The Architecture of Economic Systems: Hierarchies and Polyarchies

By Raaj Kumar Sah and Joseph E. Stiglitz*

This paper presents some new ways of looking at economic systems and organizations. Individuals' judgments entail errors; they sometimes reject good projects and accept bad projects (or ideas). The architecture of an economic system (i.e., how the decision-making units are organized together within a system, who gathers what information, and who communicates what with whom) affects the errors made by individuals within the system, as well as how those errors are aggregated.

There is a widespread belief that the performance of an economic system or organization is influenced by its internal structure. In this paper, we present some new ways of looking at the relationship between performance of an economic system and certain aspects of its structure, which we refer to as its *architecture*. The architecture (like that of a computer or electrical system) describes how the constituent decision-making units are arranged together in a system, how the decision-making authority and ability is distributed within a system, who gathers what information, and who communicates what with whom.

The two specific architectures studied in this paper are called polyarchies and hierarchies. We think of a *polyarchy* as a system in which there are several (and possibly competing) decision makers who can undertake projects (or ideas) independently of one another. In contrast, decision-making authority is more concentrated in a *hierarchy* in the sense that only a few individuals (or only one individual) can undertake projects while others provide support in decision making. These two architectures are suggestive of a market-oriented economy and a bureaucracy-oriented economy, respectively.

The aspect of organizational performance on which we focus is the quality of decision making. All individuals make errors of judgment: some projects that get accepted should have been rejected, and some projects are rejected that should have been accepted. Using an analogy from the classical theory of statistical inference, these errors correspond to Type-II and Type-I errors.

How individuals are arranged together affects the nature of the errors made by the economic system. For example, in a market economy, if one firm rejects a profitable idea (say, for a new product), there is a possibility that some other firm might accept it. In contrast, if a single bureau makes such decisions and this bureau rejects the idea, then the idea must remain unused. The same, however, is also true for those ideas that are unprofitable. As a result, one would expect a greater incidence of Type-II errors in a polyarchy, and a greater incidence of Type-I errors in a hierarchy.

The costs of acquiring and communicating information (leading to misjudgments by individuals) are the central features of the technology underlying our analysis. These costs include the direct costs (time and re-

*Yale University, New Haven, CT 06520, and Princeton University, Princeton, NJ 08544, respectively. Financial support from the National Science Foundation and the Hoover Institution, Stanford is gratefully acknowledged. An earlier version of this paper was presented at seminars at Berkeley, Chicago, Columbia, Minnesota, Pennsylvania, Rutgers, San Diego, Stanford, and Yale, and to the 1984 European meetings of the Econometric Society in Madrid. We are indebted to the participants in those seminars for their insightful comments. We are especially grateful to John Geanakoplos, Alvin Klevorick, Mark Machina, James March, Paul Milgrom, Barry Nalebuff, Michael Rothschild, and an anonymous referee for helpful suggestions.

sources) and the indirect costs that result from the inevitable contamination that occurs in the process of information communication. Communication, like decision making, is always imperfect. No individual ever fully communicates perfectly what he knows to another.

Another important feature is the limited capabilities of individuals to gather, absorb, and process information within a limited amount of time. This is why organizations, groups of individuals, may be able to do more (make better decisions) than any single individual. But the fact that communication is costly and imperfect means that an organization with two individuals, each of whom can process a given amount of information in, say, a month, is not the same as a single individual who has the capacity of processing twice that amount of information within the same time period.

The paper is organized as follows. In Section I, we present a simple model of the decision structure within a polyarchy and a hierarchy. In Section II, we assume that the nature of an individual's errors and the mix of available projects is exogenously specified, and analyze how changes in these exogenous features influence the relative performance of the two systems. In Section III, we discuss the collection and processing of information in the two systems. In particular, we show how (Bayesian) screening rules are determined, and how the two system's performances compare with these endogenously determined individuals' errors. Section IV discusses briefly some extensions of the analysis, while Section V discusses alternative interpretations and applications. Proofs of most results are omitted for brevity; these are available in our 1985a working paper.

I. The Basic Model

The problem facing the economic systems under study is to choose which of a set of projects to undertake. Each project has a net benefit, x,[1] that can be positive or negative. There are N available projects. The density function of projects is given by $g(x)$.

The task of individuals within the organization is to evaluate ("screen") the projects. We assume that the only feasible communication is whether the project is, in the judgment of the evaluator, "good" or "bad," that is, whether it should be accepted (or passed on for further evaluation) or rejected. (For now, we can think of a screener as a black box that flashes a light when it deems a project to be good.) The probability that a given individual judges a project to be good, p, is a function of its quality. We call the function $p(x)$ the *screening function*. It can take any form, provided $1 \geq p(x) \geq 0$, for all x, and the strict inequalities hold for at least some x.

