

CHAPTER 3

THE CONCEPT OF VISCOSITY

Fluid flow plays a very important part in the processing of materials. Most processes are based on the use of fluids either as raw materials, reagents, or heat transfer media. In this book, we will see many examples of processes where the rates of heat transfer, mass transfer and chemical reaction between two phases depend on the fluid flow phenomena in the system. It is therefore necessary to start our study of rate phenomena in processing systems by examining the motion of fluids.

3.1 DEFINITION AND MEASUREMENT OF VISCOSITY

The behavior of a fluid in flow is very much related to two intrinsic properties of the fluid: **density** and **viscosity**. For example, a solid body moving through a gas has to overcome a certain resistance which depends on the relative velocity between fluid and solid, the shape of the solid, the density of the gas and its viscosity. The power required to move a fluid through a conduit is a function of the fluid velocity, the diameter of the conduit and the fluid density and viscosity.

The existence and nature of viscosity can be demonstrated by suspending two horizontal, parallel plates in a liquid so that they are separated by a very small distance, Y (Fig. 3.1). Now, if the upper plate is kept stationary while the lower plate is set to motion with a velocity u_o , the layer of liquid right next to this plate will also start to move. With time, the motion of the bottom layer of fluid will cause the fluid layers higher up to also move.

As shown in Fig. 3.1, when steady-state conditions are established, the velocity of the uppermost layer, which is in contact with the stationary plate, will also be zero, while the bottom layer, in contact with the moving plate, will be moving with velocity u_o . If we measure the velocity distribution across the intermediate fluid layers, we find that velocity changes linearly with distance y from the stationary plate:

$$u_x = u_o \frac{y}{Y} \quad (3.1.1)$$

Let us assume that we can measure, e.g. by means of a calibrated spring or transducer, the horizontal force ($-F_x$, Fig. 3.1) which must be applied, in the opposite direction to u_o , to maintain the upper plate at rest; if we divide this force by the surface area of the plate, A , we find that this ratio, called the **shear stress**, is proportional to the velocity of the lower plate, u_o , and inversely proportional to the distance Y between the two plates:

$$\frac{\text{Force}}{\text{area}} = \text{shear stress} \propto \frac{u_o}{Y} = \mu \frac{u_o}{Y} \quad (3.1.2)$$

As shown in (3.1.2), the proportionality constant between shear stress and the velocity gradient u_o/Y is called the **viscosity** of the liquid, μ . Since, at steady state conditions, the velocity profile between the two plates is linear, every infinitesimal segment of the line is represented by the same relationship. Therefore, (3.1.2) can be expressed in differential form:

$$\text{Shear stress} = \tau_{y,x} = -\mu \frac{du_x}{dy} \quad (3.1.3)$$

The first subscript of the shear stress denotes the *area* on which the shear stress acts (here, an area perpendicular to the y -axis and at distance y from the origin); the second subscript represents the *direction* in which the shear stress acts (here, in the direction of velocity, x). The negative sign expresses the fact that the shear stress is applied from a region of higher velocity to a lower one (i.e., $u_{x,2} < u_{x,1}$ and $du_x < 0$).

Equation (3.1.3) is called the **Newton's law of viscosity** and states that *the shear stress between adjacent fluid layers is proportional to the negative value of the velocity gradient between the two layers.*

An alternative interpretation can be given to (3.1.3) by noting, from elementary physics, that

$$\text{Force} = \text{mass} \times \text{acceleration} = \text{mass} \times \text{velocity/time} = \text{momentum/time}$$

Therefore, (3.1.3) also states that *the rate of momentum transfer per unit area, between two adjacent layers of fluid, is proportional to the negative value of the velocity gradient between them.*

In transport theory, the *transport rate per unit surface area* of a certain quantity (e.g. of mass, momentum, heat) is called **flux**; for example, the dimensions of mass flux are $M \text{ t}^{-1} L^{-2}$. Also, the change of a certain quantity with distance is called **gradient**; for example, the dimensions of the mass gradient are $M L^{-1}$.

