terms do not appear in $\langle \mathfrak{L}\rangle_0^{(2)}$, but the presence of such terms would not alter the argument for the calculability of β. In noncompact manifolds there can be Lorentz scalars other than R which contribute (I wish to thank S. M. Christensen for this remark), but again these do not alter the argument given for the βR term.

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[10]One possibility is that the metric is not a quantum variable, but is a classical dynamical variable governed by the Euler-Lagrange equations, in which case the background metric analysis given in the text is exact. Another possibility consistent with the viewpoint of the text is that the metric is a quantum variable, with dynamics governed by a scale-invariant fundamental Lagrangian (see L. Smolin, Ref. 2). The only generalization of Eq. (4) to include a scale-invariant gravitational action is $\tilde{S} = \int d^4x\,(-g)^{1/2}\,(\mathfrak{L}_{\rm matter} + \delta C_{\mu\nu\lambda\sigma}C^{\mu\nu\lambda\sigma} + \text{counterterms})$, with $C_{\mu\nu\lambda\sigma}$ the Weyl tensor, and with δ a dimensionless coupling constant. Recent work of Stelle [K. S. Stelle, Phys. Rev. D 16, 953 (1977)] on quadratic gravitational actions suggests that this extended theory should still be renormalizable. The $\kappa^2\beta R$ term in Eq. (5) would still be calculable, even with quantum gravitational effects taken into account, but the β value calculated from the flat-spacetime matter theory would be subject to a finite, δ-dependent renormalization. This renormalization could be important if the virtual integrations contributing to β extend to energies beyond the Planck mass.

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Choosing an Expansion Parameter for Quantum Chromodynamics

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When expanded in powers of $\bar{\alpha}$ where $\bar{\alpha}$ is chosen to satisfy $\bar{\alpha}(Q_0^2) = 4\pi/\beta_0 \ln(Q_0^2/\Lambda^2) = \alpha_{\overline{\rm MS}}(Q_0^2)$ with $Q_0^2 \approx 10$ GeV^2, the quantum chromodynamics (QCD) perturbation series seems to be extremely well behaved. In particular, the large third-order corrections discussed recently by Moshe do not appear when this $\bar{\alpha}$ is used as the QCD expansion parameter.

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The behavior of the perturbation expansion in quantum chromodynamics (QCD) depends critically on the choice of the expansion parameter used to define the perturbation series. It has been noted previously that nonleading corrections to QCD predictions for deep-inelastic scattering[1,2] and for e^+e^- annihilation,[3] calculated in the minimal-subtraction (MS) scheme,[4] are significantly reduced if one expands in powers of an α defined by momentum-space subtraction[5] ($\alpha_{\rm MOM}$) or by the $\overline{\rm MS}$ scheme[2] ($\alpha_{\overline{\rm MS}}$) in which factors of $\gamma_E - \ln 4\pi$ are removed along with the divergences. This suggests that in QCD, as in quantum electrodynamics (QED), a "physically" defined coupling constant like $\alpha_{\rm MOM}$ or the closely related $\alpha_{\overline{\rm MS}}$ makes a good expansion parameter for the perturbation series. However, recently Moshe[6] has presented an interesting estimate of the third-order corrections to the QCD predictions for moments of deep-inelastic structure functions. He finds that even if the $\overline{\rm MS}$ definition of α is used, third-order corrections seem to be enormous for $Q^2 \lesssim 10$ GeV^2. Although this result is speculative, a close examination shows that large contributions to the third-order corrections come from terms like $C_{2,2}^{(N)}(\alpha/4\pi)^2\ln^2(\alpha/4\pi)$, where the constant $C_{2,2}^{(N)}$ is completely *known*, and it seems unlikely that the unknown constant terms would conspire to cancel this large known term. Moshe's work seems to indicate a disaster in QCD perturbation theory. What has gone wrong?