Two properties of the screening function are of special interest. The first is its slope, $p_x(x)$.[2] We assume $p_x(x)$ is positive, that is, a project with higher profit has a higher local probability of being accepted by a screen. Further, if p and p^1 represent two screens, and if $p_x(z) > p_x^1(z)$, then we refer to the former screen as locally more *discriminating* at $x = z$. The second important property of screens is the level of $p(x)$. If $p(z) > p^1(z)$, then we call the former screen locally *slacker*, and the latter locally *tighter*, at $x = z$. An example is the linear screening function, for which $p(x)$ can be expressed as

$$p(x) = p(\mu) + p_x(x - \mu), \tag{1}$$

where $\mu = E[x]$ is the mean of the initial portfolio. Clearly, a higher $p(\mu)$ and p_x imply globally higher slackness and discriminating capability.

If screening were perfect, then the architecture of a system has no effect on its output because all projects with $x > 0$ would

[1] This scalar valuation includes all relevant benefits and costs. Also, we are assuming that the interproject externalities are not significant (i.e., the profit from one project does not depend significantly on whether some other projects are undertaken or not), and that there is no restriction on the number of projects that can be undertaken.

[2] A letter subscript denotes the variable with respect to which a partial derivative is being taken.

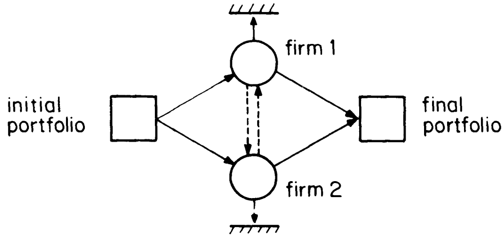

FIGURE 1. POLYARCHY

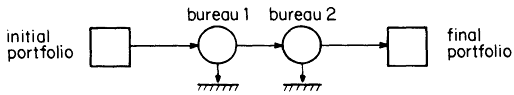

FIGURE 2. HIERARCHY

be accepted and those with $x<0$ would be rejected; that is, $p(x)=1$ if $x>0$, and $p(x)=0$ if $x\leq 0$. Without perfect screening, the architecture of the economic system determines the conditions under which a project gets selected and, hence, it affects the system's output. In the following simple model, we consider a polyarchy consisting of two firms, and a hierarchy consisting of two bureaus.

The decision process in a polyarchy and a hierarchy are depicted in Figures 1 and 2, respectively. In a polyarchy, the two firms screen the projects independently. For specificity, one may think of projects arriving randomly (with probability one-half) at one of the two firms. If a particular project is accepted by a firm, then it is no longer available to the other firm. If the project is rejected, then it goes to the other firm where, once again, it can be accepted or rejected (but firms cannot tell which of the projects that they are evaluating have been previously reviewed). Neither firm screens the same project twice, so that a project cannot cycle back and forth between firms. The portfolio of projects selected in a polyarchy therefore consists of the projects accepted separately by each of the two firms.

In contrast, in a hierarchy, all projects are first evaluated by the lower bureau (bureau 1); those that are accepted are forwarded to the higher bureau (bureau 2) and others are discarded. The projects selected by the system then are those which are selected by the higher bureau. Drawing an analogy from the design of relay circuits, the screens are placed in *series* in a hierarchy, whereas they are placed in *parallel* in a polyarchy.

In a polyarchy, the probability that a project that goes to the first firm is approved is $p(x)$. It gets rejected with probability $1-p(x)$; the probability that it then gets approved by the second firm is again $p(x)$. Hence the total probability of acceptance is $p(x)+(1-p(x))p(x)=p(x)(2-p(x))$. Similarly, in a hierarchy, the probability that a project is approved by the lower bureau is $p(x)$. The probability that the same project given to the higher bureau is approved is again $p(x)$. Hence, the probability of a project being approved is $p^2(x)$.

The probability that the project x will be accepted in the system s is denoted by $f^s(x)$, where the superscripts $s=P$ and H represent a polyarchy and a hierarchy, respectively. If individuals' decisions are independent across screens and projects, then

$$(2)\qquad f^H\equiv\left(p^H\right)^2;\qquad f^P\equiv p^P(2-p^P).$$

In the comparison of the two systems below, we assume that both systems face the same set of available projects, and that they have the same screening function. The latter assumption is dropped in Section III.

II. Comparative Performance with Identical Screening Functions

We investigate two questions in this section: what is the relative performance of the two systems, and how is it affected by the properties of the available project portfolio, and the screening function?