If we now divide and multiply the second term of (3.1.3) by the fluid density, we obtain:

$$\tau_{y,x} = -\frac{\mu}{\rho} \left(\frac{\rho du_x}{dy} \right) = -\nu \left(\frac{\rho du_x}{dy} \right) \quad (3.1.4)$$

and for incompressible flow

$$\tau_{y,x} = -\nu \frac{d(\rho u_x)}{dy} \quad (3.1.5)$$

which expresses the fact that *the momentum flux is proportional to the negative value of the gradient of mass flux.*

In the above equation, we have introduced a new term, the ratio of fluid viscosity to density, μ/ρ . This ratio is called the **kinematic viscosity** of a fluid, ν , and has the dimensions of $L^2 \text{ t}^{-1}$. As we shall see later, the properties of *thermal diffusivity* (Chapter 10) and *molecular diffusivity* (Chapter 14) have the same dimensions. Therefore, by analogy, the kinematic viscosity is also referred to as the **momentum diffusivity** of the fluid, i.e. the ability of the fluid to transport momentum.

The above discussion was based on unidirectional flow. However, it

also applies for the general case of flow in all three directions, x , y , and z , as illustrated in Fig. 3.2. The stresses

$$\tau_{y,x} \quad \tau_{y,z} \quad \tau_{x,y} \quad -\tau_{x,z} \quad \tau_{z,x} \quad \tau_{z,y}$$

are called *shear* or *tangential* stresses. The stresses $\tau_{x,x}$, $\tau_{y,y}$ and $\tau_{z,z}$ are called *normal* stresses and can be either *compressive* (+) or *tensile* (-).

By substituting the dimensions of force, area, length and velocity in (3.1.3) we find that the dimensions of viscosity are $M L^{-1} t^{-1}$. In the metric system, the unit of viscosity is the Poise ($1 P = 1 g cm^{-1} s^{-1}$), which is subdivided to 100 centipoise (cP).

Table 3.1 Comparative values of viscosity of some gases and liquids

Fluid	Temperature °C	Density kg m ⁻³	Viscosity kg s ⁻¹ m ⁻¹	Momentum diffusivity m ² s ⁻¹
Hydrogen	0	8.93×10^{-2}	8.40×10^{-6}	9.41×10^{-5}
	20.7	8.30×10^{-2}	8.80×10^{-6}	1.06×10^{-4}
	229	4.86×10^{-2}	1.26×10^{-5}	2.59×10^{-4}
	490	3.19×10^{-2}	1.67×10^{-5}	5.24×10^{-4}
	825	2.22×10^{-2}	2.14×10^{-5}	9.64×10^{-4}
Air	0	1.30×10^0	1.71×10^{-5}	1.32×10^{-5}
	18	1.22×10^0	1.83×10^{-5}	1.50×10^{-5}
	229	7.08×10^{-1}	2.64×10^{-5}	3.73×10^{-5}
	409	5.21×10^{-1}	3.41×10^{-5}	6.55×10^{-5}
	810	3.28×10^{-1}	4.42×10^{-5}	1.35×10^{-4}
	1134	2.53×10^{-1}	5.21×10^{-5}	2.06×10^{-4}
Water	0	1.00×10^3	1.79×10^{-3}	1.79×10^{-6}
	20	9.98×10^2	1.01×10^{-3}	1.01×10^{-6}
	60	9.83×10^2	4.69×10^{-4}	4.77×10^{-7}
	100	9.58×10^2	2.84×10^{-4}	2.96×10^{-7}

Molten iron	1550	7.21×10^3	6.70×10^{-3}	9.29×10^{-7}
	1600	7.16×10^3	6.10×10^{-3}	8.52×10^{-7}
	1700	7.06×10^3	5.60×10^{-3}	7.93×10^{-7}
	1800	6.96×10^3	5.30×10^{-3}	7.61×10^{-7}
High-iron slag	1200	4.50×10^3	3.50×10^{-1}	7.78×10^{-5}
Low-iron slag	1500	3.50×10^3	5.00×10^{-1}	1.43×10^{-4}

