Experiment 7:

Conservation of Momentum

1. Introduction

Momentum is one of the central quantities in physics, and less intuitive to many people than the conservation of energy. However, it can be difficult to keep track of all possible energy contributions. (What, for example, is the energy contribution that deforms the metal of two cars as they collide?) For some problems, momentum conservation is essential. We will see that specific problems, like the ballistic pendulum, require both energy and momentum conservation for their solution. Others, like the air trough, permit prediction of details of motion in cases where mechanical energy is conserved (elastic) as well as where mechanical energy is not conserved (inelastic). This lab is designed to provide you with an intuitive, yet quantitative, sense of momentum and some of its important applications.

Remark:
You must prepare some derivations (for the pendulum, the velocity ratios of the riders, and the elastic collision) at home, otherwise, you may have trouble finishing the lab on time!

2. Theory

2.1 Momentum Conservation

Momentum is a vector quantity, so it has a direction and an absolute value (magnitude). To add or subtract momenta, use the usual rules of vector addition. In this lab, we deal only with momentum in one dimension. Even in this simple case, the vector property applies, so that if two objects move in opposite directions, their momenta have opposite signs. (This is the source of most errors when using momenta, so be careful!)

Momentum is the product of mass and velocity

$$\boldsymbol{p} = m \cdot \boldsymbol{v}.$$

A fundamental property of nature is that the total momentum components of any closed system* are conserved in any physical process.

2.2 The Ballistic Pendulum

A ballistic pendulum is an instrument used to measure the velocity of a projectile. In this experiment, a metal ball strikes a pendulum, and sticks to it. The velocity of the ball can be deduced by observing the maximum height of the pendulum as it swings afterward. In

* Closed means that the sums of force components external to the specified system are negligible.

this experiment, we measure the velocity of the projectile and deduce how far the pendulum should move. This is then compared to the actual measured pendulum displacement.

It is easiest to analyze the problem by splitting it into two parts:

In the first part, consider the projectile and pendulum as two distinct objects just before they collide, and as one combined object afterward. The pendulum is initially at rest at its equilibrium point whereas the projectile travels with a velocity. When the projectile strikes the pendulum, they stick together and become a single object. In the process, the projectile transfers <u>some</u> of its energy and <u>all</u> of its momentum to the motion of the pendulum (with the attached projectile). Unfortunately, we cannot directly calculate the pendulum's subsequent motion from conservation of mechanical energy, because part of the initial kinetic energy deforms the clay (and we don't know *a priori* how much). Therefore, the concept of energy conservation will not help us for this first part. But momentum conservation will help us, since this is totally independent of the deformation of the ball.

More quantitatively, consider the projectile to have a mass m and initial velocity v, and the pendulum to have a mass M and initially no velocity. The initial total momentum is $m \times v$. When the projectile and the pendulum stick together, they have total mass $m + M$ and we define their combined velocity V. The relationship from conservation of momentum is

$$m \times v = (m + M) \times V.$$

For the second part of the problem, we consider the motion of the pendulum <u>after</u> the collision. Specifically, we measure how high (h) the pendulum swings. Here, we <u>can</u> use energy conservation[*], because there are no uncontrollable energy losses (like the energy lost when the bullet stops in and deforms the clay in the first part of the problem). Conservation of energy requires

$$\textit{Kinetic energy after the collision} = \textit{Potential energy at the top of the swing},$$

$$\tfrac{1}{2} (m + M) \times V^2 = (m + M) \times g \times h.$$

This relation permits us to calculate the velocity (just after collision) of the pendulum and, with the earlier expression, we obtain the initial velocity of the projectile.

But it is difficult to directly measure the vertical rise, h. Instead it is much easier to measure the horizontal distance, d, the pendulum swings through. (See figure.) These quantities are geometrically related, by the Pythagorean Theorem, as

$$R^2 = (R-h)^2 + d^2.$$

These three equations permit you to derive the dependence of v on the quantities m, M, g, R, d. Conversely, you can find d in terms of $m, M, g, R,$ and v.

Remark:
Derive these two relations at home before coming to the lab, since you will need them and your TA is not going to derive them for you!

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[*] Note that conservation of momentum does <u>not</u> work in the second part, because there are forces on the pendulum (gravity and the strings) from outside the pendulum which change momentum.

