

**Department of Applied Physics and Applied
Mathematics
Columbia University**

**APMA S3101D: Applied Math I - Introduction to Linear Algebra
Summer 2003**

Problem Set 2

**Note: This is for those who use the SECOND version of the
textbook. Those who have the third version of the book please go to**

ps2-v3.ps

(Due June 12, 2003)

1. Problem Set 2.6: 1¹, 7, 13

2. Problem Set 2.7: 7, 18

¹(Important) Forward elimination changes $\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}x=b$ to a triangular

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}x = c:$$

$$\begin{array}{l} x + y = 5 \\ x + 2y = 7 \end{array} \longrightarrow \begin{array}{l} x + y = 5 \\ y = 2 \end{array} \qquad \begin{pmatrix} 1 & 1 & 5 \\ 1 & 2 & 7 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 5 \\ 0 & 1 & 2 \end{pmatrix}$$

That step subtracted $l_{21} = \underline{\hspace{1cm}}$ times row 1 from row 2. The reverse step *adds* l_{21} times row 1 to row 2. The matrix for that reverse step is $L = \underline{\hspace{1cm}}$. Multiply this L times the triangular system $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}x = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ to get $\underline{\hspace{1cm}} = \underline{\hspace{1cm}}$. In letters, L multiplies $Ux = c$ to give $\underline{\hspace{1cm}}$.

3. Problem Set 3.1: 7, 10², 19

4. Problem Set 3.2: 3, 13, 15, 19³

²(a) The plane of vectors (b_1, b_2, b_3) with $b_1 = b_2$.

(b)–(f) are same as problem 10.

³Prove that U and $A = LU$ have the same nullspace when L is invertible:

If $Ux=0$ then $LUx=0$. If $LUx=0$, how do you know $Ux=0$?