A. *The Size of Final Portfolios*

The proportion of the initial portfolio selected by the two systems, n^s, is just $\int f^s(x)g(x)dx\equiv E[f^s]$. Denoting the difference in these proportions by Δn, we find that[3]

$$\Delta n = n^P - n^H > 0, \tag{3}$$

since, from (2), $f^P - f^H = 2p(x)\{1 - p(x)\} \geq 0$ for all x, and it is strictly positive for some x.

PROPOSITION 1: *A polyarchy selects a larger proportion of the available projects than does a hierarchy.*

The reason behind this result is intuitive. Consider a hypothetical situation in which the second firm in a polyarchy does not exist, and the higher bureau in a hierarchy does not exist. The proportion of projects accepted in the two systems would then be the same, namely, $E[p(x)]$. Since the second firm accepts at least some projects, and since the higher bureau rejects at least some projects, the actual proportion of projects accepted in a polyarchy must exceed that in a hierarchy. It is also obvious that this result holds for good as well as bad projects. Further, the result does not depend on how one defines good vs. bad projects, provided there is some probability that a screen will accept at least some good and some bad projects. It immediately follows that: *A polyarchy accepts a larger proportion of good as well as bad projects compared to a hierarchy, no matter how one defines good and bad projects.* Therefore, *the incidence of Type-I error is relatively higher in a hierarchy, whereas the incidence of Type-II error is relatively higher in a polyarchy.*

The above result suggests that there may be circumstances in which a polyarchy performs better than a hierarchy (when it is more important to avoid Type-I errors) and other circumstances in which a hierarchy performs better than a polyarchy (when it is more important to avoid Type-II errors).

To determine the impact of initial portfolios on the size of final portfolios, note

[3]Here as elsewhere, N, the number of available projects plays no role, and we therefore suppress it. The variables representing the performance of a system are thus normalized by the number of available projects.

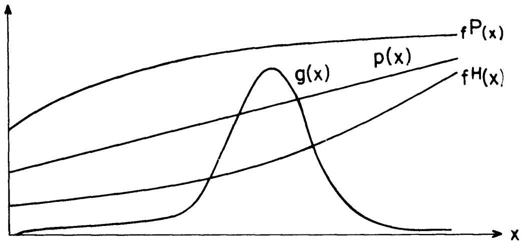

FIGURE 3. PROBABILITIES OF ACCEPTANCE IN ALTERNATIVE SYSTEMS

from (2) that $f^s(x)$ is increasing in x. Additionally, $f^P(x)$ is concave and $f^H(x)$ is convex in x, if the screening function is linear. Therefore, the standard properties of statistical dominance (under an assumption that the end points of the projects' distribution are fixed) yield the following result.

PROPOSITION 2: *A worsening in the initial portfolio in the sense of first-order stochastic dominance leads to a smaller proportion of initial projects being selected in both systems. With a linear screening function, a mean-preserving spread in the initial portfolio leads to a smaller proportion of initial projects being selected in a polyarchy, and a larger proportion being selected in a hierarchy.*

These results can be seen in Figure 3. As shown, f^P and f^H are concave and convex in x, since $p(x)$ in linear. n^s is the area above the x-axis bounded by the product of f^s and g. Naturally, this area corresponding to f^P is larger than that corresponding to f^H; and this area enlarges, for both a polyarchy and a hierarchy, if the density weight shifts from lower x to higher x. Also, if the density weight shifts from the mean to the two sides, due to a mean-preserving spread, then the area representing n^s decreases in a polyarchy and it increases in a hierarchy.

Straightforward calculations allow one to ascertain how n^s is influenced by changes in the two parameters of the linear screening function. We find the following.

PROPOSITION 3: *With a linear screening function, a higher slackness in screening*

$(p(\mu))$ raises the proportion of projects selected in both systems. And, a higher discriminating ability in screening (p_x) lowers the proportion selected in a polyarchy, whereas it raises the proportion selected in a hierarchy.

B. *Profits in Alternative Systems*

Two Types of Projects. In the case where the set of projects to be reviewed consists only of two types of projects, we can obtain a complete characterization of the conditions under which the (expected) output is higher under polyarchy or hierarchy. The initial portfolio is represented by the return on good projects, $z_1 > 0$; the return on bad projects, $-z_2 < 0$; and the proportion of good projects, α. The screening function is characterized by the probability that a good project gets accepted, denoted by $p_1 = p(3_1)$, and the probability that a bad project is accepted, denoted by $p_2 = p(-z_2)$. If $Y^s = E[xf^s]$ denotes the output, and $\Delta Y = Y^P - Y^H$ denotes the difference between the outputs of the two systems, then

$$\Delta Y = 2z_2(1-\alpha) \times [ap_1(1-p_1) - p_2(1-p_2)], \tag{4}$$

where $a = z_1\alpha / z_2(1-\alpha)$ is a summary representation of the quality of the initial portfolio.

