Math Review #2

"I just got lost in thought. It was unfamiliar territory"

"What happens if you get scared half to death twice?"

Math Review
Thursday June 12 2003

A) Introduction
   a. Symbols
   b. Operations
   c. Central Tendencies

B) Linear Algebra

C) Correlation/Regression Analysis

D) System of Equations: Linear/Quadratic

E) Applied Calculus

Basic Math Review

b. Operations

Why logarithms?

Power and product rules:

$\log_b(xy) = \log_b(x) + \log_b(y)$

$\log_b(x^n) = n\log_b(x)$

These rules motivated the introduction of logarithms (by Napier, in early 17th Century) and motivated their use in scientific computation until… computers!

Basic Math Review

b. Operations

Why logarithms?

Example: Calculate $2^{18} \times 7^5$

First use logs, then use log tables:

$$y = 2^{18} \times 7^5$$

$$\text{Log } y = \text{Log } (2^{18} \times 7^5)$$

http://www.sosmath.com/tables/logtable/logtable.html

Basic Math Review

b. Operations

a) Solve for x: $\ln(e^a) = b^x$

b) Solve for y using common logarithms (base 10): $y = 17^5$

c) Find the exponent of 10 that solves for x: $x^2 = 5.5.10^{-12}$

Basic Math Review

c. Central Tendencies

The most commonly used descriptive statistics are measures of central tendency

The ***sample mean*** (: pronounced "*x* bar") is:

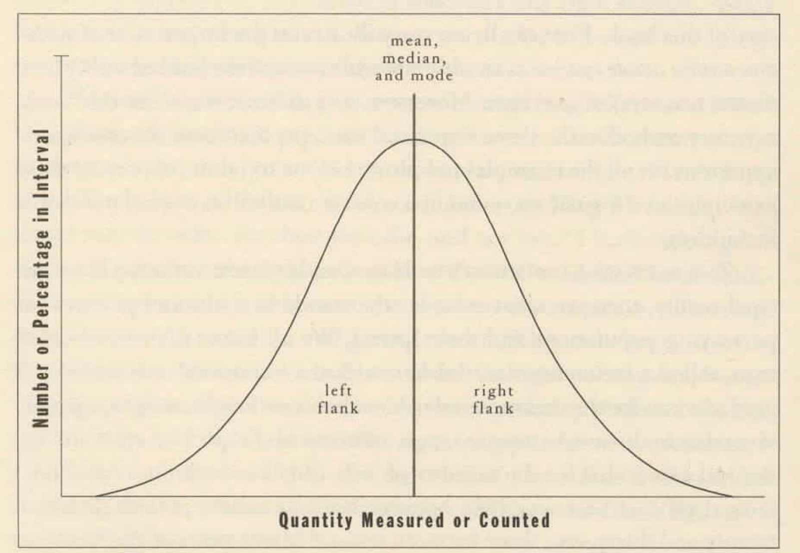
$$\bar{x} = \frac{\sum x_i}{n} = \frac{1}{n}\sum x_i$$

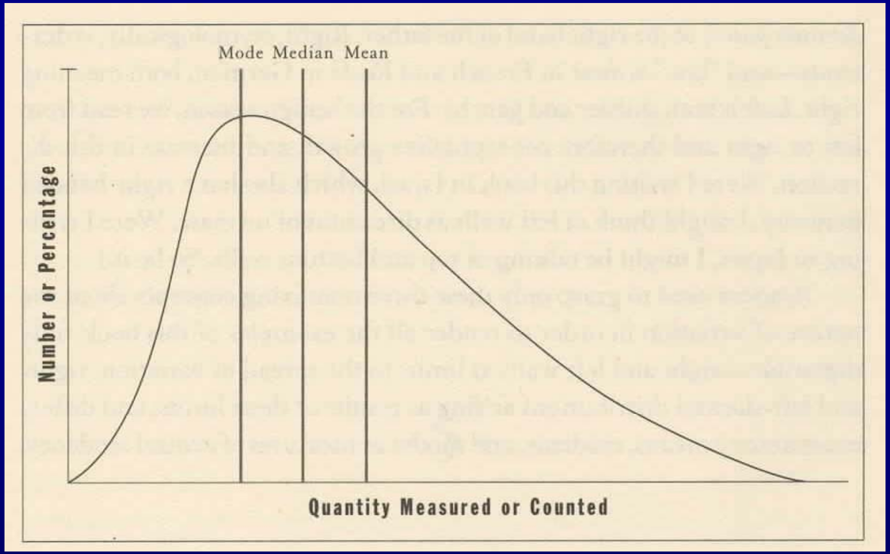
Where Σx_i represents the sum of all values in the sample and *n* represents the sample size

Basic Math Review

c. Central Tendencies

- Mean: arithmetic average
- Median: middle value of a set of values
- Mode: the data value that occurs most often