3. Experiments

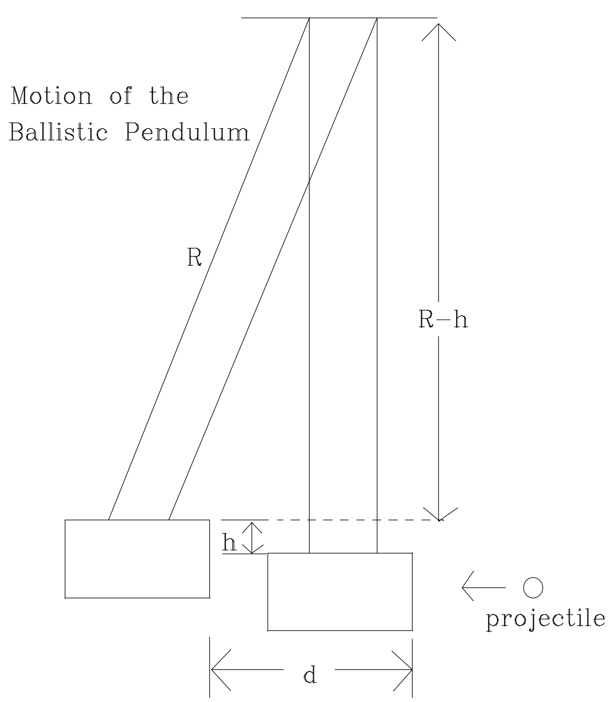
3.1 The Ballistic Pendulum I and II

Safety remark:
Do not have the gun loaded while you are measuring distances or the mass of the can!

(I) In the first apparatus, shoot a small metal ball out of a piece of tubing (gun). The pendulum consists of an open can, filled with clay, suspended by four strings. The velocity of the projectile is determined by measuring the flight time between a pair of photocells. This permits you to predict how far (d) the pendulum should swing. You measure d by observing how far the pendulum pushes a small glider. The predicted and measured values can then be directly compared.

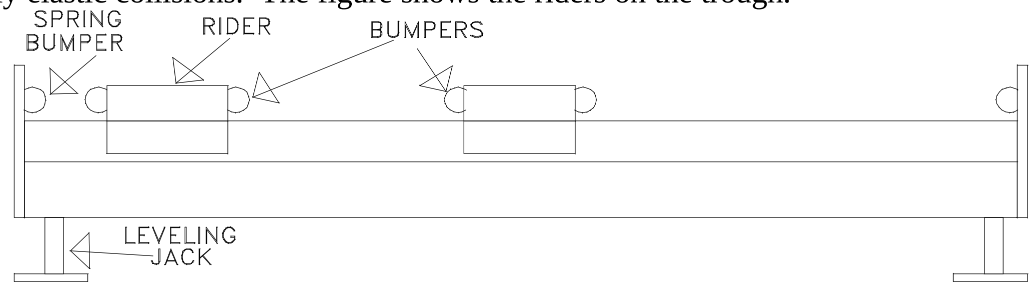
(II) The second apparatus allows you to measure how fast you can throw a clay ball (and decide if you should become a professional pitcher). Take a clay ball and throw it at the pendulum so that it sticks. By measuring how far a small glider gets pushed by the pendulum, determine d and hence also, v.

Remark:
If the clay is too stiff to perform the experiment properly, put it under the lamp provided to heat it up so that it becomes malleable again.



3.2 The Air Trough

In the air trough, we observe both elastic and inelastic reactions. The air trough provides a steady airflow so that the riders move almost frictionless along a layer of air. Flexible steel bumpers, mounted on the riders and at the ends of the trough, provide almost perfectly elastic collisions. The figure shows the riders on the trough.



The riders in the trough permit us to test momentum conservation by measuring total momentum before and after a collision. Since momentum involves both mass and velocity, we should in principle measure the mass and the velocity of each object before and after. We <u>will</u> measure the masses with a balance. But rather than measure the velocities directly, we will use a trick that simplifies the measurements.

We perform the experiments in such a way that, after the process is initiated and the riders move to opposite sides of the air trough, they touch the two ends at exactly the same time. (Clearly this requires careful positioning initially.) Since the two riders started at a common position (which we can record), we know this location relative to the two ends of the trough (called s_1 and s_2). The ratio of the velocities is just

$$v_1/v_2 = (s_1/t)/(s_2/t) = s_1/s_2.$$

This comes from simply using $s=v \times t$.

The following two experiments are performed with the air trough.

A) <u>Elastic collision between two unequal riders</u>. Select a large and a small rider. Place the large rider at rest on the air trough and send the small one moving toward it with an arbitrary velocity. See which one hits the end first. Choose the initial position of the large rider such that the two riders, in the end, hit the ends of the trough at exactly the same time.

B) <u>Inelastic separation of gliders</u>. Connect the same two gliders together using the piston and the cylinder, and place a toy cap between them. Explode the cap using an electric spark from a Tesla coil. Again choose the initial location so that the riders hit the ends at exactly the same time.

