Another example of a deep lake that undergoes irregular mixing to the bottom is Lake Tahoe in the California-Nevada Sierra mountains. Lake Tahoe has a maximum depth of 505 m, and in the coldest winters the whole water columnn approaches 4°C. The lake does not mix to the bottom every year, but only when there are severe winter storms, which occur every several years (Goldman 1988).

Throughout this discussion of lakes we have mentioned that, during stratification, deep water is isolated from the atmosphere. This is especially important because isolation from the atmosphere, accompanied by biological oxygen consumption in the deep water, results in a lowering of dissolved O_2. If the isolation is prolonged, all O_2 may be consumed and, as a result, the deep water becomes anoxic, resulting in dramatic changes in water quality. These changes are important and will be discussed later in the present chapter. However, it is important to remember that it is the physical process of stratification that initiates these chemical changes.

LAKE MODELS

Like water, other substances are added to and removed from lakes. Rivers and groundwater carry dissolved constituents into a lake, where they may undergo chemical reactions, and this is followed by removal via water flow through an outlet. One way of quantitatively treating rates of addition and removal is by means of *box modelling*. In box modelling one assumes that a portion of a lake or the whole lake is so well stirred that it is homogeneous in composition and can be treated as a uniform "box." Rates of addition (or removal) to each box are slow enough, relative to mixing within the boxes, that high concentrations of added substances do not build up around each source. [This situation is not always obeyed; see Imboden and Lerman (1978).] The concentration of a given substance in a given "box" is controlled by the relative magnitude of inputs and outputs. If inputs balance outputs, there is a steady state and concentrations do not change with time. This is analogous to (but not necessarily connected with) the situation of steady-state water content discussed earlier.

The simplest kind of box model is that of a single box representing a whole lake. In this case we have input of a dissolved substance from streams (groundwater and rainwater inputs are neglected), output by a surface outlet, precipitation and removal to bottom sediments, and addition via dissolution, or bacterial regeneration, of suspended and sedimented solids (see Figure 6.5). Rates of these processes can be represented as

F_i = rate of water inflow from streams (volume per unit time)

F_o = rate of water outflow through outlet

M = total mass of dissolved substance in the lake

R_p = rate of removal via precipitation and sedimentation (mass per unit time)

R_d = rate of addition via dissolution of solids during sedimentation or while solids rest on the bottom (mass per unit time)

C_i = concentration of dissolved substance in stream water (mass per unit volume)

C = concentration in lake water

t = time

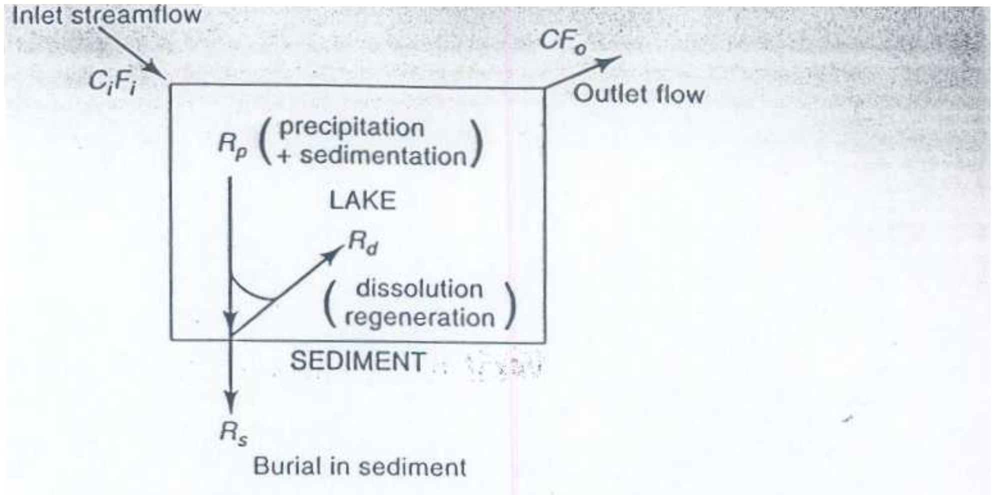

Figure 6.5. One-box model for lakes. (For explanation of symbols, see text.)