Consider a perturbative QCD calculation for the moments of the nonsinglet structure function[7] xF_3, performed with the MS scheme.[4] The β function and relevant anomalous dimensions and Wilson coefficients are expanded as

$$\beta = -g_{\rm MS}[\beta_0(\alpha_{\rm MS}/4\pi) + \beta_1(\alpha_{\rm MS}/4\pi)^2 + \beta_2(\alpha_{\rm MS}/4\pi)^3 + \dots], \tag{1}$$

$$\gamma^{(N)} = \gamma_0{}^{(N)}(\alpha_{\rm MS}/4\pi) + \gamma_1{}^{(N)}(\alpha_{\rm MS}/4\pi)^2 + \gamma_2{}^{(N)}(\alpha_{\rm MS}/4\pi)^3 + \dots, \tag{2}$$

$$C^{(N)} = 1 + C_1{}^{(N)}(\alpha_{\rm MS}/4\pi)^2 + \dots, \tag{3}$$

where

$$\alpha_{\rm MS} = g_{\rm MS}{}^2/4\pi. \tag{4}$$

One then obtains predictions for the moments (in the leading-twist approximation),

$$M_N(Q^2) = A_N[\alpha_{\rm MS}(Q^2)]^{\gamma_0(N)/2\beta_0}\{1 + D_1{}^{(N)}[\alpha_{\rm MS}(Q^2)/4\pi] + D_2{}^{(N)}[\alpha_{\rm MS}(Q^2)/4\pi]^2 + \dots\}. \tag{5}$$

A_N is an undetermined constant. The constants $D_1{}^{(N)}$ and $D_2{}^{(N)}$ are given[8] in terms of the β_i, $\gamma_i{}^{(N)}$, and $C_i{}^{(N)}$ of Eqs. (1)–(3). $D_2{}^{(N)}$ depends on the unknown parameters β_2, $\gamma_2{}^{(N)}$, and $C_2{}^{(N)}$. However, it turns out that in the expansion for $D_2{}^{(N)}$, β_2, and $\gamma_2{}^{(N)}$ are multiplied by small numbers so that their values have little effect on determining $D_2{}^{(N)}$. Furthermore, known terms contribute a sizable amount to $D_2{}^{(N)}$ so that the impact of the uncertainty in estimating $C_2{}^{(N)}$ is somewhat reduced as well. The reliability of the third-order estimate is further discussed in Ref. 6.

As discussed above, the perturbation expansion in Eq. (5) can be improved by reexpanding[2,5] in terms of a different α. In Table I, the known values[1,2] of $D_1{}^{(N)}$ for $N=4$ and the analogous constant[3] multiplying the $(\alpha/4\pi)^2$ term in $R_{e^+e^-}$ are compared for various definitions of the coupling constant

$$\alpha = \alpha_{\rm MS}[1 + k(\alpha_{\rm MS}/4\pi)]. \tag{6}$$

The table shows how the magnitudes of these higher-order corrections are reduced by use of the $\overline{\rm MS}$ or MOM definitions of α. The results suggest that the best α to use for third-order calculations might be $\alpha_{\rm MOM}$ in Feynman gauge. However, the relationship between $\alpha_{\rm MOM}$ and $\alpha_{\rm MS}$ at the order $\alpha_{\rm MS}{}^3$ is not presently known. Therefore, I will use the $\overline{\rm MS}$ definition of α which is related to $\alpha_{\rm MS}$ at this order by[9]

$$\alpha_{\overline{\rm MS}} = \alpha_{\rm MS}[1 + a\beta_0(\alpha_{\rm MS}/4\pi) + (a^2\beta_0{}^2 + a\beta_1)(\alpha_{\rm MS}/4\pi)^2], \tag{7}$$

where

$$a = \ln 4\pi - \gamma_E \approx 1.95. \tag{8}$$

The expression in Eq. (5) is now reexpanded in terms of the $\alpha_{\overline{\rm MS}}$ at this order to obtain[10]

$$M_N(Q^2) = A_N[\alpha_{\overline{\rm MS}}(Q^2)]^{\gamma_0(N)/2\beta_0}\{1 + \overline{D}_1{}^{(N)}[\alpha_{\overline{\rm MS}}(Q^2)/4\pi] + \overline{D}_2{}^{(N)}[\alpha_{\overline{\rm MS}}(Q^2)/4\pi]^2 + \dots\}. \tag{9}$$