In the SI system, viscosity is expressed in $\text{kg m}^{-1} \text{s}^{-1}$; however, sometimes viscosity values are shown in terms of the equivalent units of $\text{N m}^{-2} \text{s}$ or Pa s , where the Pascal (Pa) is the SI unit of pressure and is equal to 1 Newton per square meter (Table A-1). The SI unit of viscosity is equal to 10 Poise or 1000 centipoise.

All gases and most simple liquids, including liquid metals and other high-temperature melts, obey Newton's law of viscosity and are accordingly called Newtonian fluids. On the other hand, polymer solutions, slurries, pastes, and paints are frequently "non-Newtonian": their viscosity depends on the applied shear and, for some fluids, also on the rate at which a given shear is applied (1). The viscosities and momentum diffusivities of some gases and liquids of interest in the chemical processing of materials are compared in Table 3.1.

3.2 FACTORS AFFECTING VISCOSITY

The viscosity of Newtonian fluids is affected by temperature, pressure, and, in the case of solutions and mixtures, by composition. The effect of pressure and temperature on the viscosity of gases is illustrated in Fig. 3.3 which shows the viscosity of carbon dioxide as a function of pressure and temperature (14). The lowest viscosity is reached (Fig. 3.3) at the *critical temperature* of carbon dioxide ($T_c = 304.1 \text{ K}$). This is the highest temperature at which a gaseous substance can exist in liquid form regardless of the pressure; the corresponding *critical pressure* (P_c) for CO_2 is 73.8 bar

(about 72.9 atm). As indicated in Fig. 3.3, at high ratios of T/T_c , pressure has little effect and the gas viscosity increases with temperature. The plot of Fig. 3.3 is typical of the viscosity plots for other gases. The critical temperatures and pressures of other compounds and elements can be found in the *Chemical Engineers' Handbook* (4) and other reference books.

3.2.1 Estimation of Gas Viscosities

When experimental data on the viscosity of a particular gas are not available, its viscosity may be estimated from theoretical models. This topic has been extensively treated in other texts, the most recent of which is *The Properties of Gases and Liquids* by Reid et al (2). In general, the molecular theory of gases is sufficiently developed to allow the prediction of transport properties. Therefore, the viscosity of gases may be estimated with an accuracy sufficient for most engineering calculations, e.g. by using the following correlation for the viscosity of quasi-spherical molecules (1,2):

$$\mu = 26.7 \times 10^{-6} \frac{(MT)^{1/2}}{\sigma_c^2 \Omega_\mu} \quad (3.2.1)$$

where μ : gas viscosity, Poise

M : molecular weight, grams

T : absolute temperature, K

σ_c : collision diameter, Ångstrom

Ω_μ : collision integral (equal to 1 for non-interacting molecules)

As shown in Fig. 3.4 (2), the value of Ω_μ is a function of the group:

$$T^* = \frac{kT}{\varepsilon} \quad (3.2.2)$$

where k is the *Boltzmann constant* ($=1.38 \times 10^{-23}$ joules molecule⁻¹ K⁻¹, Table A-2) and ε the energy parameter for interaction between molecules (units: kg m² s⁻² molecule⁻¹). Numerical values of σ_c and ε are available in the literature for most common gases; otherwise the following empirical formulae may be used for estimating these quantities:

$$\varepsilon/k = 1.92 T_m \text{ and } \sigma_c = 1.222 V_m^{1/3} \quad (3.2.3)$$

$$\varepsilon/k = 1.15 T_b \text{ and } \sigma_c = 1.166 V_b^{1/3} \quad (3.2.4)$$

where T_m and T_b are the melting and boiling points of the substance in K ; V_m is the molar volume of the species at the melting point (solid), and V_b its molar volume at the boiling point (liquid), both expressed in $\text{cm}^3 \text{mol}^{-1}$. The molecular volumes of some liquid metals are shown in Table 14.3 (Chapter 14).