The rate of change of mass with time in the lake, $\Delta M/\Delta t$, is

$$\Delta M/\Delta t = C_iF_i - CF_o + R_d - R_p \tag{6.2}$$

If there is a steady state with respect to the dissolved substance, $\Delta M/\Delta t = 0$, then

$$C_iF_i - CF_o + R_d - R_p = 0 \tag{6.3}$$

Finally, if dissolution represents redissolution of the same material that was previously precipitated and sedimented toward the bottom, then

$$R_s = R_p - R_d \tag{6.4}$$

where R_s = rate of burial in sediments (mass per unit time).

Using Eq. (6.3), one can calculate, for example, from a knowledge of measured water flow and sedimentation rates, the maximum allowable input concentration (C_i) of a pollutant, P, to a lake, if the lake concentration of the pollutant (C) is not to exceed a certain level. Suppose the streamwater inflow rate (F_i) to the lake is equal to 100 m³/s (appropriate for a small river) and rainfall (minus evaporation) directly on the lake (averaged over the year) is 50 m³/s. Then the outflow rate (F_o) should be 150 m³/s in order to maintain constant lake volume. Let the sediment burial rate (sedimentation minus dissolution) (R_s) be 250 mg P/s and assume that the lake concentration (C) may not exceed 5 μg P/l (which equals 5 mg P/m³ and is sufficiently low that it should not promote eutrophication; see discussion below). Then, upon substituting in Eqs. (6.3) and (6.4),

$$C_i = \frac{CF_o + R_s}{F_i}$$

$$= \frac{(5\ \text{mg P/m}^3) \times (150\ \text{m}^3/\text{sec}) + (250\ \text{mg P/sec})}{100\ \text{m}^3/\text{sec}}$$

or

$$C_i = 10\ \mu g\ P/l$$

Two useful concepts, already applied to water itself, are those of replacement time and residence time. Replacement time is the hypothetical time necessary to replace the mass of a dissolved substance, via the present rate of stream addition, if all of the substance were suddenly removed. It gives a measure of the sensitivity of lake concentration C to changes in input concentration, C_i, or water inflow, F_i. (However, it should not be confused with the actual time necessary to change the concentration in a lake by changing the input, which is a more complicated situation and which normally requires the use of calculus). The replacement time of a dissolved substance is defined as

$$\tau_r = \frac{\text{mass in lake}}{\text{rate of stream input to lake}} = \frac{M}{C_i F_i} \quad (6.5)$$

Recalling from Eq. (6.1) that the replacement time for water is

$$\tau_w = \frac{V}{F_i}$$

(V = volume of lake) and that $M = CV$, then Eq. (6.5) can be rewritten as

$$\tau_r = \left(\frac{C}{C_i}\right) \tau_w \quad (6.6)$$

If the lake is at a steady state with respect to the dissolved substance of interest (and water), then τ_r (and τ_w) can be viewed as residence times as well as replacement times. In other words, for steady state the value τ_r represents the average time spent by a dissolved species in the lake prior to removal either via sedimentation or through the outlet.

An additional concept, that of relative residence time, (Stumm and Morgan 1981) is very useful. Relative residence time is the residence time of a given dissolved substance relative to that of water,

$$\tau_{rel} = \frac{\tau_r}{\tau_w} \quad (6.7)$$

or, from Eq. (6.6),

$$\tau_{rel} = \frac{C}{C_i} \quad (6.8)$$

Relative residence time is an indication of the type of behavior to be expected for a given substance. A relative residence time of one indicates that the substance does not react chemically in the lake ($R_d = 0$; $R_p = 0$), and it simply accompanies water as it passes through the lake. In this case the substance acts as a tracer of water motion (dissolved Cl^- is a good example). If τ_{rel} is less than one, the substance tends to undergo removal via sedimentation in the lake ($R_p > 0$), indicating its chemical reactivity. (An example is dissolved Al.) If τ_{rel} is greater than one, the substance tends to be trapped in the lake while the water that brought it in is removed. This can ake place if the substance is cycled within the lake, that is, it is precipitated and sedimented to he bottom ($R_p > 0$), then redissolved ($R_d > 0$), then reprecipitated, and so on. This is character-stic of elements involved in biological processes, for example, P, N, Si, and Ca, and such bio-ogical cycling within the lake can result in a relative residence time of each of these elements ıppreciably greater than 1.