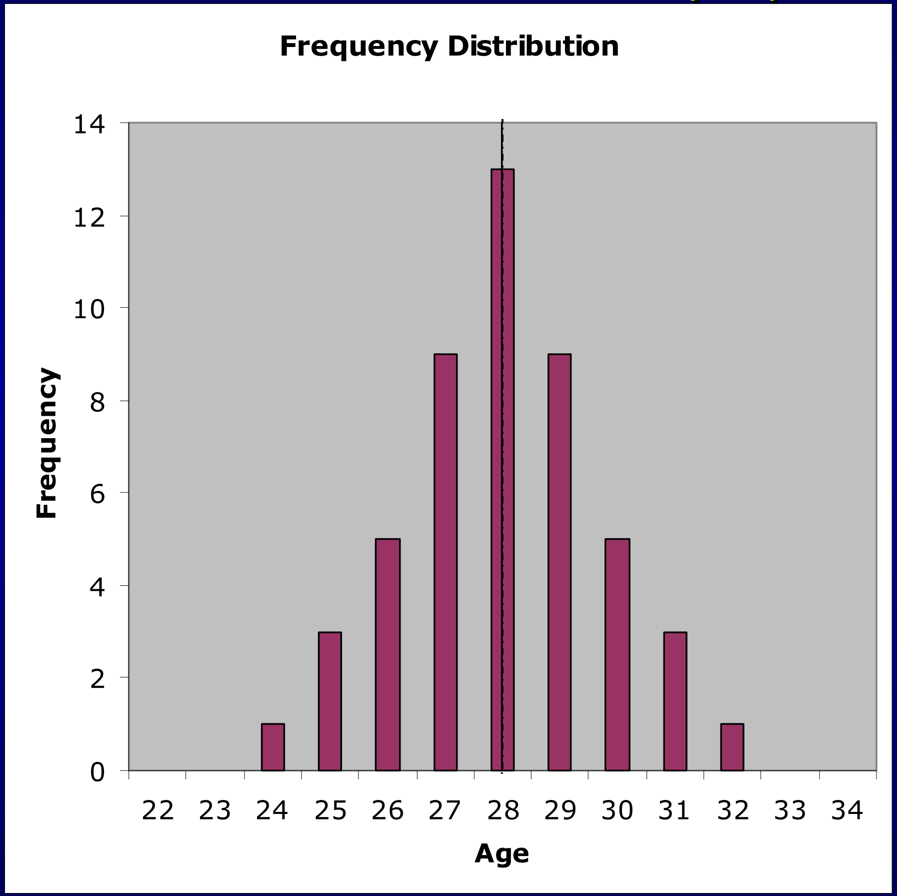




Basic Math Review

c. Central Tendencies

Let's assume we have a student population (n = 47)

