U-6016: Cost-benefit Analysis

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I. FINANCIAL APPRAISAL

A. Appraisal techniques: an heuristic overview

The basic steps in any appraisal exercise are the following:

-compile the list of alternatives, and the associated streams of costs and benefits

-for each alternative, reduce the stream of costs and benefits to a single index, i.e., choose an appraisal criterion, implicitly associated with which will be an admissibility rule

-pick the alternatives that satisfy the admissibility rule for that appraisal criterion

-if alternatives are mutually exclusive, pick the alternative that yields the highest value of the appraisal criterion, as long as it satisfies the admissibility rule

B. Discounting and present value analysis

1. Motivation and rationale

The benefits and costs of most projects are realized over an extended period of time. For instance, consider a project with a time horizon of T periods, with a stream of benefits, $B_1, ..., B_t, ..., B_T$, and a stream of costs, $C_1, ..., C_t, ..., C_T$. To appraise this project, some way must be found of appropriately comparing costs and benefits that occur at different points in time.

What is usually done is to *discount* the benefits and costs that occur in the future, i.e., to multiply costs and benefits that will be incurred/received in period t by a factor β_t where:

1) $\beta_t < 1$ for all $t > 1$

2) $\beta_t < \beta_{t-1}$ for all t

In other words, benefits received earlier are preferred to benefits that occur later, while costs incurred later are preferred to costs incurred earlier. The rationale behind discounting is fairly intuitive. Consider an example where the benefits and costs are financial.

Example: Would you prefer to receive $1000 today or a year from now?

What are some possible reasons you might prefer to receive $1000 today?

a) uncertainty about the future

b) subjective rate of time preference

c) opportunity cost of funds

We shall focus on the third reason. The presumption is that you have a benchmark alternative/fallback option which is to put the money into the bank and earn a safe/riskless rate of return.

Example: Going back to the earlier example, suppose that the prevailing interest rate on bank deposits is 10% per year. If you received the \$1000 today, you could deposit it, and with the interest you earn you'd have \$1000(1+0.10)=\$1100 a year from now. That's clearly preferable to receiving \$1000 a year from now. An alternative (but equivalent) way of arriving at the same conclusion is to note that receiving \$1000 a year from now is equivalent to receiving $\frac{\$1000}{(1+0.10)} = \$909$ today. Why? Because that's the amount, which when put in the bank, with interest added, results in \$909(1+0.10)=\$1000 a year from now.

In general, a dollar received today is worth more than a dollar received t periods in the future because a dollar received today can be invested and earn a return in the intervening periods. Let r denote the prevailing interest rate (per period) on bank deposits. A dollar received today can be deposited and will, with interest, be worth $(1+r)$ a period from now. Two periods from now the dollar will be worth $(1+r)^2$ and so on. Conversely, a dollar received a period from now is only worth $\frac{1}{(1+r)}$ today while a dollar received two periods from now is only worth $\frac{1}{(1+r)^2}$ today. To compare costs and benefits incurred/received at different points in time we therefore convert them into their *present* (i.e., current) worth. In terms of our notation, therefore:

$$\beta_t = \frac{1}{(1+r)^{t-1}}$$

So from today's perspective—note that I have chosen the time index such that the current period is treated as period $t = 1$—B_t dollars received in period t, which is $t-1$ periods from today, are worth only:

$$\beta_t B_t = \frac{1}{(1+r)^{t-1}} B_t$$

Any stream of benefits or costs can therefore be reduced to a single number which represents the *present discounted value* of that stream:

$$\begin{aligned} PVB &= \sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}} B_t \\ PVC &= \sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}} C_t \end{aligned}$$

Here PVB and PVC denote the present discounted value of benefits and costs respectively.