The values of $\overline{D}_1{}^{(N)}$ are known to be fairly small[2] (see for example Table I) except at large values of N. Estimates[11] of $\overline{D}_2{}^{(N)}$ indicate that they too are not unreasonably large. The large third-order terms found in Moshe's analysis,[6] therefore, have not yet made their appearance.

TABLE I. Coefficients of $(\alpha/4\pi)^2$ corrections in deep-inelastic scattering (M_4) and e^+e^- scattering ($R_{e^+e^-}$) for various definitions of α related by $\alpha = \alpha_{\rm MS}[1 + k(\alpha_{\rm MS}/4\pi)]$. Numerical values are from Refs. 1–3 and 5.

Scheme	k	Coefficient of $(\frac{\alpha}{4\pi})^2$ D.I.S. $N=4$	$R_{e^+e^-}$
MS	0	20.6	89.3
$\overline{\rm MS}$	16.2	6.9	24.3
MOM, Landau gauge	29.2	−3.9	−27.4
MOM, Feynman gauge	26.0	−1.2	−14.7

In order to obtain a definite prediction for the Q^2 dependence of the moments from Eq. (9) one must specify the Q^2 dependence of $\alpha_{\overline{\rm MS}}(Q^2)$. In almost all analyses, this is done by expanding $\alpha_{\overline{\rm MS}}$ itself in powers of a parameter

$$\overline{\alpha} = 4\pi/\beta_0 \ln(Q^2/\Lambda^2). \tag{10}$$

When this expansion for $\alpha_{\overline{\rm MS}}$ in powers of $\overline{\alpha}$ is substituted into Eq. (9), one reexpands to QCD perturbation series this time in powers of $\overline{\alpha}$. We have noted that the perturbation series of Eq. (9) for M_N in powers of $\alpha_{\overline{\rm MS}}$ seems well behaved. If this good behavior is to be retained when M_N is reexpanded in powers of $\bar{\alpha}$ one *must* require that $\bar{\alpha}$ and $\alpha_{\overline{\rm MS}}$ be as close to equal as possible. This can be done by choosing Λ in Eq. (10) so that

$$\bar{\alpha}(Q_0^2) = \alpha_{\overline{\rm MS}}(Q_0^2) \tag{11}$$

at some value Q_0^2 which is in the middle of the Q^2 range over which the QCD prediction is to be applied. In this analysis, I will take $Q_0^2 = 10\ \mathrm{GeV}^2$ although the precise value is not too critical. Equation (11) requires that we define Λ by

$$\Lambda^2 = Q_0^2 \exp[-4\pi/\beta_0 \alpha_{\overline{\rm MS}}(Q_0^2)]. \tag{12}$$

In other analyses, Λ is not defined as in Eq. (12) but rather is defined in such a way that it is independent of Q_0^2. However, this definition of Λ does not allow Eq. (11) to be satisfied at any reasonable value of Q_0^2, and thus the good behavior of the perturbation series of Eq. (9) is destroyed when it is reexpanded in terms of $\bar{\alpha}$. For example, using the estimated[11] value of β_2 with $\Lambda = 0.5$ GeV and four quark flavors, I find that the $\bar{\alpha}$ used in Ref. 6 did not satisfy Eq. (11) but rather was related to $\bar{\alpha}_{\overline{\rm MS}}$ at $Q^2 = 10\ \mathrm{GeV}^2$ by