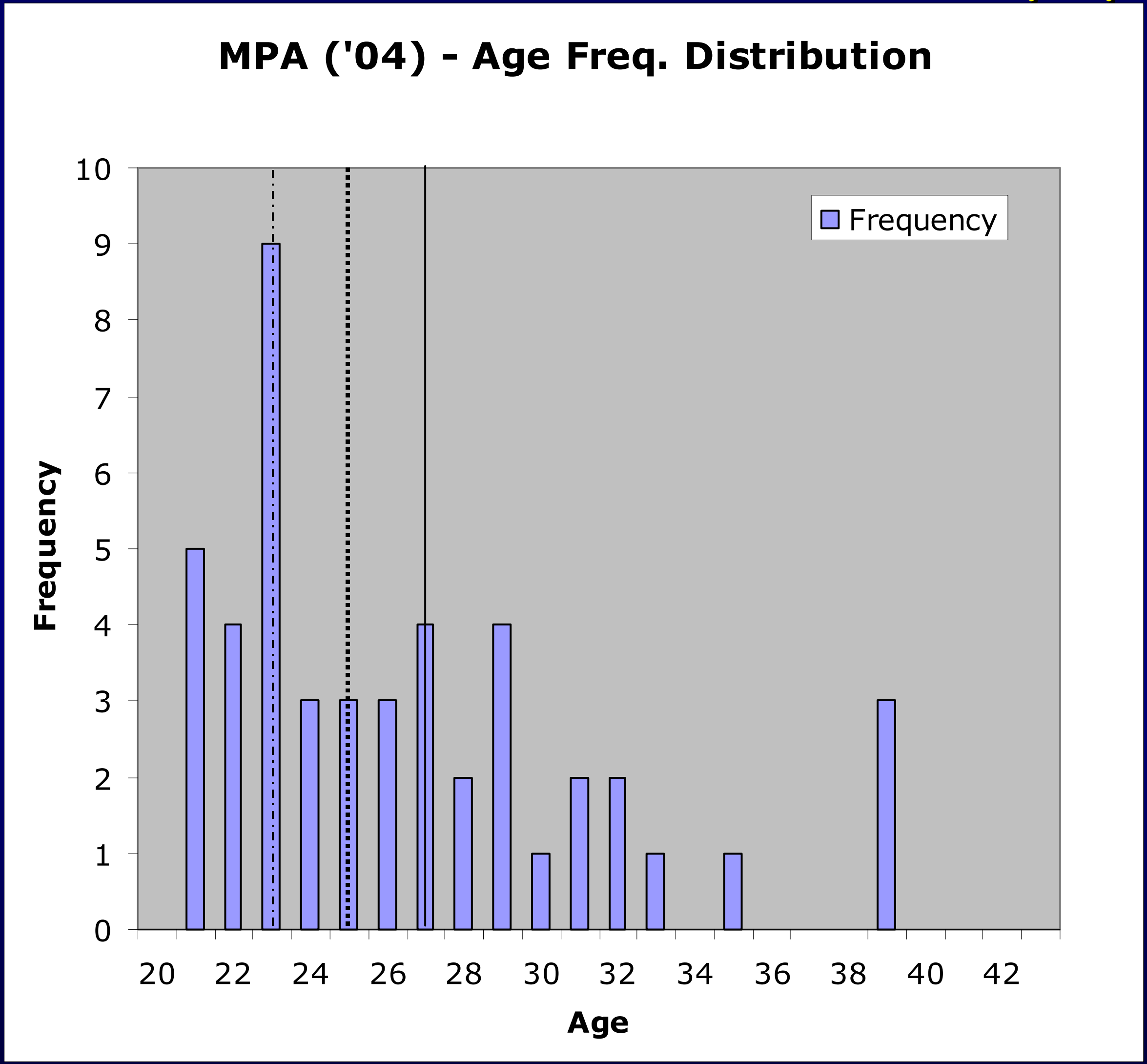


But what happens if we have an outlier (skewed distribution)?

Basic Math Review

c. Central Tendencies

Let's assume we have a real student population



Basic Math Review

B) Linear Algebra

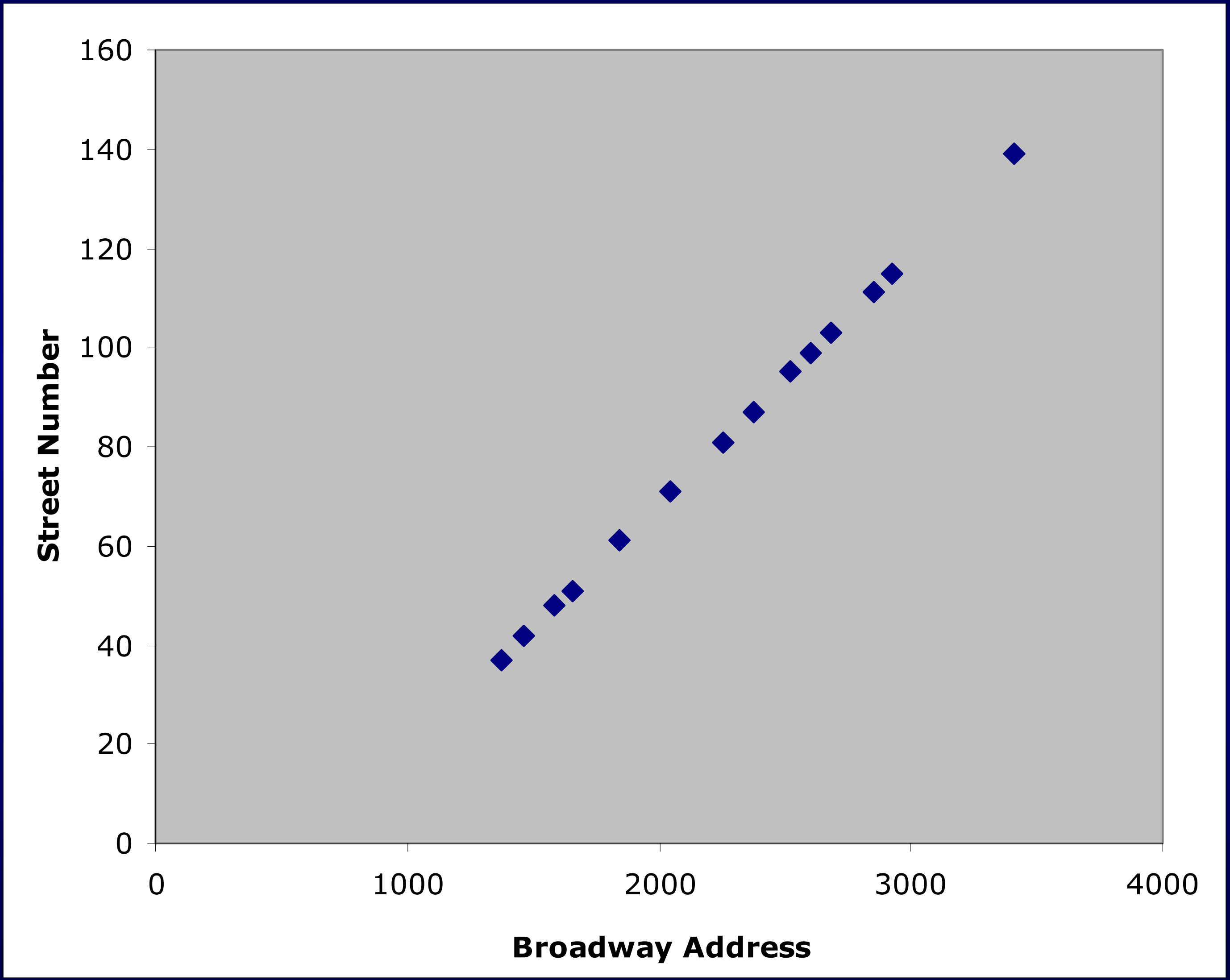
Let's play…

Address#	Street #
1372	37
1585	48
1656	51
2252	81
2379	87
2681	103
1460	42
1841	61
2045	71
2600	99
2851	111
3410	139
2521	95
2929	115

Starbucks anyone?