<u>Safety remark</u>:

Handle the Tesla coil carefully. It produces several thousand volts so that you or others can get a serious shock! It is not worth getting an electrical shock for a 1-credit course!

4. Specifics of the Experiments

4.1 The Ballistic Pendulum – I

- Measure the distance *s* between the light gates.
- Measure the length *R* of the strings.
- Measure the mass *M* of the pendulum.
- Measure the mass *m* of the projectile.
- Put the small rider behind the pendulum so that it just touches the pendulum. Read and record its position.
- Put the projectile in the tube and shoot.
- Take the projectile off the clay, clean it and reload it into the tube. Don't set back the rider!
- Reset the counter for the light gates and shoot.
- Read and record the counter and the final position of the glider.
- Calculate the velocity of the projectile from the light gate data. With this, calculate how far you predict the pendulum to have swung back. Compare this with the experimental result.
- Are your prediction and measurement equal within a reasonable error? (You need to estimate this reasonable error.)
- Describe the main sources of error in this experiment!

Remark:
The reason we shoot twice is that there is friction between the rider and the ruler on which it rests. By performing the experiment twice, we reduce the effect of this friction. (Can you explain why?)

4.2 The Ballistic Pendulum – II

- Record the mass and the length of the large pendulum as written on it.
- Measure the mass of the clay ball.
- Locate the rider so that it just touches the pendulum and record its position (d_1).
- Throw the clay ball at the pendulum such that the ball sticks to the pendulum. (You need to use some judgement about how hard to throw it and maintain enough accuracy to not miss the pendulum.)
- Record the new location (d_2). From this, calculate how far the rider moved (Δd).
- Calculate from this information how fast you threw the ball in m/s.
- Convert this speed into km/h and into miles/h.
- How fast was the ball compared to what a professional pitcher can do? (Pitchers commonly throw balls at speed exceeding 100 miles/h.)
- Make suggestions how you could improve this experiment to increase the speed with which you throw the clay ball!

4.3 The Air Trough

Remark:
Please handle the riders with care! Don't put them on the trough without air flowing and store them only on the felt covered holders provided. Don't make violent collisions! (That also compromises your data!).

- Take a small rider and a large rider. Measure their respective weights, m and M.
- Perform the elastic collision experiment (A) by colliding the moving small rider with the stationary large rider. Locate the large rider on the trough so that, after the collision, the two gliders hit the end of the trough at the same instant. (You will probably have to try this several times!)
- Measure the distances s_1 and s_2 the two gliders traveled between the collision point and the ends of the trough. Include an estimate of the uncertainty in this measurement! (Hint: This uncertainty is mostly determined by how well you can locate the correct initial position. This is probably larger than the 1mm resolution of the ruler!)
- Calculate the ratio s_1/s_2 (including uncertainties!) and compare it to a theoretical prediction from the masses of the riders!
- What would happen if the two riders had exactly equal mass?
- What would happen if you put the small glider stationary and send the large one on it? Why do we not want this to happen?
- Next, connect together the piston and the cylinder on the large and small rider. Place a toy cap between them before you put them together.
- Place them on the trough so that the individual gliders will again hit the ends of the trough simultaneously.
- Fire the cap using the Tesla coil. (Reminder: Careful with the coil!)
- Measure s_1 and s_2 including uncertainties.
- Again compare the ratio s_1/s_2 (including uncertainties) with your predicted value!
- After bouncing off the ends, do the two riders meet again at the position from which they started? Explain why or why not!
- Give the main sources of error!

5. Lab Preparation Examples

Propagation of Uncertainty:

1. Given a = 1.5 ± 0.5 m and b = 3.0 ± 0.6 m what is a/b?

Ballistic Pendulum:

2. You measure the following values for pendulum I:

s	t	R	m	M
60 cm	0.01 s	1.5 m	10 g	1 kg

What is your predicted value for Δd?

3. You measure the following values for pendulum II:

m	M	R
1.5 kg	50 kg	2.2 m

d_2		d_1		Dd
12.2 cm	-	9.7 cm	=	

What is the velocity of the clay ball?

Conversion m/s, km/h and miles/h:

4. What is 1 m/s in km/h?

5. How many miles/h is the speed of 200 km/s?

Air Trough:

6. In the elastic collision, the riders weigh m = 100g and M = 250g. What is the expected value for $v_1/v_2 = s_1/s_2$?

7. In the inelastic collision, the riders weight m = 100g and M = 250g. What is the theoretical value for $v_1/v_2 = s_1/s_2$?

Explanations:

8. You are sitting in a car of mass M. Another car with mass m = 500kg crashes into you with a relative velocity of 10 m/s. Explain in a few sentences (and maybe equations) why you feel less impact if your car has M = 2000 kg as if your car has only M = 500 kg?