2. Present values: definition and some formulae

The present discounted value of a stream of net benefits, $(B_t - C_t)$, is called the *net present value (NPV)* of that stream and is given by:

$$NPV = \sum_{t=1}^{T} \frac{(B_t - C_t)}{(1+r)^{t-1}}$$

The net present value of $\$x$, if you were to receive it today is simply x. However, the net present value today of $\$x$ that you will receive t periods from now is:

$$\frac{1}{(1+r)^t} x$$

The net present value of a *finite annuity* that pays you \$$a$ every year for T years, *starting a year from now* is:

$$NPV = \frac{a}{(1+r)} + \frac{a}{(1+r)^2} + ... + \frac{a}{(1+r)^T} = \sum_{t=1}^{T} \frac{a}{(1+r)^t}$$

The net present value of an *infinite annuity* that pays \$$a$ starting a year from now, in every future year is:

$$\begin{aligned} NPV &= \frac{a}{(1+r)} + \frac{a}{(1+r)^2} + ... \\ NPV(1+r) &= a + \frac{a}{(1+r)} + \frac{a}{(1+r)^2} + ... \\ NPV(1+r) &= a + NPV \\ NPV &= \frac{a}{r} \end{aligned}$$

3. Choice of discount rate

Note that in calculating present discounted values, the choice of r, the interest rate (also known as the *discount rate*) is crucial. There is a huge literature on what the appropriate choice should be, which we will not go into. For private investment decisions the issues have to do with the opportunity costs of capital, costs of financing, etc. Standard corporate finance texts discuss the many details. For public cost-benefit analyses, the question is whether interest rates observed in existing capital markets are the appropriate discount rates from a societal perspective. Chapter 5 of the textbook discusses the various issues involved. For the purposes of this course, simply think of r as the interest rate that can be earned from a safe alternative investment such as a bank savings account. Note also, that there is no reason why r has to be the same in all periods. If the analyst anticipates changes in the interest rate in each future period, that can be taken into account by modifying the formulae for the present discounted values.

4. Treatment of inflation

The prospect of inflation, i.e., a rise in the general price level, raises two related issues:

a. First, you have to now be sure to take into account the possibility of inflation in projecting costs and benefits that occur in the future.

b. Second, you have to keep track of and distinguish between variables that are expressed in nominal terms versus variables that are expressed in real terms

The general rule you should follow is to be consistent in using either nominal or real values in all aspects of your analysis. If you express costs and benefits in nominal terms—i.e., in terms of the prices in effect in the year in which the costs or benefits are realized—then you should use the nominal interest rate to discount those costs and benefits. On the other hand if you choose to express costs and benefits in real terms—i.e., in terms of a constant set of prices, for e.g., today's prices—you should use the real interest rate to discount future costs and benefits.

For further details and an illustrative example look at Chapter 6 of the unpublished manual by Harberger and Jenkins that is on reserve.

C. Appraisal criteria

There are several different appraisal criteria that are used in project appraisal. We shall go through four of them:

1. net present value
2. internal rate of return
3. benefit-cost ratio
4. payback period

1. Net present value

• *Definition*

The net present value (NPV) of a stream of costs and benefits is simply the difference between the present discounted value of benefits and costs:

$$NPV \equiv PVB - PVC = \sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}}(B_t - C_t)$$

• *Admissibility rule*

A project is said to be *admissible*, i.e., worth undertaking if the $NPV > 0$. Note that this implies that the project generates a better return than investing in the benchmark alternative.

• *Admissibility rule with mutually exclusive alternatives*

When comparing admissible mutually exclusive projects, the decision rule changes to: pick the project with the highest NPV.

• *Advantages/disadvantages*

The NPV of a project is the cleanest and most reliable indicator of the project's feasibility. The main drawback of the NPV as the criterion for appraising projects is the fact that we need to specify a discount rate. In many situations it may be difficult to determine what the appropriate discount rate should be.

