MICROECONOMICS AND POLICY ANALYSIS - U8213
Professor Rajeev H. Dehejia
Class Notes - Spring 2001

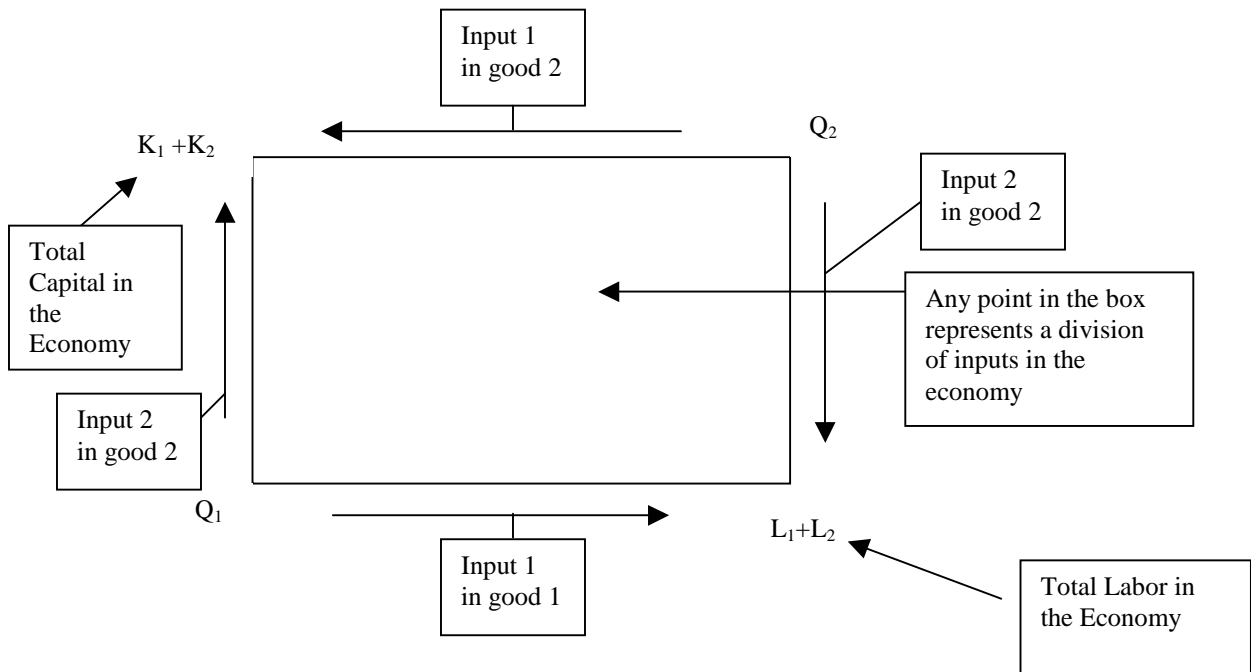## General Equilibrium and welfare with production
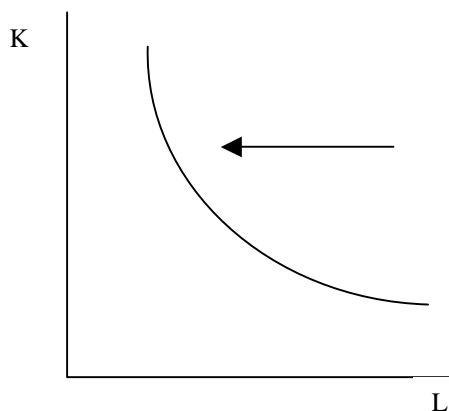**Wednesday, January 24th and Monday, January 29th**
*Reading:* PR Chapter 16

## The Edgeworth Box and Production
-The Edgeworth box in production operates similar to consumption

Input 1
in good 2

$K_1 + K_2$

$Q_2$

Input 2
in good 2

Total
Capital in
the
Economy

Any point in the box
represents a division
of inputs in the
economy

Input 2
in good 2

$Q_1$

$L_1 + L_2$

Input 1
in good 1

Total Labor in
the Economy

## Isoquant for a Firm

K

The slope of the isoquant is the **Marginal
Rate of Technical Substitution (MRTS)** -
the rate at which firms can use inputs to
produce an output. The isoquant itself
represents the different combinations of
capital and labor that produce the same level
of output

L

# What division of inputs between 2 firms is an efficient division?

$Q_2$
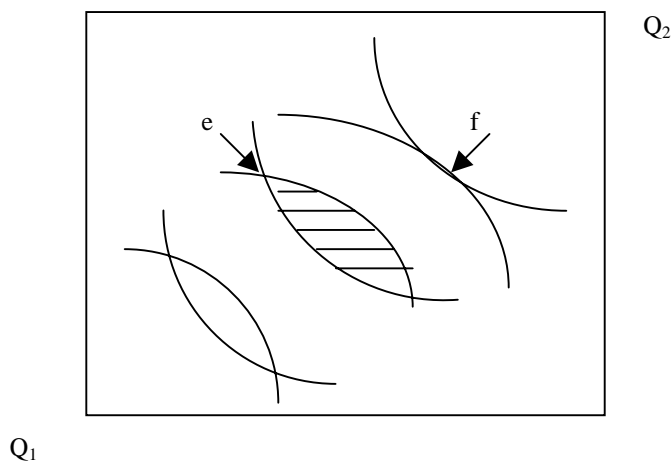


Point e is not an efficient allocation of inputs between the two firms. At e there are gains to the reallocation of inputs. Both firms can increase their level of output in the shaded region.