An improvement in the initial portfolio in the present model is represented by a larger a (i.e., a larger α or a larger z_1/z_2). It follows from (4) that a worse initial portfolio implies that the relative performance of a polyarchy, compared to a hierarchy, is worse. This is simply because the relative advantage of a hierarchy is in rejecting bad projects, whereas the relative advantage of a polyarchy is in accepting good projects. If the initial portfolio worsens, then the former advantage becomes increasingly more important and the relative performance of a hierarchy improves. However, we must caution that the probability that a project is accepted or rejected by a screen (i.e., the rules for project acceptance and rejection), might be affected by the mix of available projects, among other things. One might suspect, for instance, that if there was a large proportion of bad projects, screening would become relatively tighter in a polyarchy, and this might improve its relative performance. Endogenous screening functions with such properties are discussed later.

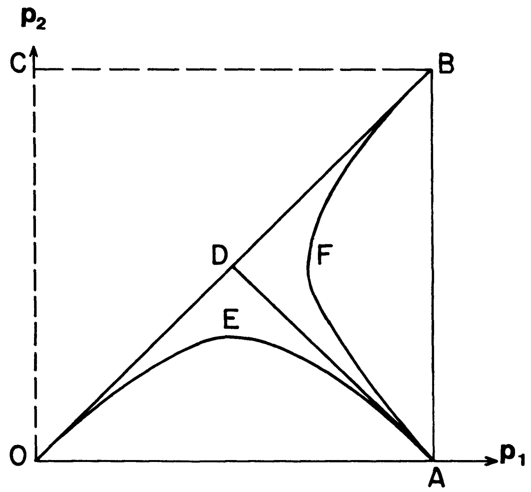

FIGURE 4. COMPARISON OF A POLYARCHY AND A HIERARCHY

The above expression also allows us to demarcate the parameter space into two regions: one in which a polyarchy has a higher output than a hierarchy, and the other in which the reverse holds.

Figure 4 summarizes the results. We are concerned only with the area below the 45° line, since screens have some discriminating capability; that is, $p_1 > p_2$.

First consider the case where the initial portfolio is moderately good; that is, $a = 1$. This happens, for instance, if the initial portfolio has equal number of good and bad projects $(\alpha = 1/2)$ and if the gains and losses from the two types of projects are symmetric $(z_1 = z_2)$. In this case, a polyarchy has a higher output if

$$1 - p_1 > p_2. \tag{5}$$

In Figure 4, thus, a polyarchy performs better in the area *ODA* and the reverse holds in the area *ADB*.

This result has a simple explanation. Recall that $(1-p_1)$ is a screen's Type-I error, the probability of rejecting a good project; and p_2 is a screen's Type-II error, the probability of accepting a bad project. Now, if a screen is more likely to reject a good project than to accept a bad project, that is, if (5) holds, then it must be the case that a polyarchy (which gives a second chance to the rejected projects) would do better.

If the initial portfolio is worse, that is, $a<1$, then, from (4), we find that the parameter space is separated by a hyperbola like OEA, which is inside the region ODA. A polyarchy has a higher profit within the region OEA, and the reverse holds outside of it. The region OEA shrinks as the initial portfolio becomes worse, and it coincides with the line OA if $a \approx 0$. The opposite case, in which the initial portfolio is better (i.e., $a>1$) has a parallel implication. A polyarchy then has a higher profit outside of the region AFB, and the reverse holds inside it.

There is another way in which the results can be seen intuitively. Suppose that we subjected each project to two screenings. Clearly, if both screens indicated that the project was bad, the project should be rejected, and if both indicated that the project was good, it should be accepted. A tradeoff arises in those cases where there is a mixed review. Whether a project with a mixed review should be undertaken depends on the profit from such a project. The probability of a good project getting a mixed review is $2p_1(1-p_1)$, while the probability of a bad project getting a mixed review is $2p_2(1-p_2)$. Hence the expected profit from projects with mixed reviews is the same as (4). Now, if it turns out that the expression (4) is positive, it means that projects with mixed reviews should be accepted; this is precisely what polyarchy ensures. Similarly, if it turns out that (4) is negative, it means that the projects with mixed reviews should be rejected, and this is precisely what hierarchy ensures.