The effect of temperature on the viscosity of some commonly used gases is shown in Fig. 3.5 (6). With regard to the viscosity of mixtures of gases, the following semi-empirical equation can be used to represent with reasonable accuracy the viscosity of a gas mixture at low or moderate pressures:

$$\mu_{mix} = \frac{\sum X_i \mu_i (M_i)^{1/2}}{\sum X_i (M_i)^{1/2}} \quad (3.2.5)$$

where X_i : mole fraction of component i of viscosity μ_i

M_i : molecular weight of component i

The use of such equations for predicting the viscosity is illustrated in the following example.

Example 3.2.1.

Let us estimate the viscosity of a gas mixture consisting of 20% Zn vapor, 50% N_2 , and 30% CO at 1000°C .

The density of zinc at its melting point ($T_m=692 \text{ K}$) is approximately 7.14 g cm^{-3} . Therefore, its molar volume at that temperature is calculated to be $= 65.38/7.14 = 9.16 \text{ cm}^3/\text{mole}$ and the corresponding $\sigma_c=2.55 \text{ \AA}$ (see (3.2.3)). Also, from (3.2.3), we obtain $\varepsilon/k= 1.92 \times 692=1329$. Therefore, at the given temperature of 1273 K , the value of T^* is calculated to be equal to 0.958 and $\log T^*=-0.019$. From Fig. 3.4, the corresponding value of $\log \Omega_\mu$ is approximately 0.21 , and $\Omega_\mu=1.63$. By introducing the calculated values of σ_c and Ω_μ in (3.2.1), we obtain the viscosity of zinc:

$$\mu_{Zn} = \frac{26.7 \cdot 10^{-6} \times (65.37 \cdot 1273)^{0.5}}{2.55^2 - 1.63} = 727 \cdot 10^{-6} \text{ Poise}$$

Also, from the available compilations for gas viscosities, we obtain

$$\mu_{N_2} = 500 \times 10^{-6} \text{ P}$$

$$\mu_{CO} = 480 \times 10^{-6} \text{ P}$$

We can now set up the following table and use (3.2.5), to calculate the viscosity of the gas mixture:

$$\mu_{mix} = \frac{3200 \times 10^{-6}}{5.85} = 558 \times 10^{-6} \text{ P}$$

Species	Mole Fraction	M	$\mu_i, \text{ P}$	$X_i M_i^{1/2}$	$X_i M_i^{1/2} \mu_i$
Zn	0.2	65.37	727×10^{-6}	1.62	1178×10^{-6}
N ₂	0.5	28.02	500×10^{-6}	2.65	1323×10^{-6}
CO	0.3	28.00	480×10^{-6}	1.59	761×10^{-6}
$\Sigma:$				5.85	3262×10^{-6}

3.2.2 Estimation of Liquid Viscosities

Models for predicting liquid viscosities are much less developed than for gases and their use is limited to a qualitative description of observed behavior. The behavior of liquids is markedly different from gases in that the viscosity *decreases* with increasing temperature; the relationship can be expressed in the form of an Arrhenius-type equation

$$\mu = \mu_o e^{\frac{E_\mu}{RT}} \quad (3.2.6)$$

where μ_o : viscosity at some reference temperature

E_μ : temperature coefficient for viscosity

The viscosity temperature coefficient, E_μ , has the physical significance of activation energy (ΔG_μ , Gibbs free energy, Chapter 18), so that the mobility ($1/\mu$) of liquid molecules is a temperature-activated process while that of gas molecules is not. An example of the Arrhenius-type dependence of viscosity on temperature is given in Fig. 3.6 for pure silica (5). Figs 3.7 (17) and 3.8 (13) show a similar behavior for a large number of metals, fused salts, and other liquids of interest in materials processing.

