Problem Set 4

Note: This is for those who use the THIRD version of the textbook. Those who have the second version of the book please go to ps2-v2.ps

(Due June 26, 2003)

1. Problem Set 4.2: 4, 5, 13, 22

2. Problem Set 4.3: 12

3. Problem Set 4.4: 1, 12, 18, 23

4. Consider approximating the cubic \( y_3(x) = x^3 + 2x \) by a quadratic function \( y_2(x) = c_1 + c_2x + c_3x^2 \) in the interval \(-2 \leq x \leq 2\). To do this, sample the cubic at points \( x = (-2,-1,0,1,2) \) to obtain corresponding values of the function \( y_3(x) \) at those points. Then perform a least squares fit of these “data” to the quadratic function \( y_2 \) to obtain \( (c_1, c_2, c_3) \).

5. (Weighted least squares (10 pts)) Compute the least squares solution \( \hat{x} \) by minimizing the following cost function with respect to \( x \):

\[
J(x) = (y - Ax)^T R^{-1} (y - Ax) + (x - x_o)^T W^{-1} (x - x_o)
\]
In the above, \( R \) and \( W \) are invertible, symmetric matrices known as the observation noise covariance matrix and the prior covariance matrix, respectively. \( x_o \) represents a prior guess of \( x \) (i.e., our best estimate of \( x \) before any observations \( y \) become available) and \( W \) represents the uncertainty in this guess. The matrix \( R \) represents the noisiness of the observations.