PUAF 8216 Microeconomics Section 3
Lab Summary 8
10 November 2000

1. Discounting Example. **I plan to have a child today and send him / her to a 4-year college in 18 years' time after I retire. Suppose I expect that my real annual salary, *M*, will remain constant over the next 18 years, that real college tuition remains constant at 0.8*M* per year, and that the real rate of return is 5% per annum.**

a. What proportion of your annual income, *M*, must you save for the next 18 years in order to pay 4 subsequent years of college tuition of 0.8*M* per year?

Assume that all payments start in year 1, and therefore approximate the denominator $R=1-r_e$ but not $r_e=1-R$.

Intuition: We want the total stream of savings to exactly pay for the total stream of tuition spending. That is to say, we want to equate the present discounted values of 18 years of savings to 4 subsequent years of tuition spending ($PDV_{savings}=PDV_{tuiton}$).

$r = 0.05$ $R = 0.05$ $r_e = \frac{1}{1+R} = \frac{1}{1.05}$

Savings stream, *saving bM per year for 18 years*

$$\text{PDV}_{\text{savings}} = \frac{bM\left(1-r_e^{\,n}\right)}{1-r_e} \approx \frac{bM\left(1-\left(\frac{1}{1.05}\right)^{18}\right)}{0.05} = 11.690bM$$

Tuition stream, *spending 0.8M per year for 4 subsequent years*

4 years of future payments discounted to Year 18 dollars — discount 18 years till today

$$\text{PDV}_{\text{tuition}} = \frac{0.8M\left(1-r_e^{\,n}\right)}{1-r_e} \times r_e^{18}$$

$$\approx \frac{0.8M\left(1-\left(\frac{1}{1.05}\right)^{4}\right)}{0.05} \times \left(\frac{1}{1.05}\right)^{18} = 1.179M$$

Equating the two,

$$\text{PDV}_{\text{savings}} = \text{PDV}_{\text{tuition}}$$
$$11.690bM = 1.179M$$
$$b = 0.100$$

This means that you must save 10.0% of your income for 18 years in order to have 80% of your income to spend on each of the 4 subsequent years of college tuition.

b. Suppose instead that I saved 15% of my salary instead. How many years of college can I put my child through?

We can still use the equations in part (a), except that we substitute b = 0.15 and we use N instead of 4 years of college.

Savings stream, *saving 0.15I per year for 18 years*

$$\text{PDV}_{\text{savings}} = \frac{0.15M\left(1-r_e^{\,n}\right)}{1-r_e} \approx \frac{0.15M\left(1-\left(\frac{1}{1.05}\right)^{18}\right)}{0.05} = 1.753M$$

Tuition stream, *spending 0.8I per year for N years*

20 years of future payments discounted to Year 30 dollars — discount 30 years till today

$$\text{PDV}_{\text{tuition}} = \frac{0.8M\left(1-r_e^{\,N}\right)}{1-r_e} \times r_e^{\,18}$$

$$= \frac{0.8M\left(1-\left(\frac{1}{1.05}\right)^{N}\right)}{0.05} \times \left(\frac{1}{1.05}\right)^{18}$$

$$= 6.648M - 6.648M(0.952)^N$$

Equating the two,

$$\text{PDV}_{\text{savings}} = \text{PDV}_{\text{tuition}}$$

$$1.753M = 6.648M - 6.648M(0.952)^N$$

$$6.648M(0.952)^N = 6.648M - 1.753M = 4.895M$$

$$0.952^N = \frac{4.895M}{6.648M} = 0.736$$

$$N \log 0.952 = \log 0.736$$

$$N = \frac{\log 0.736}{\log 0.952} \approx 6$$

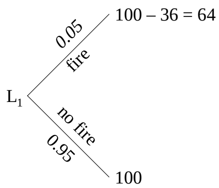
This means that if you save 15% of your income for 18 years, you can afford to send your child to 6 years of college in a college that charges 0.8M tuition per year.

2. Insurance Example. **Suppose I have a house worth $100. There is a 5% chance that it catches fire and inflicts $36 of damage. My friend (who starts with $100 cash) offers me an insurance scheme such that, (a) if my house does not catch fire, I will pay her an insurance premium α, and (b) if my house does indeed catch fire she will refund me the insurance premium and pay for the damages as well. If my utility of wealth is given by $U = \sqrt{w}$, then:**

a. What is the maximum premium α that I am willing to pay?

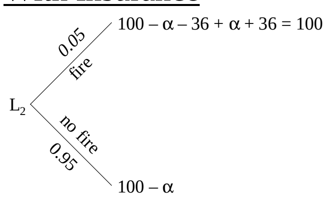
Intuition: We want to find the value of α that makes me indifferent between buying and not buying insurance. That is, E·U(no insurance) = E·U(insurance).

No Insurance



$$E\cdot U = 0.05\sqrt{64} + 0.95\sqrt{100} = 9.9$$

With Insurance



$$E\cdot U = 0.05\sqrt{100} + 0.95\sqrt{100-\alpha}$$
$$= 0.5 + 0.95\sqrt{100-\alpha}$$

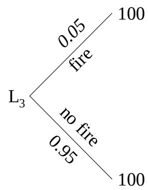
$$E\cdot U(\text{no insurance}) = E\cdot U(\text{insurance})$$
$$9.9 = 0.5 + 0.95\sqrt{100-\alpha}$$
$$0.95\sqrt{100-\alpha} = 9.4$$
$$\sqrt{100-\alpha} = \tfrac{9.4}{0.95}$$
$$100 - \alpha = \left(\tfrac{9.4}{0.95}\right)^2 = 97.91$$
$$\alpha = 100 - 97.91 = 2.09$$

b. What is the minimum premium β that my friend is willing to accept?

Intuition: We also want to make the friend indifferent between offering insurance and not offering insurance. Here, I did not specify the friend's utility function, so we leave it in its general form U=U(w).

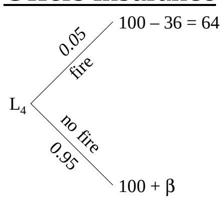
Does not offer insurance

$$E \cdot U = 0.05 \cdot U(100) + 0.95 \cdot U(100)$$
$$= U(100)$$



Offers insurance

$$E \cdot U = 0.05 \cdot U(64) + 0.95 \cdot U(100+\beta)$$



Try this out yourself in Problem Set 5 using Jane's utility function of wealth.