2. Internal rate of return

• *Definition*

The internal rate of return (IRR) of a stream of benefits and costs is the the rate of return or discount rate, i, at which that stream has a $NPV = 0$. Algebraically:

$$\sum_{t=1}^{T} \frac{1}{(1+i)^{t-1}}(B_t - C_t) = 0 \Leftrightarrow i = IRR$$

• *Admissibility rule*

Using the IRR criterion, a project is said to be admissible only if $IRR > r$, the discount rate. This follows directly from the definition of the IRR and the rule used to determine admissibility with the NPV criterion.

• *Advantages/disadvantages*

The main advantage of the IRR is that it offers a way of comparing projects when it is unclear what the appropriate discount rate ought to be. There are however several problems with the IRR as a criterion for choosing projects.

> -projects with net cashflow/benefit profiles that go from being negative to positive (or vice-versa) *more than once* during the lifetime of the project *may have multiple IRRs.*
>
> -even when an unique *IRR* exists, the *IRR* may be misleading as a criterion for choosing between *mutually exclusive* projects that are:
>
> ...significantly different in scale/size
>
> ...of different lengths

3. Benefit-cost ratio

• *Definition*

The benefit-cost ratio (BCR) of a project is defined as:

$$BCR \equiv \frac{PVB}{PVC}$$

• *Admissibility rule*

The admissibility rule when using the benefit-cost ratio is simple: projects with $BCR > 1$ are admissible. Any project with $NPV > 0$ will satisfy this criterion.

• *Advantages/disadvantages*

As with the *IRR*, the *BCR* can be misleading as a guide to choosing between mutually exclusive projects. There are two reasons for this:

> -first, the decision as to whether particular expenditures are classified as costs or are used to reduce the stated benefits is often based on fairly arbitrary accounting rules. With the *NPV* this classification is irrelevant. With the *BCR* it can artificially inflate or deflate the ratio of benefits to costs.
>
> -second, the *BCR* may be misleading as a criterion for choosing between projects of significantly different scale

4. Payback period

For projects that have the conventional cashflow profile associated with investments, i.e., an initial outlay (net cash-outflow) followed by net cash-inflows, the payback period offers a measure of how quickly the initial outlays are recovered. The theoretical basis for this criterion is weak. There are no clear rules for determining admissibility. It is however a popular criterion, perhaps because in settings where there is significant uncertainty, it provides an intuitive (though potentially misleading) feel for how quickly an investment will be repaid.

D. Operational issues

The *NPV* criterion, applied properly, provides a reliable guide to making investment decisions. It is worthwhile however to keep in mind several operational issues that are often overlooked in practice.

1. Considering mutually exclusive alternatives

In appraising projects, it is very important to consider mutually exclusive alternatives that are, by definition, being foregone if the project under appraisal is undertaken. Such mutually exclusive alternatives arise when;

a. the selection of projects *is limited by an overall budget constraint.* It becomes important then to recognize that in approving a particular project whose NPV may be positive, you are implicitly foregoing (because of the overall budget constraint) an alternative project whose NPV may be larger

b. the *scale* of the project is potentially alterable. Again, even if the scale being considered yields a positive NPV, it is worth exploring whether a smaller scale (or larger scale) project may have a larger NPV

c. the *timing* (i.e., when to undertake the project) of a project is alterable. While the impetus in many situations is to initiate projects immediately if the appraisal indicates a positive NPV, there may be instances where delaying the start of the project may raise the NPV.

d. the project being appraised can be split up into *smaller separable components*, or combined with other projects.

2. Treatment of opportunity costs

3. Treatment of sunk costs

E. Review: constructing cashflow profiles

In carrying out a financial appraisal, it is important to focus on cashflows. Most of the details of constructing cashflow statements from balance sheet information, and income and expense statements are covered in accounting courses. In this course we shall mostly ignore these details/complications. However, there are some basic issues/concepts you ought to be comfortable with, and these are:

1. cashflows vs. income/expenditure

2. treatment of depreciation

3. accounts receivable/accounts payable

4. working capital

5. cashflows from different perspectives

6. liquidity planning