Basic Math Review

B) Linear Algebra



Basic Math Review

B) Linear Algebra

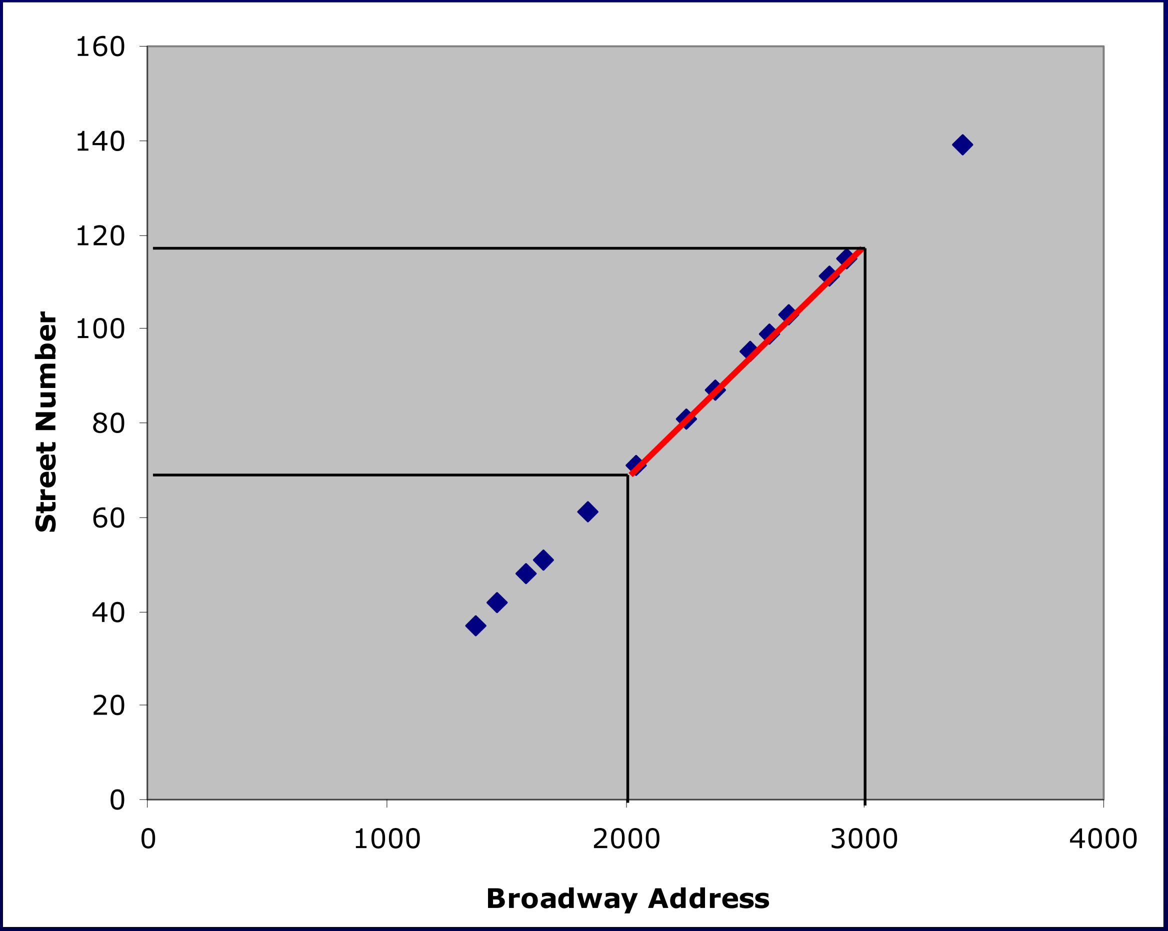
The "slope" (m) of a line is its rate of change:

Slope:

$\Delta y / \Delta x$

or

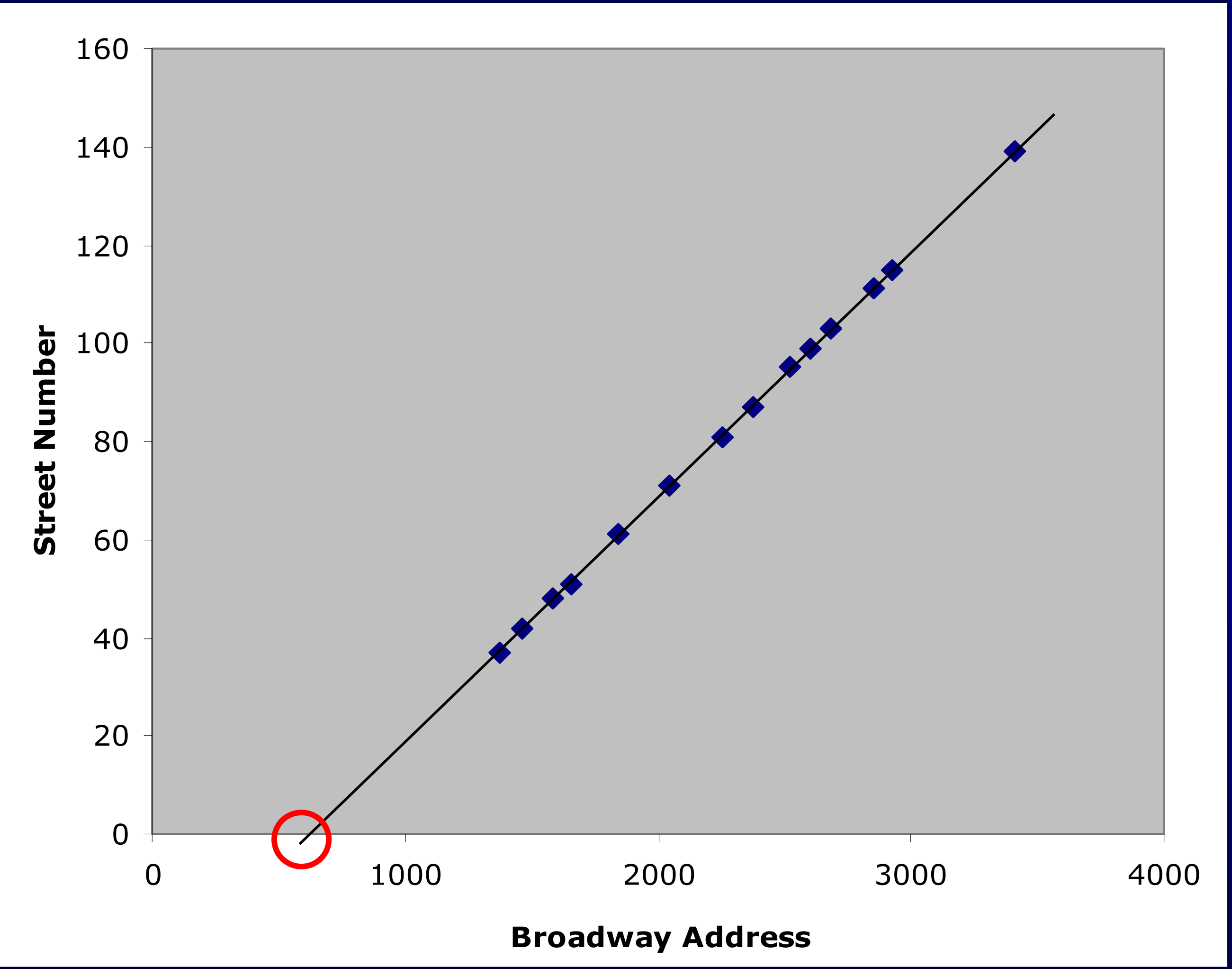
$(y_2 - y_1)/(x_2 - x_1)$



Basic Math Review

B) Linear Algebra

The "intercept" (b) of a line is the point where x = 0

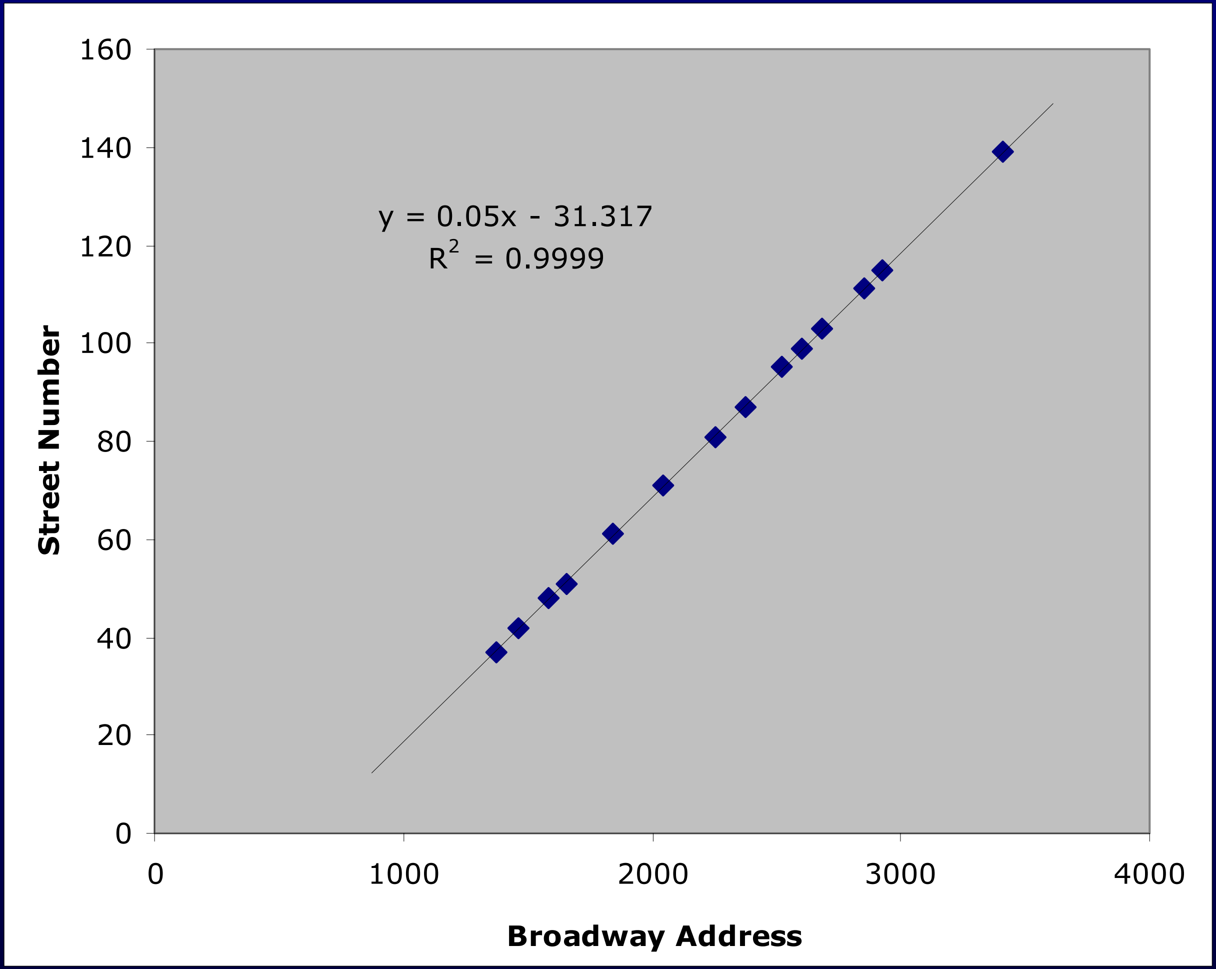


Basic Math Review

B) Linear Algebra

The function $f(x) = y = \mathrm{m}x + b$

You can use it to make predictions



Basic Math Review

B) Linear Algebra: System of equations

a) $3x - y = -7$

$5y + 5 = -5x$

b) $4x - y - 1 = 0$

$2x = 17 - y$

c) $x + 2y = 1$

$5x + 3y = 26$

d) $3x + 4y = 2$

$2y = 4 - 3/2x$

Basic Math Review

B) Linear Algebra

Let's assume we have a <u>real</u> fish population

Weight (lb)	Length (in)
1.18	13.53
1.35	14.5
1.71	13.5
1.72	16.03
1.99	16.42
2.02	15.83
2.58	15.72
4.26	21.1
4.5	21.47
7.31	22.96
7.99	24.39
8.1	23.17

Any question regarding this data set?

Basic Math Review

B) Correlation

The ***sample mean*** is:

$$\bar{x} = \frac{\sum x_i}{n} = \frac{1}{n}\sum x_i$$

Sum of squares for variable x. This statistics quantifies the spread of variable x:

$$SS_{XX} = \sum_{i=1}^{n}(x_i - \bar{x})^2$$

