

1. Income and Substitution Effects of a Price Change. For any good X, a change in its absolute price P_x will produce an observable change in the quantity demanded x . The observed effect is the sum of:

- Pure substitution effect. The change in the absolute price of X produces a change in the correlative price of another good Y (even though the absolute price P_y remains unchanged). The pure substitution effect on the individual is to substitute between Y and X according to these new relative prices so that he is exactly indifferent.
- Pure income effect. The change in the absolute price of X produces a correlative change in income available to spend on all goods X and Y (even though the absolute income M remains unchanged). The pure income effect is to reoptimize his consumption bundle of X and Y in response to this relative change in income so that he maximizes his utility.

The Slutsky equation allows us to calculate these effects separately:

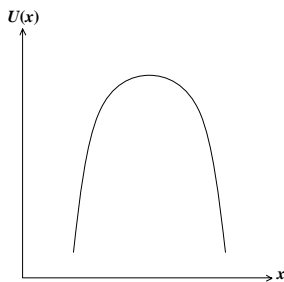
$$\underbrace{\frac{\Delta x}{\Delta P_x}}_{\text{observed effect}} = \underbrace{\frac{\Delta x}{\Delta P_x} \Big|_{U=\text{constant}}}_{\text{pure substitution effect}} - \underbrace{x \frac{\Delta x}{\Delta M}}_{\text{pure income effect}}$$

The negative sign is due to the fact that a price change produces an opposite correlative effect on income. That is, when prices increase, relative incomes decrease and vice versa.

2. First and Second Order Conditions. Sometimes we are asked to maximize (or minimize) a function $U(x)$ with respect to x . To do this, we need to find the first and second derivatives of the function $U(x)$ to solve and confirm our answers.

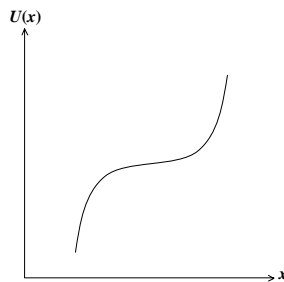
- The first order condition, $\frac{dU}{dx} = 0$, helps us find the value(s) of x for which the function $U(x)$ is either maximum, minimum or inflexive.
- The second derivative, $\frac{d^2U}{dx^2} = \frac{d}{dx} \frac{dU}{dx}$, help us determine exactly what shape the

function when $\frac{dU}{dx} = 0$.



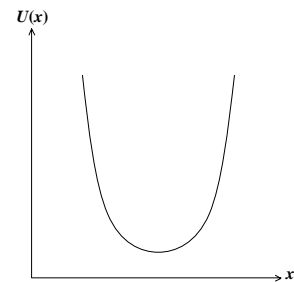
$$\frac{d^2U}{dx^2} < 0$$

maximum



$$\frac{d^2U}{dx^2} = 0$$

inflexion



$$0 < \frac{d^2U}{dx^2}$$

minimum

3. Labor Supply Problem. Usually, you will be asked to maximize the utility of consumption and leisure. Given (i) wage rate w , (ii) the maximum time amount of time available for work h , and (iii) utility function of consumption and leisure $U(c,l)$:

- Write out the consumption function, $c = w(h - l)$.
- Substitute c into U so that U is expressed as a function of only one variable l .
- Apply the first and second order conditions to U to find the value of l that maximizes U .
- From this value of l , we can calculate c .

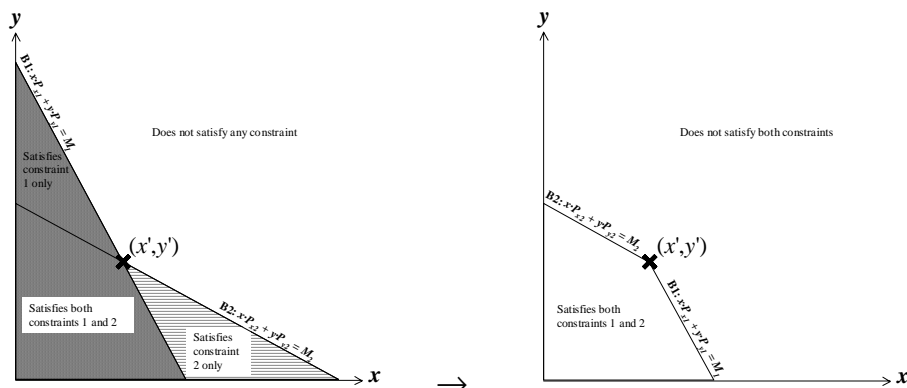
4. Utility Maximization Problem for a Kinked Budget Curve. Sometimes, you may find that the question may impose more than one (usually two) budget constraints that cannot be expressed as a single function. It may be useful to decompose the two constraints as two separate budget functions. The following outline the broad steps that we covered in the lab example:

- Find the two main constraints and express them as two separate budget functions.

B1: $x \cdot P_{x1} + y \cdot P_{y1} = M_1$ Keep constraint 1 and relax constraint 2.

B2: $x \cdot P_{x2} + y \cdot P_{y2} = M_2$ Keep constraint 2 and relax constraint 1.

The lower boundary of the two budget curves is the aggregate budget curve because it is the only area under which both constraints 1 and 2 are satisfied.



- Find the point of intersection of B1 and B2, and let this point be (x', y') . Let the final non-boundary utility-maximizing solution be (x^*, y^*) . Then we know that:

If (x^*, y^*) falls on B2, then $0 < x^* \leq x'$ and $y' \leq y^* < \frac{M_2}{P_{y2}}$.

If (x^*, y^*) falls on B1, then $x' \leq x^* < \frac{M_1}{P_{x1}}$ and $0 < y^* \leq y'$.

- Now treat B1 and B2 as separate budget curves and apply the utility-maximizing condition separately. Let the bundle for B1 be (x_1, y_1) and for B2 be (x_2, y_2) .

If $x' \leq x_1 < \frac{M_1}{P_{x1}}$ and $0 < y_1 \leq y'$, then (x_1, y_1) is the solution.

If $0 < x_2 \leq x'$ and $y' \leq y_2 < \frac{M_2}{P_{y2}}$, then (x_2, y_2) is the solution.

If neither (x_1, y_1) nor (x_2, y_2) meet the conditions above, then the non-boundary solution is the corner bundle (x', y') .