U8216 Microeconomics and Policy Analysis
Fall 2000
Problem Set 5

1. The date is January 1, 2000. A zero coupon bond is a bond that pays out nothing until it comes due and then it pays out its face value. Suppose that a zero coupon bond with a $1,000 face value becomes due on January 1, 2015. Given an unchanging interest rate of 15%, what will this bond sell for today?

2. Assume the interest rate is 10%. A plot of land can be used in one of two ways. Option 1 is to use the land for a parking lot. There are no startup costs in building the lot, and it yields a net return of $8,000 per year forever starting next year. Option 2 is to build a house, at a cost of $50,000 now with a net return of $12,000 a year forever (also starting next year).

   a. How should the land be used? Is the answer the same if the interest rate goes down to 5%? Explain.

   b. Does your answer to (a) change if, instead of a net return of $12,000 forever on the house, you get a return of $12,000 today and a net return of $12,000 for 30 years (starting in 1 year)?

3. Suppose you are 30 and have just graduated with your MPA. You plan to work for exactly 30 years, retire for 20 years, and then drop dead. Suppose your salary from today till retirement is constant in real terms (i.e. inflation-adjusted). You wish to retire with an annual income that is 60% of your salary. Today a government bond pays a real return of about 2% annually (the market rate is about 5%, but the inflation rate is about 3%). Suppose the rate will be constant forever (in fact, real rates display little variation in most circumstances).

   a. Figure out how much money as a fraction of your salary you will need to save in order to meet your retirement goals? (Hint: figure everything out in terms of present values today.)

   b. Suppose you save 15% of your salary. How many years of retirement will your savings fund? (Hint: solve for an unknown N, where N is the number of years of retirement. You will need to use logarithms.)

   c. The US Social Security system is funded by contributions of about 15% of salary (up to a limit, but benefits are also limited so you can ignore this complication). The interest rate and benefit levels are close to those assumed above. Should Clinton start a campaign to encourage old people to start smoking? Suppose we let people invest their contributions in private sector securities that pay higher rates but are just as safe for long-term investments. Why would this strategy help the situation (suppose that the returns on government securities and private sector securities is not changed by the policy)? If the government is buying these securities, who will not be buying them? Does this matter?

4. Winfred Whiz is playing on a game show. He must choose between two offers. The first offer is a payment of $2,000, which he can take for simply being on the show, or he can enter a gamble. In the gamble, he chooses one of two curtains that conceal two items. He makes a draw for curtain 1 or 2 from a hat and then receives the gift behind the curtain picked. He knows that behind one curtain is an automobile valued at $4,000 and behind the other curtain is a set of encyclopedias valued at $500. If his initial wealth is $1,000 and his utility function can be described by:

$$U(w) = 1 - \frac{1000}{w},$$

then what must be the probability of drawing the car for Winfred to be indifferent between the two choices?

5. Betty Bat loves the New York Yankees. She has followed their exploits since she was five years old. In three of the past four years (1996, 1998, 1999) they won the World Series, and Betty thinks they can do it again next year. Betty has just thought up a clever plan. She has $1,000 of savings that she has hidden under her bed. She could spend $600 of the $1,000 in making Yankees championship paraphernalia: buttons, cups, pens, and so on. Then, if the Yankees win, she estimates that she would earn $1,500. If the Yankees lose, she won't be able to sell any of her stock. Betty figures that the Yankees have a 0.6 chance of winning the next World Series. Betty's utility function is given by:

$$U(w) = \sqrt{w}\,.$$

a. If Betty is an expected utility maximizer, will she make the $600 investment into Yankee championship gadgets?

b. Calculate the certainty equivalent of Betty's clever prospect.

c. Suppose that a friend offers her insurance. He says to Betty, "If you pay me F dollars whether or not the Yankees win, then, in the event that the Yankees lose, I will pay you $1,500, the amount that you would have earned had the Yankees won the World series. If the Yankees win, I will pay you nothing." What is the maximum value of F that Betty is willing to pay for the insurance policy? If Betty's friend is risk-neutral, will he gain by this venture? Explain.

d. If Betty purchases the insurance on the $600 investment, might a moral hazard problem arise in which Betty shirks on her efforts to sell the Yankee merchandise? Why or why not?

6. Jack has a house worth \$100. There is a 5% chance that his house will catch a fire that will cause \$64 of damage. Jill has \$100 cash and is willing to offer Jack some form of insurance scheme as follows: Jack pays Jill a premium α. If there is no fire, Jill keeps the premium. If the house catches fire, Jill will refund α and pay for the fire damages. If their utility functions of wealth is given by:

$$U_{\text{Jack}}(M) = \sqrt{M}$$
$$U_{\text{Jill}}(M) = M^{\frac{5}{6}}$$

(a) What is the maximum premium that Jack is willing to pay?

(b) What is the minimum premium that Jill is willing to accept?

(c) Can they reach a deal?

7. (*optional*) Ms Gamble currently has an income of \$25,000. Assign the utility number 100 to this income level and the utility number 85 to the income level \$20,000. It is known that Ms Gamble would be willing to pay a maximum of \$5,000 for a lottery ticket that yields \$10,000 with a probability of $\frac{3}{5}$ (and yields zero otherwise). What is the utility number appropriate to the income level of \$30,000? Explain.

8. (*optional*) Suppose now that using the lot for the house costs the same and yields the same return as in the previous question, 2b (let the interest rate be 10%). The parking lot, if successful, now yields \$16,000 forever with probability 0.5, or nothing forever with probability 0.5. Suppose your utility function of wealth is given by $U(I) = \sqrt{\frac{I}{1000}}$ and you have no other income or wealth.

a. Compute the expected return of investing in the parking lot.

b. Which option will you pursue now? Compare your answer to that of problem 2b and justify your answer.

c. Suppose you may now try out the land as a parking lot for 1 year. If successful, you will always use the lot as a parking lot. If a flop, you will build the house next year with the return and cost specified as before. Compute the present value of each possible outcome. Which option will you pursue now? Explain.

9. (*optional*) Given a venture with a 50-50 chance of succeeding, and a utility function of $U=\sqrt{I}$, where I is income, then a venture where I is \$25 or \$9 is exactly as good as a sure income of how much?

10. (*optional*) After making a $100,000 killing in cattle futures, you are considering investing your winnings in a portfolio of two alternative investments. The first is a risk-free bond that pays 10% always. The second is a stock that has a 0.4 probability of paying 15%, a 0.3 probability of paying 18%, and a 0.3 chance of paying a 6% return.

a. Find the expected return, variance, and standard deviation of the stock.

b. Show graphically the risk-return combinations of the different portfolios that you could buy (i.e. the budget line between expected return and risk as measured by the standard deviation of the portfolio). What is the slope of the budget line? Interpret the meaning of the slope.

c. Suppose that you decide to invest $70,000 in the stock and $30,000 in the bond. What is the expected return and standard deviation of the portfolio you have chosen?