A General Project Portfolio. Before concluding this subsection, we briefly consider an initial portfolio consisting of a continuum of projects. Recall from (2) that $f^P - f^H = 2p(1-p)$. Then, $\Delta Y = 2E[x\psi]$, where $\psi = p(1-p)$. To determine the effect of a change in the initial portfolio, we note

$$\text{(6)} \qquad \psi_x = (1-2p)p_x;$$
$$\psi_{xx} = -2p_x^2 + (1-2p)p_{xx}.$$

Since ψ can be either a concave or convex function of x, the effect of a mean-preserving change in the initial portfolio is, in general, ambiguous. If the range of x is small, however, then using (6), we obtain

$$\text{(7)} \quad E[x\psi] \approx \psi(\mu)\mu + \psi_x(\mu)E[x(x-\mu)]$$
$$= p(\mu)((1-p(\mu))\mu + (1-2p(\mu))$$
$$\times p_x(\mu)E\left[(x-\mu)^2\right].$$

Hence, if $\mu \geq 0$, and $p(\mu) < \frac{1}{2}$, then a polyarchy has a larger output than hierarchy; further, an increase in the variance of the portfolio improves the relative performance of polyarchy, regardless of the value of μ.

When the screening function is linear, then additional comparative statics results can be easily obtained. For instance, if the initial portfolio contains projects symmetrically distributed around zero mean, then a polyarchy performs better or worse than a hierarchy depending simply on whether $p(\mu)$ is less than or more than one-half, that is, whether the screening is tight or slack. A higher mean or a greater negative skewness of the initial portfolio, on the other hand, improves the relative performance of a polyarchy.

III. Endogenous Screening Rules

The individual decision makers in the above model can be interpreted to be Bayesian, with each of them receiving a binary (imperfect) signal concerning the quality of the projects. More generally, individuals observe a much richer set of signals that they have to interpret; they have to decide, in other words, under what conditions they will recommend that the project be undertaken. Assume, for instance, that the project evaluator observes

$$y = x + \theta. \tag{8}$$

Project evaluators use reservation levels for screening: a project is accepted if its observed profit is above the reservation level, R, and it is rejected otherwise.[4] Assume θ is distributed independently of x and denote the distribution function of θ by $M(\theta)$ and its density by $m(\theta)$. The screening function, then, is given by

$$p(x, R) \equiv Prob[y \geq R] = 1 - M(R - x). \tag{9}$$

The above expression yields $p_x \geq 0$, and $p_R \leq 0$: the probability that a project is accepted by a screen is increasing in the quality of the project, and it is decreasing in the reservation level. Increasing R increases the probability of a good project being rejected (Type-I error) and decreases the probability of a bad project being accepted (Type-II error). The reservation level R is chosen to balance off these errors.

In a polyarchy, denote the two firms by superscripts i and j. For firm i, R^i is the reservation level, $p^i \equiv p(x, R^i)$ is the screening function, and Y^{iP} denotes the output; then

$$Y^{iP} = E\left[xp^i(2 - p^j)\right]/2; \qquad Y^P = Y^{1P} + Y^{2P}. \tag{10}$$

We now turn to a comparison of the reservation levels under the two systems, and using these results, to a comparison of performance. The reservation level in a hierarchy, R^H, maximizes $Y^H = E[xp^2]$; that is, it satisfies

$$Y_R^H = 2E[xpp_R] = 0. \tag{11}$$

[4]The optimal policies can be characterized in terms of a single reservation level only if some mild regularity conditions are satisfied by the nature of the error terms.

To emphasize the independence (and potential competition) between the two firms in a polyarchy, we assume that their reservation levels are determined without coordination. We focus on the symmetric Nash optimum; from (10), the corresponding reservation level, R^P, is characterized by

$$E[x(2 - p)p_R] = 0. \tag{12}$$

As a benchmark, we also note that in the case of a *coordinated polyarchy* (where the reservation level for the firms is set to maximize the combined output Y^P), the reservation level, R^C, is obtained by equating

$$Y_R^P = 2E[x(1 - p)p_R] \tag{13}$$

to zero. Using expressions (11) to (13), one can show the following:[5]

PROPOSITION 4: $R^C > R^P > R^H$. *That is, the screening in a polyarchy is more conservative than that in a hierarchy, but less conservative than that in a coordinated polyarchy.*

This result has an intuitive explanation. While in a hierarchy, the lower bureau knows that its decisions are rechecked at the upper bureau; and the upper bureau knows that all projects it receives have been checked at the lower bureau; in a polyarchy, each firm knows that its decision will not be rechecked; and to make matters worse, it knows that the set of projects which it is examining includes many that have already been examined elsewhere, and have been rejected. This conservatism is reflected in market economies by firms insisting on a high "expected" re-

