi(1)]^{-1} e^{-8\pi^2/g^2}\, 2^{15}\pi^6 g^{-8} \mu_0{}^8 \rho^3\, d\rho\, d^4 z , \tag{2.4}$$

where

$$\Pi(1) = (\mu_0 \rho)^{2/3} e^{0.443\,307} , \tag{2.5}$$

μ_0 is the renormalization mass, and ρ and z are collective coordinates representing the size and position of the pseudoparticle. The coupling constant is defined by its value at the renormalization point, $g = g(\mu_0)$. The integration over the scalar and pseudoscalar fields has also been done by 't Hooft[10] resulting in an additional multiplicative factor

$$[\Pi(1)]^{-1} . \tag{2.6}$$

Finally, we must perform the integration over the Dirac spinor field after continuing the Dirac Lagrangian to Euclidean space. This continuation has been discussed carefully by Peccei and Quinn in Ref. 14. The Euclidean-space functional integral to be evaluated is[14]

$$\int \delta(\psi_a)\delta(\psi_a^\dagger) \exp\left(\int d^4x [-\tfrac{1}{2}(\psi_a^\dagger M_{ab}\psi_b + \psi_a M_{ab}^* \psi_b^\dagger) + \bar{K}_a \psi_a + \psi_a^\dagger K_a] \right) , \tag{2.7}$$

where

$$M_{ab} = \gamma_\mu (\partial_\mu \delta_{ab} + g\epsilon_{acb} A_\mu^{c(\text{cl})}) . \tag{2.8}$$

The integral can be evaluated by defining a set of normal modes as appropriately normalized solutions to the eigenvalue equation

$$M_{ab}\phi_{bi} = \lambda_i \phi_{ai} . \tag{2.9}$$

An important feature of this equation is that for $i = 1, 2, 3$, and 4, $\lambda_i = 0$.[24,25] These zero-frequency mode solutions play a key role in our analysis and will be explicitly displayed in Sec. III. They are γ_5 eigenstates, right-handed for the antipseudoparticle and left-handed for the pseudoparticle. It is interesting to note that these zero-frequency mode solutions are the supersymmetry partners of the gauge field zero-frequency modes.[17,24,25] However, the gauge field zero-frequency modes are eliminated by the introduction of collective coordinates, while these Dirac modes remain. We do not consider the possibility of a family of solutions related by supersymmetry and parametrized by an anticommuting collective coordinate. It is this difference in interpretation which results in the breakdown of supersymmetry.

The functional integral of Eq. (2.7) is evaluated by expanding the field ψ_a in normal modes assigning one anticommuting parameter per normal mode.[15] The integration over these anticommuting parameters is then performed using the usual integration rules for such variables.[26] The result is

$$\left(\prod_{i=5}^{\infty}\lambda_i\right)(\bar{K}_a\phi_{a1})(\phi_{a2}^\dagger K_a)(\bar{K}_a\phi_{a2})(\phi_{a1}^\dagger K_a) \times (\bar{K}_a\phi_{a3})(\phi_{a4}^\dagger K_a)(\bar{K}_a\phi_{a4})(\phi_{a3}^\dagger K_a) + O(K^{10}) , \tag{2.11}$$

where

$$(\bar{K}_a\phi_{ai}) = \int d^4x\, \bar{K}_a(x)\phi_{ai}(x - z, \rho) , \tag{2.12}$$

and likewise for $(\phi_{ai}^\dagger K_a)$, with $O(K^{10})$ indicating terms which will only contribute to Green's functions with ten or more Fermi fields. Such terms will not interest us here.

The result (2.11) must, of course, be renormalized and evaluated. This has been done by 't Hooft with the result that after renormalization,

$$\prod_{i=5}^{\infty}\lambda_i \to \frac{1}{\mu_0{}^4}[\Pi(1)]^2 . \tag{2.13}$$

Combing (2.4), (2.6), (2.11), and (2.13) gives us the fermion generating functional,

$$Z[\bar{K}_a,K_a]=2^{15}\pi^6\int \rho^3 d\rho d^4z\, e^{-8\pi^2/g^2} g^{-8}\mu_0{}^4[(\bar{K}_a\phi_{a1})(\phi^\dagger_{a2}K_a)(\bar{K}_a\phi_{a2})(\phi^\dagger_{a1}K_a)(\bar{K}_a\phi_{a3})(\phi^\dagger_{a4}K_a)(\bar{K}_a\phi_{a4})(\phi^\dagger_{a3}K_a)]+O(K^{10}) . \tag{2.14}$$