The dependence of viscosity on the composition of liquid solutions is illustrated by Fig. 3.9 for molten zinc-aluminum solutions (7); and by Fig. 3.10 for iron-carbon alloys (8). It can be seen that the behavior of molten alloys cannot be described by simple addition rules.

In many materials processing systems, it is of considerable importance to maintain liquid viscosities at a sufficiently low level so as to ensure satisfactory mass transfer rates and to facilitate charging and discharging of liquid materials. In the case of some liquids, such as metal oxide and silica solutions (slags), this minimum viscosity may exist only over a very narrow composition range so that very careful control may be required.

In the pyrometallurgical production of metals, the unwanted components in the raw feed are driven off either as gaseous components (e.g. SO_2 or CO_2) or in the form of a liquid silicate solution ("slag"). Because of the importance of slag fluidity in metallurgical processes, there have been many experimental investigations of slag viscosity. In addition to SiO_2 , steelmaking slags contain CaO , MgO , Al_2O_3 and other compounds in smaller concentrations. Pure silica has a very high viscosity (Fig. 3.5) but the addition of other metal oxides, with the exception of Al_2O_3 , decreases the slag viscosity. At low alumina concentrations, Al_2O_3 is equivalent to SiO_2 but at $X_{\text{Al}_2\text{O}_3}$ above 0.05, its effect depends on the molar ratio of $\text{Al}_2\text{O}_3/(\text{CaO}+\text{MgO})$ present in the slag (Fig. 3.11, ref.9). Once the effective concentration of $X_{\text{Al}_2\text{O}_3, eq}$ is established, it can be added to the X_{SiO_2} content of the slag and Fig. 3.12 (9) can be used to determine the viscosity of a particular slag.

Example 3.2.2

A blast furnace slag at 1500°C contains 39.8% CaO, 2.5% MgO, 15.2% Al₂O₃ and 40.7% SiO₂ (by weight). Use the plots of Fig. 3.11-3.12 to estimate the slag viscosity.

From the given composition, we calculate the molar composition of the slag to be as follows:

$$X_{CaO} = 0.44 ; X_{MgO} = 0.04 ; X_{Al_2O_3} = 0.09 ; X_{SiO_2} = 0.42$$

Also, we calculate the ratio $X_{Al_2O_3}/(X_{CaO}+X_{MgO})$ to be equal to 0.19. Fig. 3.11 shows that at this ratio and for $X_{Al_2O_3} = 0.09$, the value of $X_{Al_2O_3,eq}$ is 0.14; reference to Fig. 3.12 shows that the viscosity of the slag containing $X_{SiO_2}+X_{Al_2O_3,eq}=0.56$ is approximately 7.1 Poise.

In contrast to slags produced in the ironmaking process, slags from non-ferrous smelting (production of Cu, Pb, Zn, Ni, etc.) contain a large amount of iron as FeO and Fe₃O₄ and are usually more fluid than steelmaking slags; the iron content of copper smelting slags can vary from 30 to 40% Fe.

Work by Johannsen and Wiese (10) on synthetic copper smelting slags has shown that FeO, CaO, MgO, and Fe₃O₄ have an equivalent effect on viscosity; also, in the concentration range encountered in copper production, the effects of silica and alumina are additive. These data were correlated (11) by means of a "viscosity modulus", K_v , which was defined as follows:

$$K_v = \frac{\%FeO + \%Fe_3O_4 + \%CaO + \%MgO}{\%SiO_2 + \%Al_2O_3} \quad (3.2.2)$$

The resulting plot is shown in Fig. 3.13 (11), which also shows the wide variation between the viscosity of synthetic slags and industrial slags which contain other impurities (15). Other metal oxides, such as copper and zinc, may be included in the numerator of the (3.2.2. Manganese and chromium oxides may be assumed to act as Al₂O₃ and be included in the denominator. A recent study by Battle and Hager (12) has confirmed the usefulness of a similar slag modulus for estimating the viscosity of lead and zinc smelting slags (Fig. 3.14).

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