[5]To show this, we define $c(x) = (1 - p(x))/p(x)$, and observe that the turning points as well as global maximum of $c(0)Y^H$ are identical to those of Y^H for any constant $c(0)$, and that $d(c(0)Y^H - Y^P)/dR = 2Expp_R[c(0) - c(x)] < 0$, since $c_x(x) < 0$ and $p_R < 0$. Assume to the contrary: that $R^H > R^C$. Then from $Y_R^P > c(0)Y_R^H$, it follows that $Y^P(R^H) - Y^P(R^C) > c(0)\{Y^H(R^H) - Y^H(R^C)\}$. The left-hand side is negative and the right-hand side is positive. This contradiction establishes that $R^C > R^H$. The uncoordinated polyarchy can be thought of as maximizing $Ex[2p - .5p^2]$. The result that $R^C > R^P > R^H$ follows along exactly parallel lines to our proof that $R^C > R^H$.

turn in order to undertake a project. For example, firms often have a decision rule that only projects with an expected return in excess of 20 percent be undertaken, but the actual average returns are considerably smaller. Firms know that to attain the required return, they have to set high reservation levels. Our analysis also shows, as one would expect, that firms in an uncoordinated polyarchy do not take into account the negative externality that they exert on one another (i.e., each firm worsens the portfolio which the other firm faces) as much as they would were their reservation levels coordinated.

Comparative Statics of Reservation Levels. An immediate implication of (11)–(13) is the following.

PROPOSITION 5: *An unambiguous increase in the relative proportion of bad projects increases the reservation levels under both hierarchy and polyarchy. That is, if* $\partial g(x,\beta)/\partial\beta \lesseqgtr 0$ *as* $x \gtrless 0$, *then* $dR^s/d\beta > 0$ *for* $s = P, H,$ *and* C.

Thus, a worsening of the portfolio of available projects leads to tighter screening (lower probabilities of acceptance).

Another critical determinant of the reservation levels is the quality of information based on which projects are selected. Intuitively, we would expect a worsening of the quality of information to lead to higher reservation levels. For simplicity, let the noise associated with observing a project depend on x. Now consider a new distribution of θ, which first-order stochastically dominates the original distribution for $x < 0$. Because with the new distribution, we are more likely, at any reservation level, to accept a bad project, we refer to the new information as noisier than the original. It is easy to show

PROPOSITION 6: *If the screening is tight, noisier information induces an increase in the reservation level of a hierarchy.*[6]

Comparison of Performance. Because the reservation levels are set differently in the different systems, the *individuals'* Type-I and Type-II errors are different; this makes the comparison of the system performance more difficult than that in the previous section. We present three sets of results, focusing on the special case where there are two types of projects.

(a) Suppose a polyarchy has a larger output than a hierarchy when both systems use the hierarchy's (optimal) reservation levels. Then clearly, a *coordinated* polyarchy, using its reservation levels, will perform even better. Moreover, if Y^P is locally a concave function of R,[7] then Proposition 4 implies that an uncoordinated polyarchy using its own reservation levels, will also outperform hierarchy.

Thus, *when* $a = 1$, *and the reservation levels for hierarchy are such that the screening probabilities fall within the area ODA in Figure* 4, *then the output of a polyarchy is larger.* By the same reasoning, whenever the reservation levels for polyarchy are such that the screening probabilities fall within the area *ADB*, then a hierarchy has a larger output. Analogous interpretations apply when $a \neq 1$. Note that one cannot reach a verdict on the relative performance (using this approach) if hierarchy's screening probabilities fall within *ADB*, or if polyarchy's fall within *ODA*.

When will hierarchy's screening probabilities be such that at those probabilities, polyarchy outperforms hierarchy? First, consider the case where a is slightly less than unity. Then, $p_1 = p_2 = 0$ if we observe a completely uninformative signal. By continuity, using the above results, *if the signal concerning the quality of the project is sufficiently bad, then polyarchy outperforms hierarchy.* This is a

[6] This definition of an increase in noise is not the standard one in the statistical decision literature. Under the stated conditions, for all values of $x < 0$, $p(x, R)$ is increased for all R, while $p_R(xR)$ is decreased at the optimal R, provided only that the error density function for large θ is increased. This result establishes that each local maximum to Y^H is shifted to the right. To ensure that the global maximum is reduced, we need to assume either concavity, or to impose somewhat stronger conditions on how the distribution of noise changes.