We have completed the gauge field integration by integrating over pseudoparticle size and position. The one-loop β function for this model is[17]

$$\beta(g)=-\frac{g^3}{4\pi^2} , \tag{2.15}$$

so we see that the result (2.14) obeys the renormalization-group equation which is a good check of our calculation. *Note that the various factors* $\Pi(1)$ *have canceled indicating the cancellation of vacuum bubbles in the presence of the pseudoparticle.* This cancellation occurs due to the supersymmetry of the model and has previously been noted in conventional perturbative calculations as well.[23]

The antipseudoparticle contribution to the generating function is identical to that of Eq. (2.14) except that the modes ϕ_{a1}, ϕ_{a2}, ϕ_{a3}, and ϕ_{a4} with negative chirality are replaced by corresponding right-handed modes. Since the pseudoparticle and antipseudoparticle contributions are identical except for opposite chirality we will focus our attention in the next section just on the pseudoparticle contribution of Eq. (2.14).

Equation (2.14) and the corresponding contribution from the antipseudoparticle discussed above verify the results stated in Sec. I. Since the generating functional involves at least four factors of K_a and four factors of $\bar{K}_a$, the pseudoparticle contribution will only survive, after the fermion sources are set equal to zero, for Green's functions involving at least four ψ's and four $\bar{\psi}$'s. The introduction of boson sources obviously does not modify this argument which only depends on the number of fermion zero-frequency modes. Finally, the nonvanishing pseudoparticle contribution to G_8 discussed in Sec. I is clearly present in Eq. (2.14) and will be displayed in more detail in the next section.

III. THE FERMION 8-POINT FUNCTION

The first term of Eq. (2.14) gives us the pseudoparticle contribution to the fermion generating functional from which we can read off the fermion 8-point function. In order to simplify our notation we will work in a representation in which γ_5 is diagonal, i.e.

$$\gamma_5=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \tag{3.1}$$

and

$$\gamma_\mu=\begin{pmatrix}0 & \alpha_\mu\\ \bar{\alpha}_\mu & 0\end{pmatrix} , \tag{3.2}$$

where

$$\alpha_\mu=(-i\vec{\sigma},1) \quad \bar{\alpha}_\mu=(i\vec{\sigma},1) . \tag{3.3}$$

The zero-frequency modes of the preceding section can now be written in terms of γ_5 eigenstates as[24,25]

$$\phi_{ai}=\begin{pmatrix}0\\ \eta_{ai}\end{pmatrix}, \quad i=1,2,3,4 \tag{3.4}$$

with

$$\eta_{a1}=\frac{\sqrt{2}\rho^2}{\pi}\frac{1}{[(x-z)^2+\rho^2]^2}\sigma_a u , \tag{3.5}$$

$$\eta_{a2}=\frac{\sqrt{2}\rho^2}{\pi}\frac{1}{[(x-z)^2+\rho^2]^2}\sigma_a v , \tag{3.6}$$

$$\eta_{a3}=\frac{\rho}{\pi}\frac{1}{[(x-z)^2+\rho^2]^2}\sigma_a\bar{\alpha}\cdot(x-z)u , \tag{3.7}$$

and

$$\eta_{a4}=\frac{\rho}{\pi}\frac{1}{[(x-z)^2+\rho^2]^2}\sigma_a\bar{\alpha}\cdot(x-z)v , \tag{3.8}$$

where u and v are arbitrary orthogonal two-component spinors normalized to

$$u^\dagger u=v^\dagger v=1 , \quad u^\dagger v=0 . \tag{3.9}$$

Actually the u and v spinors appearing in Eqs. (3.5) and (3.6) could be different from those in Eqs. (3.7) and (3.8), but since our final result will be independent of any particular choice for these constant spinors, we will leave them the same for notational simplicity. Note that in agreement with previous statements the zero-frequency modes (3.4) are purely left-handed.

Let us also write the source K_a in terms of its chiral components,

$$K_a=\begin{pmatrix}R_a\\ L_a\end{pmatrix}. \tag{3.10}$$

Then, using the anticommuting property of the sources, the orthonormal properties of the u and v constant spinors, and Eqs. (2.12), (2.14), (3.4)–(3.8), and (3.10), we can write the first term of Eq. (2.12) which expresses the pseudoparticle contribution to the fermion 8-point function as

