MICROECONOMICS AND POLICY ANALYSIS - U8213
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Class Notes - Spring 2001

Imperfect Competition
Wednesday, March 21st
Reading: Pindyck/Rubinfeld Chapter 12

Strategic Interaction – figure out how rival firms' actions affect the firm's own actions

Duopoly – two sellers rather than one.

Example
Imagine a case of a duopoly where the profits of the two firms can be represented by:

$$\Pi_1 = pq_1 - C(q_1) \qquad\qquad \Pi_2 = pq_2 - C(q_2)$$

At first glance it appears that both firms operate independently. But they don't. What is the source of their interaction? It is with price. There is one market clearing price that depends on both of their output decisions.

A monopolist does not take the price of a good as given but takes the price from the demand for quantity produced. In this case, profit can be represented using the demand function:

$$\Pi_1 = D(q_1 + q_2)q_1 - C(q_1) \qquad\qquad \Pi_2 = D(q_1 + q_2)q_2 - C(q_2)$$

Each firm cannot decide their output unless they know the other firm's output. In the case of a duopoly, profit becomes very sensitive to the other firm's output. One way to solve the problem is to use the concept of the Nash Equilibrium. The quantity produced by each firm is determined by each firm assuming the action of the other firm will be the Nash Equilibrium.

The Cournot model

Each firm treats the output of its competitors as fixed, and all firms decide simultaneously how much to produce (Pindyck/Rubinfeld, p. 431). The Cournot equilibrium, introduced by the French economist Augustin Cournot in 1838, is interesting because it is the pre-cursor to the Nash Equilibrium and is itself an example of a Nash Equilibirum.

Let the Nash Equilibrum = (q_1*, q_2*)

Assume the demand function to be:
$P = 100 - Q$

And the cost function for each firm to be:
$C(q_1) = q_1$
$C(q_2) = 2q_2$

Then you can maximize profits for firm 1:

$\Pi_1 = D(q_1 + q_2)*q_1 - C(q_1)$

$\text{Max } \Pi_1 = (100 - q_1 - q_2)*q_1 - q_1$
$\quad q_1$

$\text{Max } \Pi_1 = 100q_1 - q_1^2 - q_1q_2 - q_1$
$\quad q_1$

$\dfrac{\delta\Pi_1}{\delta q_1} = 100 - 2q_1 - q_2 - 1$

$q_1 = \dfrac{(100 - q_2 - 1)}{2}$ ⟵ Reaction function for q_1

Same thing for Π_2

$\Pi_2 = D(q_1 + q_2)*q_2 - C(q_2)$

$\text{Max } \Pi_2 = (100 - q_1 - q_2)*q_2 - 2q_2$
$\quad q_2$

$\text{Max } \Pi_2 = 100q_2 - q_2^2 - q_1q_2 - 2q_2$
$\quad q_2$

$\dfrac{\delta\Pi_2}{\delta q_2} = 100 - 2q_2 - q_1 - 2$

$q_2 = \dfrac{(100 - q_1 - 2)}{2}$ ⟵ Reaction function for q_2

Now find the Nash equilibrium by solving for q_1 and q_2 using the reaction functions:

Substitute the reaction function for q_2 into the reaction function for q_1

$q_1 = \dfrac{100 - \dfrac{(100 - q_1 - 2)}{2} - 1}{2}$

$q_1 = \dfrac{100 - 50 + 1/2q_1 + 1 - 1}{2}$

$q_1 = 25 + 1/4q_1 = \dfrac{25 * 4}{3} = 33 \text{ and } 1/3$

substitute q_1 into the reaction function for q_2

$q_2 = \dfrac{100 - (33.33333) - 2}{2} = \dfrac{200 - 6}{6} = 32$ and $1/3$

However, the firms may figure out that they may be more profitable by colluding.
Firms would make a joint decision based on maximizing joint profits.

In general firms will be more profitable by colluding (i.e., maximizing joint profits). In general, the sum of their joint profits will be greater than the sum of their Cournot-Nash Equilibrium profits. Why? In the latter case, each firm is not taking into account the influence that its pricing decision has on its rival.