Basic Math Review

B) Correlation

Sum of squares for variable y. This statistics quantifies the spread of variable y:

$$SS_{YY} = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

Basic Math Review

B) Correlation

Sum of the cross-products. This statistics is analogous to the other sums of squares except that it quantifies the extent to which the two variables go together or apart:

$$SS_{XY} = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

Basic Math Review

B) Correlation

Fish Data:

Weight (lb)	Length (in)
1.18	13.53
1.35	14.5
1.71	13.5
1.72	16.03
1.99	16.42
2.02	15.83
2.58	15.72
4.26	21.1
4.5	21.47
7.31	22.96
7.99	24.39
8.1	23.17

SSxx: 78.5

SSyy: 182.0

SSxy: 113.8

The correlation coefficient is:

$$r = \frac{SS_{XY}}{\sqrt{(SS_{XX})(SS_{YY})}}$$

Here r = 0.95

Basic Math Review

B) Correlation: Fish Data

Weight (lb)	Length (in)
1.18	13.53
1.35	14.5
1.71	13.5
1.72	16.03
1.99	16.42
2.02	15.83
2.58	15.72
4.26	21.1
4.5	21.47
7.31	22.96
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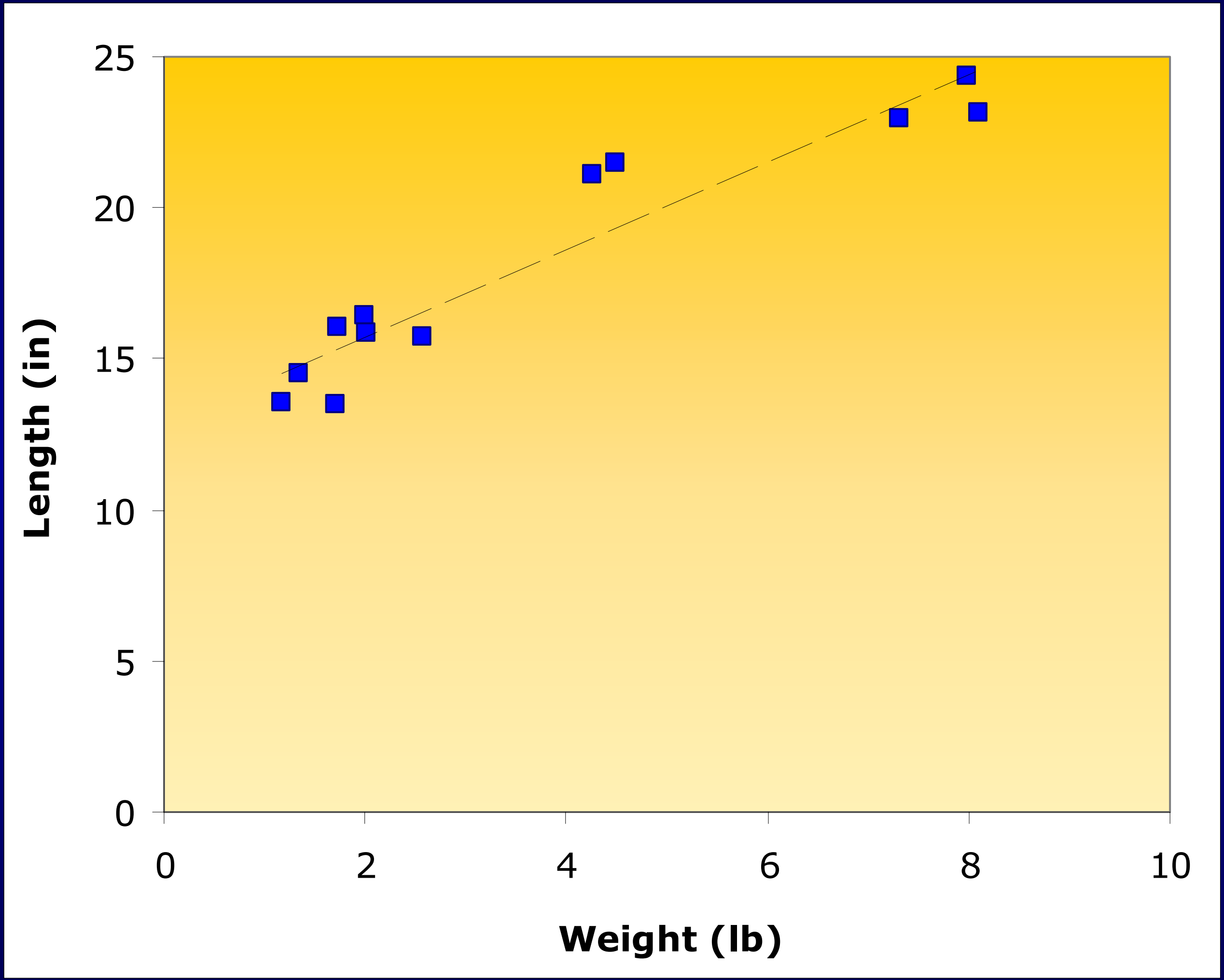