$$Z_8[\bar{K}_a, K_a] = \frac{2^{15}}{\pi^2} \int \frac{d\rho}{\rho^{17}} \frac{\mu_0^{\,4}}{g^8} e^{-8\pi^2/g^2} \int d^4z \left(\prod_{j=1}^{8} d^4x_j\right) \Big\{ [R^\dagger_{a'}(x_5)\sigma_{a'}\sigma_2\sigma_a L_a(x_1)][R^\dagger_{b'}(x_6)\sigma_{b'}\sigma_2\sigma_b L_b(x_2)]$$
$$\times [R^\dagger_{c'}(x_7)\bar{\alpha}\cdot(x_7 - z)\sigma_{c'}\sigma_2\sigma_c \alpha\cdot(x_3 - x)L_c(x_3)]$$
$$\times [R^\dagger_{d'}(x_8)\bar{\alpha}\cdot(x_8 - z)\sigma_{d'}\sigma_2\sigma_d \alpha\cdot(x_4 - z)L_d(x_4)]$$
$$\times f\left(\frac{x_1 - z}{\rho}\right) f\left(\frac{x_2 - z}{\rho}\right) f\left(\frac{x_3 - z}{\rho}\right) f\left(\frac{x_4 - z}{\rho}\right) f\left(\frac{x_5 - z}{\rho}\right)$$
$$\times f\left(\frac{x_6 - z}{\rho}\right) f\left(\frac{x_7 - z}{\rho}\right) f\left(\frac{x_8 - z}{\rho}\right) \Big\}, \qquad (3.11)$$

where

$$f(t) = 1/(t^2 + 1)^2 . \qquad (3.12)$$

Note the chiral structure of the sources in our result (3.11) in agreement with the chirality violation predicted by Eq. (1.9).

We can also express our result in momentum space by writing the sources as

$$R^\dagger_a(x) = \int d^4p\, e^{ip\cdot x} r^\dagger_a(p) \qquad (3.13)$$

and

$$L_a(x) = \int d^4p\, e^{ip\cdot x} l_a(p) . \qquad (3.14)$$

Then we find

$$Z_8[\bar{K}_a, K_a] = 2^{27}\pi^{18} \int d\rho\, \rho^{15} \frac{\mu_0^{\,4}}{g^8} e^{-8\pi^2/g^2} \int \left(\prod_{j=1}^{8} d^4p_j\right) \Big\{ \delta^4\left(\sum_{j=1}^{8} p_j\right) [r^\dagger_{a'}(p_5)\sigma_{a'}\sigma_2\sigma_a l_a(p_1)][r^\dagger_{b'}(p_6)\sigma_{b'}\sigma_2\sigma_b l_b(p_2)]$$
$$\times \left[r^\dagger_{c'}(p_7)\bar{\alpha}\cdot\frac{\partial}{\partial p_7}\sigma_{c'}\sigma_2\sigma_c \alpha\cdot\frac{\partial}{\partial p_3} l_c(p_3)\right]$$
$$\times \left[r^\dagger_{d'}(p_8)\bar{\alpha}\cdot\frac{\partial}{\partial p_8}\sigma_{d'}\sigma_2\sigma_d \alpha\cdot\frac{\partial}{\partial p_4} l_d(p_4)\right]$$
$$\times [F(|p_1|\rho)F(|p_2|\rho)F(|p_3|\rho)F(|p_4|\rho)$$
$$\times F(|p_5|\rho)F(|p_6|\rho)F(|p_7|\rho)F(|p_8|\rho)] \Big\} , \qquad (3.15)$$

where

$$F(t) = 2\int_0^\infty ds \frac{s^2 J_1(s)}{(s^2 + t^2)^2} \qquad (3.16)$$

Note that off the mass shell ($|p_i| \neq 0$ for $i = 1, 2, \ldots, 8$) the integration over the pseudoparticle size is finite.

IV. SUPERGRAVITY AND EXTENDED SUPERGRAVITY

It is interesting to speculate that gravitational pseudoparticles may induce supersymmetry breaking in supergravity and extended supergravity. Hawking[27] has noted that the Taub-Newman-Unti-Tamburino solution of general relativity, when continued to Euclidean space, provides a pseudoparticle solution with Pontryagin index two. In order to find zero-frequency modes for spin-$\frac{1}{2}$ fermion fields, one must have a gravitational pseudoparticle with Pontryagin index 24. In supergravity the appropriate starting point would be zero-frequency solutions of a spin-$\frac{3}{2}$ fermion in the field of a gravitational pseudoparticle. If such solutions exist, they could well provide a mechanism for chiral and supersymmetry breaking in models of supergravity.[28]

Note added in proof. In background field gauge the ghost contribution to Eq. (1.12) vanishes.

ACKNOWLEDGMENTS

The authors wish to thank Professor H. Pendelton and Dr. E. Poggio for helpful discussions, and the Aspen Center for Physics where this work was completed.

*Research supported by ERDA under Contract No. E(11-1)3230.
†Address Sept. 1, 1977: SLAC, Stanford, California 943054.
‡Research supported by the NSF under Grant No. PHY-76-02054.
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[28] Compare with the mechanisms proposed in Ref. 7.