When the isoquants of the two firms are tangent they are at an efficient point. This is true at point f.

$Q_1$

$Q_2$
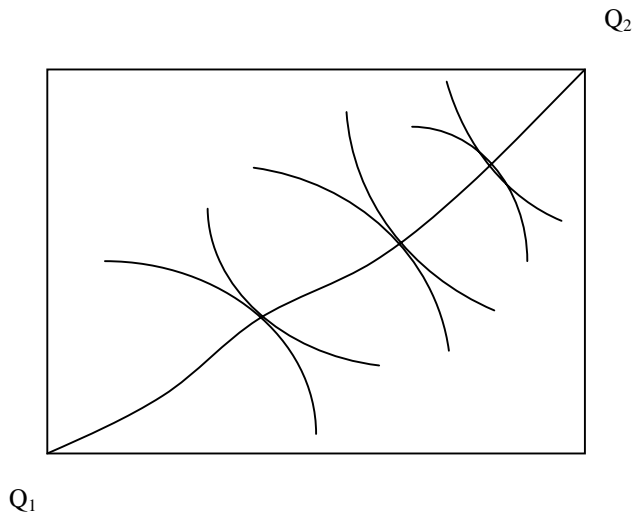


The set of efficient points in the Edgeworth box occur at every point where the isoquants of the two firms are tangent. At this points the **MRTS₁=MRTS₂**. The set of all points where the MRTS of both firms are equal is the **CONTRACT CURVE**. The shape of the curve depends on the level of technology used by the firms.

$Q_1$

Any point on the contract curve corresponds to a diagram of outputs. Plugging input values into the production function gives the amount of output produced. If you plot each point from the contract curve (left) on the diagram on the right you get the **Production Possibilities Frontier (PPF)** which is the combination of 2 goods that can be efficiently produced in an economy. At these points there are no inputs being wasted.

$L_2$                                         PPF
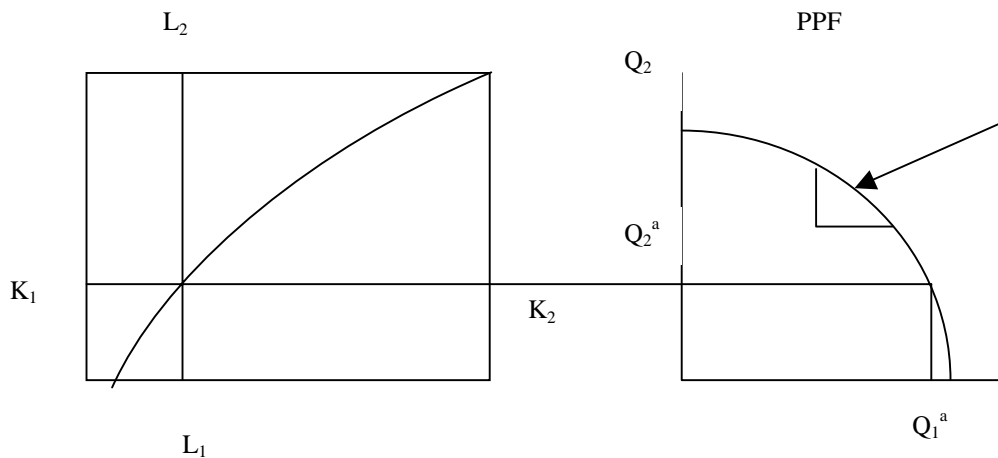


The slope of the PPF is the **Marginal Rate of Transformation (MRT)**. This is the rate at which you can trade the production of one good for another. If input use is implicit this is the trade-off between the production of goods.

