“Exponential Asymptotics for Line Solitons in Two dimensional Periodic Potentials”

As a first step toward a fully two-dimensional asymptotic theory for the bifurcation of solitons from infinitesimal continuous waves, an analytical theory is presented for line solitons, whose envelope varies only along one direction, in general two-dimensional periodic potentials. For this two-dimensional problem, it is no longer viable to rely on a certain recurrence relation for going beyond all orders of the usual multiscale perturbation expansion, a key step of the exponential asymptotics procedure previously used for solitons in one-dimensional problems. Instead, we propose a more direct treatment which not only overcomes the recurrence-relation limitation, but also simplifies the exponential asymptotics process. Using this modified technique, we show that line solitons with any rational line slopes bifurcate out from every Bloch-band edge; and for each rational slope, two line-soliton families exist. Furthermore, line solitons can bifurcate from interior points of Bloch bands as well, but such line solitons exist only for a couple of special line angles due to resonance with the Bloch bands. In addition, we show that a countable set of multiline-soliton bound states can be constructed analytically. The analytical predictions are compared with numerical results for both symmetric and asymmetric potentials, and good agreement is obtained.
With Jon Weare (Chicago), we study the continuum limit of a family of kinetic Monte Carlo models of crystal surface relaxation that includes both the solid-on-solid and discrete Gaussian models. With computational experiments and theoretical arguments we are able to derive several partial differential equation limits identified (or nearly identified) in previous studies and to clarify the correct choice of surface tension appearing in the PDE and the correct scaling regime giving rise to each PDE. We also provide preliminary computational and analytic investigations of a number of interesting qualitative features of the large scale behavior of the models. The PDE models involved are fully non-linear Fourth order diffusion type equations with many interesting geometric features. We will given time discuss recent progress analyzing properties of solutions to such PDE.