The correlation coefficient is positive

Basic Math Review

B) Correlation:

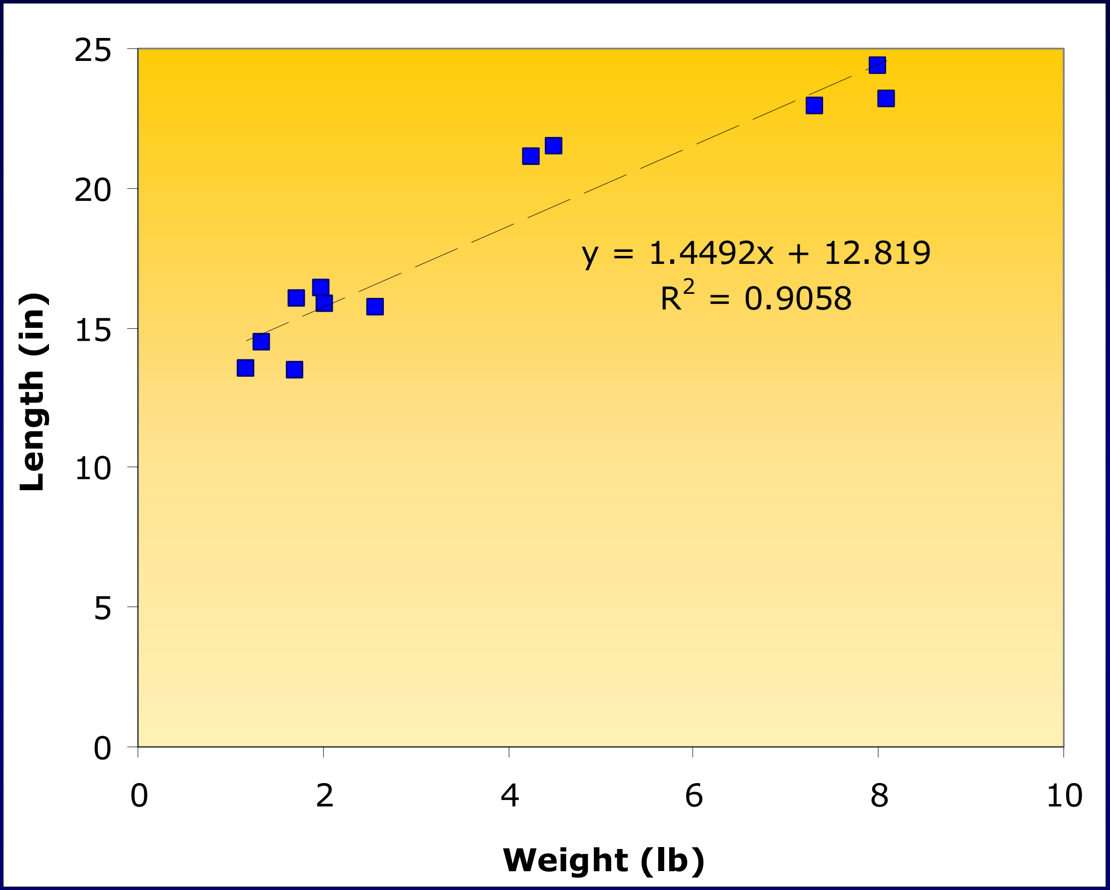
the correlation coefficient has no inherent value, and in the exception of strong relationships as in the case presented, r is hard to use to determine correlational strength. Another statistics is much more useful: the coefficient of determination (r^2)

Weight (lb)	Length (in)
1.18	13.53
1.35	14.5
1.71	13.5
1.72	16.03
1.99	16.42
2.02	15.83
2.58	15.72
4.26	21.1
4.5	21.47
7.31	22.96
7.99	24.39
8.1	23.17

Basic Math Review

B) Correlation:

Here $r^2 = 0.91$



This statistic quantifies the proportion of the variance of one variable that is explained by the other – Functional?