Max $(\Pi_1 + \Pi_2) = (100 - q_1 - q_2)*q_1 - q_1 + (100 - q_1 - q_2)*q_2 - 2q_2$
q_1, q_2

$\dfrac{\delta(\Pi_1 + \Pi_2)}{\delta q_1} = 100 - 2q_1 - q_2 - 1 - q_2 = 0$

$\dfrac{\delta(\Pi_1 + \Pi_2)}{\delta q_2} = 100 - q_1 - 2q_2 - 2 - q_1 = 0$

We observe that there is not enough information to solve for q1 and q2 to maximize joint profits? Why? Looking at the cost structure of each firm more closely we observe that they have constant marginal cost. This means that to maximize joint profits, for any desired quantity, it should be produced by the firm with the lower cost. This is an example of a corner solution, where taking derivatives and setting them equal to zero doesn't help us find the maximum. To solve this, we set q2=0 and then maximize profits.

Max $(\Pi_1 + \Pi_2) = (100 - q_1)*q_1 - q_1$
q_1

$\dfrac{\delta(\Pi_1 + \Pi_2)}{\delta q_1} = 100 - 2q_1 - 1 = 0$

$q_1 = 49.5$
$q_2 = 0$

We can confirm that joint profits from collusion are greater than the sum of Cournot profits. This means that the firms could reach an agreement to collude which would make them both better off as a result of the agreement. However, note that such an agreement would difficulties of enforcement. For example, given that q1=49.5, what does firm 2 want to produce? That is given by the reaction function for firm 2, and it's not zero.

From the first example (Cournot-Nash Equilibrium model), the profit would be:

$\Pi_1 = 1111.09$
$\Pi_2 = 1045.43$

From the second example, when the firms collude the total profit of the two firms would be more than if they competed:

$$\Pi_1 + \Pi_2 = 2450.25$$

The Bertrand Model

Instead of basing decisions on quantities produced, firms can make decisions based on price. The Bertrand Model treats the *price* of its competitors as fixed, and all firms decide simultaneously what *price* to charge (Pindyck/Rubinfeld, p. 437)

Firms will continue to undercut each other until the price is set at the marginal cost. It can't have a price equilibrium above the marginal cost because firms will continue to undercut until the price is at the marginal cost.

Note how sensitive the predictions of the model is depending on whether we specify quantity competition (Cournot) or price competition (Bertrand). In the latter case, with just two firms we are driven to P=MC. With Cournot, as we have more and more firms we would closer to P=MC, but it would take a large number of firms.

Pricing Games (the Stackelberg Model)

Solve the decision making process backwards. Assume that one firm can set its output before the other firms do.
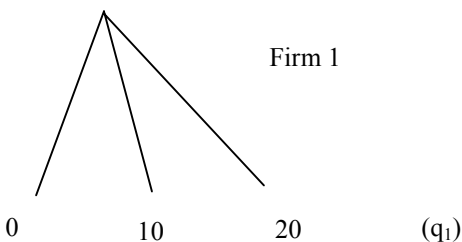
Firm 1

| Firm 1 makes a decision on what quantity to produce (q_1) |

0   10   20   (q_1)
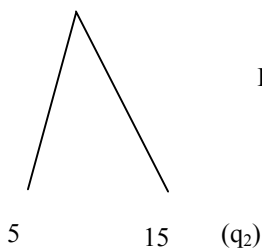
Firm 2

| Based on Firm 1's quantity (q_1), Firm 2 decides what quantity to produce (q_2). |

5   15   (q_2)

| Firm 1 looks at Firms 2 response (q_2), then will decide q_1. |

$$\Pi_1 = (100 - q_1 - q_2)*q_1 - q_1$$

substitute for q_2

$$\Pi_1 = (100 - q_1 - \frac{100 - q_1 - 2}{2}) * q_1 - q_1$$

$$\Pi_1 = 100 - q_1{}^2 - 50q_1 + 1/2q_1{}^2 + q_1 - q_1$$

$q_1 = 50$ $q_2 = 24$

$\Pi_1 = 1274$ $\Pi_2 = 576$

The first mover will be able to produce more, knowing that the other firm will have to produce less to maximize profits. The disadvantage of pricing first is that the second firm can undercut you.

Monopolistic competition (Chap. 12)

This last model relaxes the assumptions that goods across firms are homogeneous. In reality we know this is not true. Each firm differentiates their product. For example, each wine is unique. Firms also take advantage of geographical locations to differentiate their products. So, since each firm's product is unique, you can expect firms to enjoy monopolistically competitive profits. Firms can be modeled as monopolistic because they are the only firm producing that good with those particular attributes.