Department of Applied Physics and Applied Mathematics

Columbia University

APMA S3101D: Applied Math I - Introduction to Linear Algebra

Summer 2003

Problem Set 4

Note: This is for those who use the SECOND version of the textbook. Those who have the third version of the book please go to ps2-v3.ps

(Due June 26, 2003)

1. Problem Set 4.2: 4[1], 5, 13, 22

2. Problem Set 4.3: 12

3. Problem Set 4.4: 1, 12, 18[2], 22

4. Consider approximating the cubic $y_3(x) = x^3 + 2x$ by a quadratic function $y_2(x) = c_1 + c_2x + c_3x^2$ in the interval $-2 \leq x \leq 2$. To do this, sample the cubic at points $x = (-2, -1, 0, 1, 2)$ to obtain corresponding values of the function $y_3(x)$ at those points. Then perform a least squares fit of these "data" to the quadratic function y_2 to obtain (c_1, c_2, c_3).

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[1] Construct the projection matrices P_1 and P_2 onto the lines through the a's in Problem 2. Is it true that $(P_1 + P_2)^2 = P_1 + P_2$? This $would$ be true if $P_1P_2 = 0$.

[2] Find orthogonal vectors A,B,C by Gram-Schmidt from a,b,c:

$a = (1, -2, 0, 0)$, $b = (0, 1, -1, 0)$, $c = (0, 0, 1, -1)$

A,B,C and a,b,c are bases for the vectors perpendicular to $d = (1, 1, 1, 1)$

5. (*Weighted least squares* (10 pts)) Compute the least squares solution $\hat{\mathbf{x}}$ by minimizing the following *cost function* with respect to $\mathbf{x}$:

$$J(\mathbf{x}) = (\mathbf{y} - \mathbf{A}\mathbf{x})^{\mathrm{T}}\mathbf{R}^{-1}(\mathbf{y} - \mathbf{A}\mathbf{x}) + (\mathbf{x} - \mathbf{x_o})^{\mathrm{T}}\mathbf{W}^{-1}(\mathbf{x} - \mathbf{x_o})$$

In the above, $\mathbf{R}$ and $\mathbf{W}$ are invertible, symmetric matrices known as the observation noise covariance matrix and the prior covariance matrix, respectively. $\mathbf{x_o}$ represents a *prior* guess of $\mathbf{x}$ (i.e., our best estimate of $\mathbf{x}$ before any observations $\mathbf{y}$ become available) and $\mathbf{W}$ represents the uncertainty in this guess. The matrix $\mathbf{R}$ reprsents the *noisiness* of the observations.