Basic Math Review

B) Linear Algebra

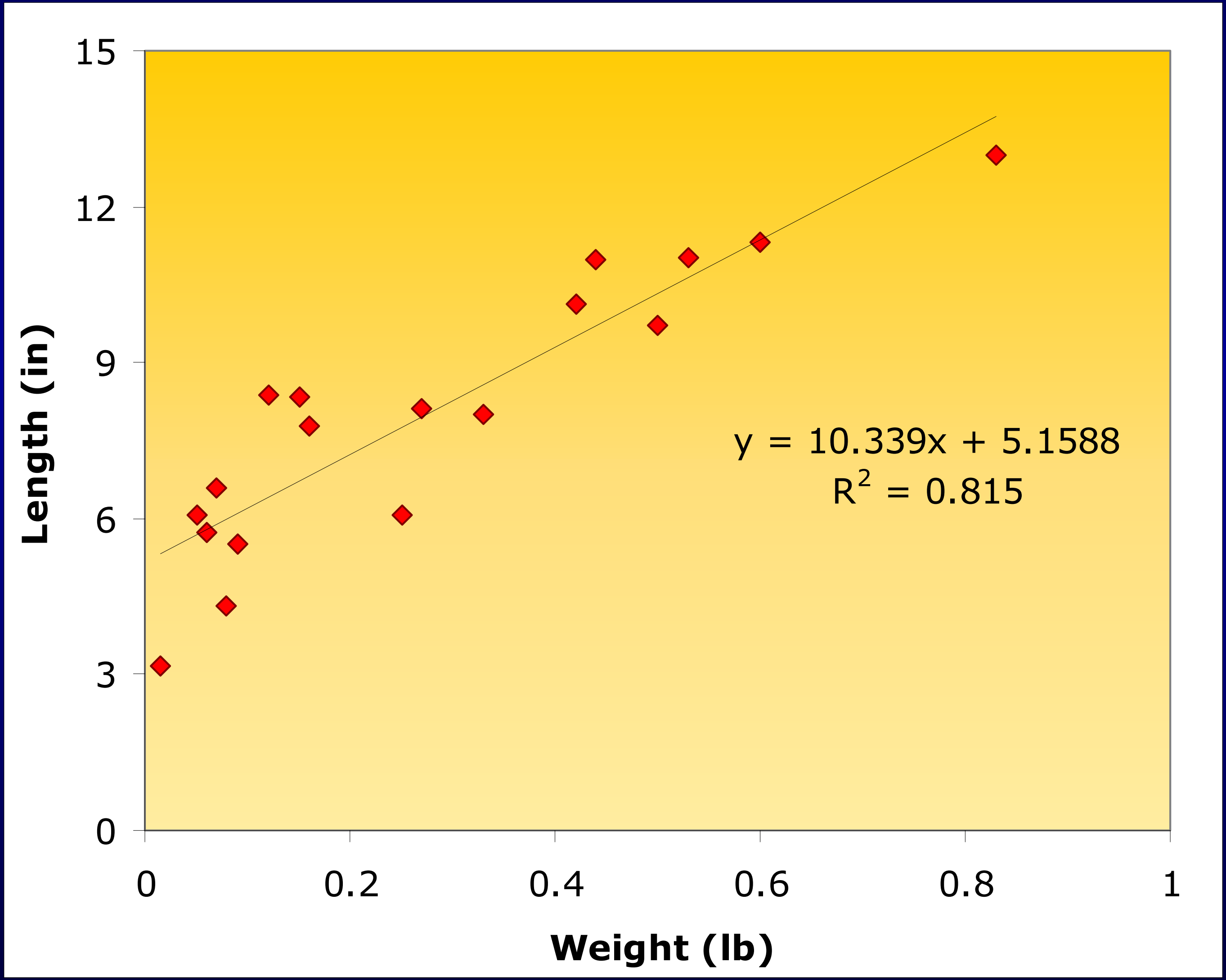
Forgot a section of the fish data set

Weight (lb)	Length (in)
0.015	3.16
0.05	6.07
0.06	5.72
0.07	6.57
0.08	4.32
0.09	5.52
0.12	8.39
0.15	8.32
0.16	7.79
0.25	6.05
0.27	8.11
0.33	8
0.42	10.13
0.44	10.97
0.5	9.72
0.53	11.02
0.6	11.33
0.83	13

Basic Math Review

B) Correlation:

Here $r^2 = 0.82$



Friday

- Water I
- Math Review:
  - More on linear (and non-linear) relationships
  - System of Equations: Linear/Quadratic
  - Applied Calculus

- Don't forget the website AND the math sheets!

http://www.columbia.edu/~pl2065/courses/mpa.htm