

1. Identities and Conditions. It is very important to distinguish between an *identity* (a definition which holds true all the time) and a *condition* (which is true only in a specific situation). For our purposes:

- The Marginal Rate of Substitution, $MRS = -\frac{U_x}{U_y} = -\frac{MU_x}{MU_y} = -\frac{dU/dx}{dU/dy}$, is an *identity* that always tells us the slope of the indifference curve. It tells us how much extra of good y we must be compensated with in order to stay indifferent if we were to give up one unit of good x .
- The slope of the budget curve, $-\frac{P_x}{P_y}$, is an *identity* that tells us how much extra of good y we can afford if we were to give up one unit of good x .
- The equality, $MRS = -\frac{P_x}{P_y}$, is a *condition* that only holds true at the utility-maximizing consumption bundle.

2. Utility Maximization Problem. In order to solve for specific values for x and y , you must use four things. Always check that you have made use of:

- P_x , the price of good x .
- P_y , the price of good y .
- M , the budget constraint.
- U , the utility function.

3. Solving a Two-Good Utility Maximization Problem. Given M , P_x , P_y and U :

- Find MRS in terms of x and y .
- Equate MRS to $-\frac{P_x}{P_y}$. Find either “ y in terms of x ” or “ x in terms of y ”.
- Substitute into the budget constraint, M , so that we only have one unknown.
- Solve for the unknown (either x or y depending on which you chose earlier).
- Once you have solved for a numerical value for either x or y , then you can further solve for the other unknown (from the result in the second step).

4. Solving a Three-Good Utility Maximization Problem. Given M, P_x, P_y, P_z and U :

- Find U_x, U_y and U_z .
- Knowing the general utility-maximizing condition $\frac{U_x}{P_x} = \frac{U_y}{P_y} = \frac{U_z}{P_z}$, work with two pairs of equalities separately in order to express two unknowns as a function of the third. (For example, express x and y in terms of z .)
- Substitute into the budget constraint, M , so that we only have one unknown.
- Once you have solved for a numerical value for that unknown then you can further solve for the other two.

5. Revealed Preferences without a Utility Function. Suppose we are given an individual's budget constraints M_0, P_{x0} and P_{y0} , and we are told that he is maximizing his utility at the bundle (x_0, y_0) . Then we are told that the budget constraints have changed to M_1, P_{x1} and P_{y1} , and that he now consumes the bundle (x_1, y_1) . How do we decide if (x_1, y_1) is a rational bundle?

- Is the new bundle affordable under the old budget constraints?
i.e. $x_1 \cdot P_{x0} + y_1 \cdot P_{y0} \leq M_0$?
- Is the old bundle affordable under the new budget constraints?
i.e. $x_0 \cdot P_{x1} + y_0 \cdot P_{y1} \leq M_1$?

If the answers to both those questions are yes, then the new bundle (x_1, y_1) must be irrational. From the first part, we know that (x_0, y_0) is revealed-preferred to (x_1, y_1) . That is to say, whenever (x_0, y_0) and (x_1, y_1) are affordable, he must rationally choose (x_0, y_0) over (x_1, y_1) . From the second part, we know that both (x_0, y_0) and (x_1, y_1) are affordable under the new budget constraints. Therefore, he must rationally choose (x_0, y_0) . Since he chose (x_1, y_1) instead, then he must be irrational.