Simple one-box models, although applicable to all lakes to express their *average* properties, ıre most accurate as representations of shallow lakes that do not undergo stratification. For the nore usual case of stratified lakes, a two-box model is more appropriate (e.g., Imboden and Ler-nan 1978; Stumm and Morgan 1981). One box is used to represent the epilimnion and the other ɔox the hypolimnion. This is shown in Figure 6.6. In the two-box model we have *fluxes* between he reservoirs (boxes) as well as inputs and outputs for the whole lake. Figure 6.6 represents the ituation expected for a biological element such as phosphorus or nitrogen. There is input of dis-olved material by streams to the epilimnion and output via an outlet. There is exchange of water containing the dissolved substance of interest between hypolimnion and epilimnion, which is epresented by up and down (short) arrows. (Actual exchange occurs sporadically during sea-onal overturn, but for modelling this is averaged over a year.) Finally, there are chemical reac-ions; in this case these include removal of the substances from the epilimnion via precipitation ınd transfer downward by sedimentation, injection of a portion to solution in the hypolimnion, ınd burial of the remainder.

Mathematical representation of the rates in a two-box lake model is similar to that present-ed above for a one-box lake. Besides the parameters defined for the one-box model we also have he following:

F_U = rate of water transfer from hypolimnion to epilimnion

F_D = rate of water transfer from epilimnion to hypolimnion

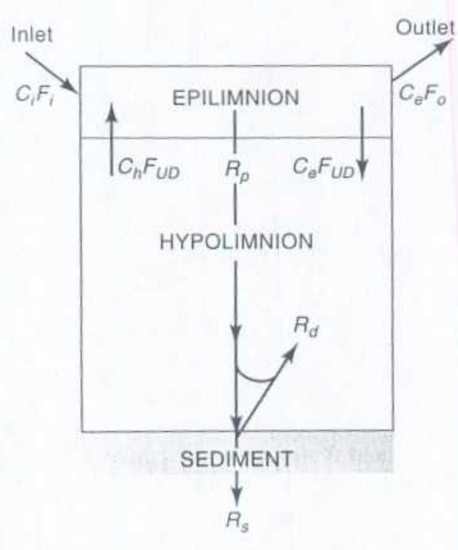

Figure 6.6. Two-box model for lakes (see text).

R_p = rate of removal, by precipitation and sedimentation, from epilimnion

R_d = rate of addition, via dissolution, to hypolimnion

M_e = mass dissolved in epilimnion

C_e = concentration in epilimnion (mass per unit volume)

M_h = mass dissolved in hypolimnion

C_h = concentration in hypolimnion (mass per unit volume)

If we have steady state with respect to water for both boxes (constant volumes of the epilimnion and hypolimnion, V_e and V_h, with time), then

$$F_U = F_D$$

and we can refer to either rate as F_{UD} (see Figure 6.6).

Using the above definition, and referring to Figure 6.6, we obtain the rates of change of mass in solution for each box $\Delta M_e/\Delta t$ and $\Delta M_h/\Delta t$, by summing inputs and outputs:

$$\frac{\Delta M_e}{\Delta t} = C_i F_i - C_e F_o + (C_h - C_e)F_{UD} - R_p \quad (6.9)$$

$$\frac{\Delta M_h}{\Delta t} = R_d - (C_h - C_e)F_{UD} \quad (6.10)$$

If in addition, we have steady state with respect to both the dissolved substance and its solid precipitated form, and dissolution represents redissolution of sedimenting solids, we have

$$C_i F_i - C_e F_o + (C_h - C_e)F_{UD} - R_p = 0 \quad (6.11)$$

$$R_d - (C_h - C_e)F_{UD} = 0 \quad (6.12)$$

$$R_s = R_p - R_d \quad (6.13)$$

From these equations we can thus gain information on rates of processes from measurements of other rates and concentrations and, in this way, the equations are most useful.

BIOLOGICAL PROCESSES IN LAKES AS THEY AFFECT WATER COMPOSITION

Photosynthesis, Respiration, and Biological Cycling

One of the main reasons why hypolimnia and epilimnia differ in chemical composition is that each is greatly, but differently, affected by biological processes. The starting point is photosynthesis.