

Name: \_\_\_\_\_

MIDTERM  
SIPA  
Columbia University

U8216  
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This midterm contains two sections:

- Short answer questions: 7, worth 4 points each. You must answer each of these in the space provided. You can work on each short problem for about 5 minutes. Some intelligible, relevant, and correct work might lead to part marks.
- Problems: You must answer problems 1 and 2 (18 points, 25 minutes each), but can choose two of three problems among 3, 4, and 5 (9 points, 12 minutes each). Answer these problems in blue books.

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**Short Answer Questions:** Answer in the space provided.

**1**

For the utility function,  $U(x,y)=x^{1/2}y^{1/2}$ , which of the following statements is true?

True    False

\_\_\_\_\_    \_\_\_\_\_    (3,5)    (4,4)

\_\_\_\_\_    \_\_\_\_\_    (1,4) ~ (2,2)

\_\_\_\_\_    \_\_\_\_\_    (3,5)    (1,9)

\_\_\_\_\_    \_\_\_\_\_    (100,4)    (5,5)

**2**

Given the utility function  $U(x_1,x_2) = x_1 + 2x_2$ , the marginal rate of substitution (MRS) of  $x_2$  for  $x_1$  is:

\_\_\_\_\_.

**3**

Beth consumes two goods,  $x_1$  and  $x_2$ , and her income is \$120. The price of  $x_1$  is \$10, and the price of  $x_2$  is \$5. Her budget line intersects the  $x_2$  axis at consumption bundle

$(x_1,x_2)=$  \_\_\_\_\_.

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**4**

Beth faces the same prices and income as in question 3. All that is known about her preferences is that her indifference curves exhibit a diminishing marginal rate of substitution and that her marginal rate of substitution of  $x_2$  for  $x_1$  at the bundle (7,10) is 1. Then her utility maximizing bundle is:

(A) On the budget line to the right of (7,10).

(B) (7,10)

(C) On the budget line to the left of (7,10).

(D) (6,12).

(You may wish to draw yourself a little graph.)

**5**

Princeton University is losing \$20 million per year. The trustees of the university want to increase fees to cover part of the deficit. The president of the student body thinks that decreasing fees will help to cover the deficit. On the basis of this information:

The trustees think that demand is price : elastic inelastic

The student president thinks that demand is price: elastic inelastic.

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**6**

In an exchange economy, at the initial allocation Minh will exchange 3 bread for 1 cheese and remain indifferent, and Mukki will exchange  $1/2$  a bread for 1 cheese (and remain indifferent). When they engage in a mutually agreeable trade:

- | Yes | No  |                                 |
|-----|-----|---------------------------------|
| ___ | ___ | Minh will acquire more bread.   |
| ___ | ___ | Minh will acquire more cheese.  |
| ___ | ___ | Mukki will acquire more bread.  |
| ___ | ___ | Mukki will acquire more cheese. |

**7**

Once both individuals have traded to a point on the contract curve, which of the following is false:

- (A) it is impossible for both individuals to have more of both goods.
- (B) it is possible to make one individual better off.
- (C) there is no feasible change that would make both individuals better off.
- (D) it is possible to make both individuals better off.

**Long Problems:****1** (18 points)

Suppose that Horatio has preferences represented by the utility function  $U(C,L) = C^{1/2}L^{1/2}$ , where  $C$  is dollars spent on other goods and  $L$  is hours of leisure. Recall that the budget constraint he faces is

$$wL + C = 16w$$

where  $w$ , the wage rate, is \$20 per hour.

- A. Compute the marginal rate of substitution. (3 points)
- B. Equate the marginal rate of substitution to the appropriate price ratio (recall that the effective price of leisure is  $w$  and the price of a dollar spent on other goods is \$1. Simplify. (3 points)
- C. Solve for the utility maximizing consumption of leisure and other goods. (4 points)
- D. Given your answer in (C) how much labor does Horatio supply at current wage? (2 points)
- E. Suppose the wage rate rises to \$40 per hour. What is Horatio's supply of labor? (2 points)
- F. What can we conclude about income and substitution effects for a change in wage rates for these preferences? (2 points)
- G. Graph Horatio's labor supply function. (2 points)

**2** (18 points)

Mike and Jane are stranded on a dessert island with only two commodities, waffles ( $W$ ) and chocolates ( $C$ ). Mike's utility for these two goods can be expressed by the function  $U_M(C_M, W_M) = C_M + W_M$  (where  $C_M$  and  $W_M$  are the quantities of chocolates and waffles consumed by Mike) and Jane's utility can be expressed by  $U_J(C_J, W_J) = C_J^{1/2}W_J^{1/2}$  (where  $C_J$  and  $W_J$  are the quantities of chocolates and waffles consumed by Jane). There are only 10 chocolates and 10 waffles on the island. Currently, Mike has 2 chocolates and 5 waffles.

- A. Draw an Edgeworth Box diagram, label the axes, and show the initial endowment. (3 points)
- B. Determine the MRS for both Mike and Jane, and evaluate them at the endowment point (3 points).
- C. Is the endowment point a Pareto optimal allocation? Why or why not? (2 points)
- D. If they are willing to trade from the endowment point, in what direction will they trade? If they are not willing to trade from the endowment point, why not? It might help you to sketch in Mike's and Jane's indifference curves at the endowment point, given the MRS's you have computed in (B). (2 points)

- E. What points of exchange equilibrium are there (i.e., what is the contract curve)? (4 points)  
Recall: you must equate the general expressions for Mike's and Jane's MRS's.
- F. Using a graphical argument, show the competitive equilibrium allocation. What is the market-clearing price and what is the final bundle consumed by each? (4 points)

**Answer 2 of the following 3 questions.**

**3** (9 points)

Below is a Prisoners' Dilemma game for two players. Both individuals have been accused of a crime and have to make a decision: each can either *confess* or *not confess* to the crime. If one person confesses while the other doesn't, he/she faces no jail time, while the other faces a stiff sentence. If both prisoners confess, they each face a medium sentence, whereas if both do not confess, each faces a minimal sentence. The payoffs to each combination of actions are given in the box below. (Note that payoffs are given in terms of jail sentences, so *lower numbers are preferred to higher ones.*)

Payoff = (Prisoner 1, Prisoner 2)		Prisoner 2	
		Confess	Not Confess
Prisoner 1	Confess	5,5	0,15
	Not Confess	15,0	1,1

- A. What is the dominant strategy for each player? (3 points)
- B. What is the Nash equilibrium of this game? (3 points)
- C. Is the Nash equilibrium Pareto efficient? (3 points)

**4** (9 points)

Congratulations, you have just won the New York State Lottery. The lottery commission has given you the option of the following 3 prizes:

a lump sum payment of \$3.6 Million today;  
10 annual payments of \$550,000 a year starting today; or  
\$300,000 a year for life, starting today.

Assuming a discount rate of 10% and no inflation which option would you chose? Why? (Part marks are possible, so show your work.)

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5 (9 points)

Maya's demand for plantains is given by:

$$D(P_P) = 10 - P_P,$$

where  $P_P$  is the price of plantains.

- A. If  $P_P=5$ , then what is her demand for plantains? (2 points)
- B. Graph Maya's demand for plantains. (Label at least two points.) (3 points)
- C. The price of plantains increases from 5 to 6. What is the price elasticity of demand at the starting value? (4 points)