[7] That is, $Y^P_{RR} < 0$ within the region $R^H < R < R^C$.

somewhat surprising result: one might have thought that with poor information, the second screening provided by hierarchy would be more valuable. But the reservation levels adjust so much, the resulting screening is so tight, that the second chance provided by polyarchy is more important than the second review provided by hierarchy.

Next, note from Proposition 5 that an improvement in the portfolio (i.e., an increase in a) leads to larger acceptance probabilities. Our earlier analysis, on the other hand, showed that a larger a increases the range of p_1 and p_2 within which polyarchy dominates a hierarchy. We therefore ascertain conditions under which, nonetheless, it can be established that polyarchy performs better than a hierarchy. For this, the internal optimum in a hierarchy, (11), is restated in the present case (where the initial portfolio consists of only two types of projects) as

$$ap_1 p_{1R} = p_2 p_{2R}. \tag{14}$$

Now define $k \equiv [m(R^H - z_1)/M(R^H - z_1)]/[m(R^H + z_2)/M(R^H + z_2)]$. Using (4), then, the following can easily be established.

PROPOSITION 7: *If $k \leq 1$, polyarchy performs better than hierarchy.*

To see what is entailed, consider the case where $m(\theta)$ is unimodal and a is large, so that reservation levels are low, sufficiently low that $R + z_2$ and $R - z_1$ are below the mode. Then $k < 1$ provided that for $R - z_1 < \theta < R + z_2$, $m(\theta)/M(\theta)$ is increasing in θ (i.e., $m_\theta > m^2/M$). Analogous results can be derived illustrating conditions under which at polyarchy's optimally chosen screening probabilities, hierarchy outperforms polyarchy.

(b) We can derive an alternative, somewhat weaker set of sufficient conditions under which one or the other system performs better by taking into account the fact that screening is tighter in a polyarchy. Let a polyarchy choose its reservation level such that the (expected) number of projects it undertakes is the same as that chosen optimally by a hierarchy. This is clearly not optimal, but if we can show that with this nonoptimal reservation level, polyarchy performs better than hierarchy, then a coordinated polyarchy will surely perform better with reservation levels optimally chosen.

If R^T denotes the reservation level at which a polyarchy chooses the same number of projects as that chosen by a hierarchy using reservation level R^H, then $R^T > R^H$. For brevity, we also define $m^P(x) \equiv m(R^T - x)$, and $m^H(x) \equiv m(R^H - x)$; M^P and M^H are defined analogously. Now, the above polyarchy performs better than hierarchy if the aggregate screening function is more discriminating for the former; that is, if $f_x^P > f_x^H$ in the relevant range[8] (equivalently, if $m^P M^P > m^H(1 - M^H)$). This will be true if the screening is very tight (in which case M^H is close to one), provided only that the difference between m^P and m^H is not too large, which, in turn, will be true provided m_θ is not too large. Moreover, with very tight screening, $R^P \approx R^C$ (i.e., the externality effect becomes negligible). It follows therefore that (under the conditions stated above) a hierarchy is outperformed by a coordinated as well as an uncoordinated polyarchy.

Much weaker conditions are required to establish the above result if R^T is not much larger than R^H. A sufficient condition in this case is that the screening is moderately tight, that is, a screen's probability of accepting a project is less than one-half.[9]

(c) We have investigated in detail the case of uniform distributions of errors, with mean zero, for the case of symmetric projects ($z_1 = z_2$). In this case: *A polyarchy has a*

[8] The probability that a project is accepted by a polyarchy is $f^P(R^T, x) = 1 - M^2(R^T - x)$, where $x = z_1$ and $-z_2$ for good and bad projects, respectively. The corresponding probability in a hierarchy is $f^H(R^H, x) = \{1 - M(R^H - x)\}^2$. With the same number of projects being accepted in the two systems, polyarchy performs better if $f^P(R^T, z_1) > f^H(R^H, z_1)$, or, if $f^P(R^T, -z_2) < (f^H, -z_2)$. An equivalent condition is $f^P(R^T, z_1) - f^P(R^T, -z_2) > f^H(R^H, z_1) - f^H(R^H, -z_2)$, which is satisfied if $f_x^P > f_x^H$ in the relevant range.

[9] This is because if $R^T \approx R^H$ then the required inequality (see fn. 8), $f^P(R^T, z_1) - f^P(R^T, -z_2) > f^H(R^H, z_1) - f^H(R^H, -z_2)$, is satisfied provided M is larger than $1/2$, within the range $R^H - z_1$ to $R^T + z_2$.

higher (lower) profit than a hierarchy if the proportion of good projects in the initial portfolio is less (more) than one-half. Obviously, if one hypothesizes that unprofitable ideas typically outnumber the profitable ones in a portfolio, then the present example suggests that a polyarchy is a superior institutional arrangement.