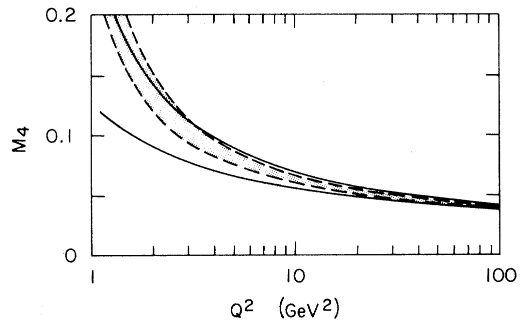

FIG. 1. The lower solid curve is the lowest-order QCD prediction for the $N=4$ moment of xF_3 with arbitrary overall normalization. The upper solid curve is the QCD prediction including known second-order terms as well. The shaded area indicates the estimated third-order prediction including a 50% error on all unknown constants. For these curves $\Lambda = 0.5$ GeV and four quark flavors have been used.

$$\bar{\alpha} = \alpha_{\overline{\rm MS}}[1 + 21.1(\alpha_{\overline{\rm MS}}/4\pi) + 575.0(\alpha_{\overline{\rm MS}}/4\pi)^2]. \tag{13}$$

The introduction of large numbers like 21.1 and 575.0 into the perturbation expansion was then what produced the large third-order corrections in Moshe's analysis.[6] When Λ is chosen as in Eq. (12) so that Eq. (11) is satisfied, we have

$$\alpha_{\overline{\rm MS}}(Q^2) = \bar{\alpha}(Q^2)\left\{1 + \left[\frac{\beta_1}{\beta_0}\ln\frac{\bar{\alpha}(Q^2)}{\bar{\alpha}(Q_0^2)}\right]\left[\frac{\bar{\alpha}(Q^2)}{4\pi}\right] + \left[\left(\frac{\beta_2}{\beta_0} - \frac{\beta_1^2}{\beta_0^2}\right)\left(1 - \frac{\bar{\alpha}(Q_0^2)}{\bar{\alpha}(Q^2)}\right) + \frac{\beta_1^2}{\beta_0^2}\left(\ln^2\frac{\bar{\alpha}(Q^2)}{\bar{\alpha}(Q_0^2)} + \ln\frac{\bar{\alpha}(Q^2)}{\bar{\alpha}(Q_0^2)}\right)\right]\left(\frac{\bar{\alpha}(Q^2)}{4\pi}\right)^2 + \cdots\right\}. \tag{14}$$

By examining this expression, one can again see the importance of imposing condition (11). In Moshe's analysis,[6] factors of $\{(\beta_1/\beta_0)\ln[\bar{\alpha}(Q^2)/4\pi]\}^2 \approx 450$ (at $Q^2 = 10\ \mathrm{GeV}^2$) appeared at third order and led to the large terms $C_{2,2}^{(N)}\ln^2[\bar{\alpha}(Q^2)/4\pi]$ discussed above. When condition (11) is applied these are replaced by $\{(\beta_1/\beta_0)\ln[\bar{\alpha}(Q^2)/\bar{\alpha}(Q_0^2)]\}^2 \approx 0$. However, the most important point is that Eq. (11) is a *natural* consequence of the assumption that $\alpha_{\overline{\rm MS}}$ is a reasonable expansion parameter for QCD.

The final QCD predcition for $M_N(Q^2)$ is obtained by substituting $\alpha_{\overline{\rm MS}}$ of Eq. (14) into Eq. (9). The results for the $N=4$ moment with $\Lambda = 0.5$ GeV and four quark flavors are shown in Fig. 1. The lower solid curve is the lowest-order QCD prediction where the arbitrary normalization $A_N = 1$ has been chosen. The upper solid curve shows the result of adding in the known second-order corrections. Finally, the shaded region indicates the estimated QCD prediction including first-, second-, and third-order terms. A 50% error has been included for all unknown parameters. The perturbation series seems remarkably well behaved.[12]

My conclusion is that, although disastrous results can be obtained by use of arbitrary choices of the expansion parameter, the QCD perturbation series[13] in powers of $\bar{\alpha}$ where $\bar{\alpha}$ is chosen to satisfy

$$\bar{\alpha}(Q_0^2) = 4\pi/\beta_0 \ln(Q_0^2/\Lambda^2) = \alpha_{\overline{\rm MS}}(Q_0^2) \tag{15}$$

with $Q_0^2 \approx 10\ \mathrm{GeV}^2$ shows[14] no anomalously large terms at least up to order $\bar{\alpha}^3$.