2

## A Numerical Example of the Contract Curve

$Q_1 = K_1{}^{1/2} L_1{}^{2/3}$

$Q2 = K_2{}^{2/3} L_2{}^{1/3}$

Production Functions for both firms

To compute the contract curve you need to set $MRTS_1 = MRTS_2$

$$MRTS_1 = \frac{MP_L}{MP_k} = \frac{2/3K^{1/2} L^{-1/3}}{1/3K^{-1/3} L^{2/3}}$$

$$MRTS_1 = \frac{2K}{L}$$

$$MRTS_2 = \frac{K}{2L}$$

$MRTS_1 = MRTS_2$

$$\frac{2K_1}{L_1} = \frac{K_2}{2L_2}$$

\*Remember: There is a total amount of labor and capital in the economy (like a budget constraint). In this case the constraint must be
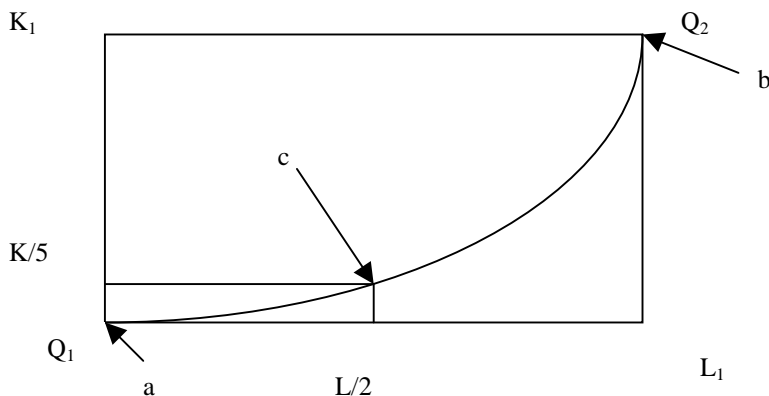
$\frac{(K_1 + K_2)}{2(L - L_1)}$ = total capital in economy
= total labor in the economy

This should be substituted in for the MRTS of firm 2.

When you plug in the constraint and solve the formula for the contract curve is:

$$K_1 = \frac{-KL_1}{3L_1 - 4L}$$

Plugging points into the equation you can graph the contract curve:



$L_1 = 0$ then $K_1 = 0$ (a)
If $L = L$ then $K = K$ (b)
If $L = L/2$ then $K = K/5$ (c)

## What is the point of the PPF?

1) The PPF is useful in understanding international trade
2) These ingredients are sufficient to make an argument that a market economy operates efficiently - There are three allocation rules.

**ALLOCATION RULE #1** (A single firm across goods)
A firm with fixed resources has allocated those resources efficiently if it has them fully employed and the MRTS between inputs is the same for every output the firm produces.

If a firm is producing multiple outputs the use of inputs across these outputs should be efficient. In other words, for outputs A, B, and C the $MRTS_A = MRTS_B = MRTS_C$ for all goods a firm produces.

Why?
For a cost minimizing firm:
$$MRTS_A = \frac{MP^A_L}{MP^A_K} = \frac{W}{R} \quad \longleftarrow \quad \text{Wage Rate}$$

Rental Rate/Price of Capital

and

$$MRTS_B = \frac{W}{R}$$

If W and R are equal within a firm then $MRTS_A = MRTS_B = MRTS_C$ for a profit-maximizing firm. If it weren't true then a firm could reduce costs by transferring a unit of labor and capital to the production of another good.

**ALLOCATION RULE #2** (A single good across different firms)
If production is to be efficient resources should be allocated so that the marginal product of any resource in the production of a particular good is the same no matter which firms produces that good (Law of one Price: Firms face the same input prices).

In other words, for firms 1 and 2 and good A,

$$MRTS^1_A = MRTS^2_A = \frac{W}{R}$$

Even across firms - if labor is being more productive at the margin in one firm than another then shifting this unit will make the economy more efficient (This will be demonstrated further in the input market lecture).

**ALLOCATION RULE #3** (Multiple firms and multiple goods)
If two or more firms produce the same outputs they must operate at those points on their PPF at which the MRT across goods is equal across firms.

In other words, for firms 1 and 2 and goods A and B, $MRT^B_{A1} = MRT^B_{A2}$.

Why?

$$MRT = \frac{MC_A}{MC_B}$$ ⎤— Rate of MC of producing the 2 goods

The MRT is the trading of good A for good B, therefore, the slope of the PPF is the MC of producing the goods.

$$MRT^B_{A1} = \frac{MC_A}{MC_B} = \frac{P_A}{P_B} = MRT^B_{A2} = \frac{MC_A}{MC_B}$$

Because each firm faces the same price the MRT will be equated across firms.

## **Why is the market production economy efficient?  How do we know the mix of goods produced is what is desired by society**?

$$\frac{P_A}{P_B} = MRS \quad \text{(Prices are given in the market)}$$

$$\frac{MU_A}{MU_B} = MRS = \frac{P_A}{P_B} = \frac{MC_A}{MC_B} = MRT$$

↗

Utility
Maximizing
motive

Or in other words,  $MRS^B_{A1} = MRT^B_A$.
The value at the margin is equal to the value at the margin of those goods to the firm.  Goods are valued at the margin in the same way.

Argument for
the efficiency
of markets.
⎰ **1$^{st}$ fundamental theorem of welfare economics**:  A market economy will produce a pareto efficient optional allocation.

**2$^{nd}$ fundamental theorem of welfare economics:**  Any pareto optimal allocation could be supported as the equilibrium of a competitive market with redistribution and transfers.

Keep in mind, however, that there are market imperfections, especially in free trade, where these rigorous conditions are not true.

## **Vocabulary**

MRT   Marginal Rate of Transformation
MRTS Marginal Rate of Technical Substitution

PPF Production Possibilities Frontier
R Rental Rate/Price of Capital
W Wage Rate