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[7]I chose the nonsinglet moment to avoid the complication of operator mixing and take xF_3 just for definiteness. The results are only changed slightly for other nonsinglet structure functions like $F_2^p - F_2^n$.

[8]Explicity,

$$D_1^{(N)} = C_1^{(N)} + \frac{\gamma_1^{(N)}}{2\beta_0} - \frac{\gamma_0^{(N)}\beta_1}{2\beta_0^2},$$

$$D_2^{(N)} = C_2^{(N)} + \frac{\gamma_2^{(N)}}{4\beta_0} - \frac{\beta_2\gamma_0^{(N)}}{4\beta_0^2} + \frac{\beta_1^2\gamma_0^{(N)}}{4\beta_0^3} - \frac{\beta_1\gamma_1^{(N)}}{4\beta_0^2} + \frac{\gamma_1^{(N)2}}{8\beta_0^2} - \frac{\gamma_1^{(N)}\beta_1\gamma_0^{(N)}}{4\beta_0^3} + \frac{\beta_1^2\gamma_0^{(N)2}}{8\beta_0^4} + C_1^{(N)}\left(\frac{\gamma_1^{(N)}}{2\beta_0} - \frac{\beta_1\gamma_0^{(N)}}{2\beta_0^2}\right).$$

[9]J. Collins and A. Macfarlane, Phys. Rev. D 10, 1201 (1974). For a discussion of this result in a simply calculable model see L. F. Abbott and M. T. Grisaru, to be published.

[10]Explicitly,

$$\bar{D}_1^{(N)} = D_1^{(N)} - \tfrac{1}{2}a\gamma_0^{(N)},$$

$$\bar{D}_2^{(N)} = D_2^{(N)} - a\left\{\frac{\gamma_0^{(N)}\beta_1}{2\beta_0} + \beta_0\left(1 + \frac{\gamma_0^{(N)}}{2\beta_0}\right)D_1^{(N)}\right\} + a^2\left[\frac{\gamma_0^{(N)}\beta_0}{4} + \frac{\gamma_0^{(N)2}}{8}\right].$$

[11]I estimate the parameters β_2, $\gamma_2^{(N)}$, and $C_2^{(N)}$ by assuming that the ratio between first- and second-order coefficients is equal to that between second- and third-order terms. Thus, I take as my estimated values $\beta_2 = (\beta_1/\beta_0)\beta_1$, $\gamma_2^{(N)} = [\gamma_1^{(N)}/\gamma_0^{(N)}]\gamma_1^{(N)}$, and $C_2^{(N)} = (C_1^{(N)}/1)C_1^{(N)}$.

[12]Figure 1 should be compared with Fig. 2 of Ref. 6.

[13]The problem of choosing an expansion parameter is not unique to QCD. For example, in QED if one choosed to reexpand the usual perturbation series for the magnetic moment of the electron in terms of $\alpha_{\rm MS}^{\rm QED}$ one also finds anomalously large third-order corrections.

[14]The choice $Q_0^2 = 10$ GeV^2 is not crucial. Various values of Q_0^2 are acceptable provided that they lie in a region of Q^2 in which perturbation theory is applicable.

Search for Narrow $\bar{p}p$ States

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We have searched for narrow states of the $\bar{p}p$ system in the mass range from 2000 to 2400 MeV/c^2 in the inclusive channel $\pi^- + (p \text{ or C}) \to \bar{p}p + X^0$. No statistically significant enhancements in the data have been observed.

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High-mass baryon-antibaryon resonances with narrow widths have been predicted by theories based on nuclear potential models,[1] duality,[2] or quark confinement.[3] Despite confusing and even contradictory evidence[4,5] concerning such resonances, the importance of these ideas warrants