Remarks:

(*i*) The assumption of limited communication plays a critical role in our analysis. We assumed that firms in a polyarchy cannot (do not) communicate at all, and the bureaus in a hierarchy communicate only binary signals (whether they think a project is good or bad); they cannot communicate their actual information concerning the characteristics of the projects. We believe that although the extent of information sharing varies under different circumstances, it is seldom perfect, and our model has been constructed to capture the consequences of this.

(*ii*) The architecture of the economic system itself conveys some information to its constituents, which they use in setting decision rules. For example, in our analysis of polyarchy, in which the firms do not share any information with one another, each firm knows that some of the projects it receives are those rejected by the other firm and, consequently, the portfolio of projects faced by a firm is not an exact replica of the initial portfolio, but has been modified by the other firm. This implicit information is partly used in determing optimal reservation levels.

IV. Extensions

The basic components of our model are the screening function, the distribution of available projects, and the system's architecture, with its associated decision rule. Each of these components can be generalized (as we have partly done in our referenced papers). In the preceding section, we endogenized the screening function, given the available information. By allocating more resources to information acquisition, more informative signals can be obtained. The level of spending on information acquisition within each level of the hierarchy (by each firm within a polyarchy) must then be endogenously determined. In general, the levels of expenditure at each level of the hierarchy will not be the same: a higher quality screening at the lower bureau improves the portfolio to be evaluated by the higher bureau, but it also costs more because a larger number of projects are evaluated at the lower level. Similar issues arise in the assignment of individuals of differing abilities within a hierarchy (in the present paper all individuals have the same ability). The architecture of a system may also influence the mix of available projects because the likelihood of acceptance for projects of various types may well affect research incentives.

Alternative "architectures" that we have investigated include committees; that is, groups of individuals of different sizes who use particular decision rules (for instance, majority voting) to approve projects. (The decision-making units investigated in the present paper can be viewed as the limiting case of committees of one.) These polar architectures, in turn, can be viewed as building blocks for complex organizations and economies which are mixtures of hierarchies, polyarchies, and committees.

We can also investigate the consequences of alternative decision rules. We have followed the natural presumption that a project is not undertaken unless it is approved by the organization; within a hierarchy, by both bureaus; within a polyarchy, by at least one firm. We could, of course, *imagine* quite different organization of decision making: for instance, a hierarchy in which all projects are accepted except those which get vetoed by both bureaus and a polyarchy in which a project is accepted unless vetoed by one of the units.

We refer to organizations operating according to the veto rule as a veto hierarchy and polyarchy, in contrast to those we have analyzed earlier, which we refer to as an acceptance hierarchy and polyarchy. It can be easily shown that, in the absence of costs of coordination: *An acceptance polyarchy (hierarchy) is equivalent to a veto hierarchy (polyarchy)*.

Note, however, that the coordination requirements may be markedly different depending on the architecture and the nature of decision rules. For instance, an acceptance polyarchy does not require any informational coordination among its constituent units; a firm does not need to inform other firms concerning the projects it has accepted or rejected. This is an important aspect of the independence of firms within market-like systems, which we stressed earlier. In comparison, in a veto polyarchy in which one unit can veto a project from being undertaken, each unit must inform other units which projects it has rejected. Similarly, in a veto hierarchy the lower bureau must send all projects to the higher bureau. If there are significant costs to informational coordination (significant noise in information transmission), then it is clear that an acceptance polyarchy may have an advantage over other organizational forms.

V. Applications and Conclusions

Our analysis has focused on alternative ways of structuring decision making that might be applied to any organization. The comparative statics propositions indicate the "objective" circumstances under which each form might be observed, if the choice of organizational form is being made explicitly or implicitly. Not only is such a comparative view relevant to corporate decision making, but also to decision making within the public sector: for instance, to on-going controversies over the organization of the military (three branches, more or less independent, or one unified armed force). Similarly, alternative political structures (with their systems of checks and balances) can be viewed as alternative architectures to balance the consequences of different types of human errors.

An important application of our approach, as we indicated in the beginning, is in the comparison of alternative ways of organizing economic systems, the polyarchical structure capturing certain central elements of market economies, the hierarchical structure those of the more centralized economies. The organization of decision making (and the corresponding errors, costs, and consequences) has played little or no role in some of the most important previous work in this area. There is clearly more at stake in the choice of an economic system than, for instance, a comparison of alternative algorithms for arriving at a once and for all allocation of society's resources, that was emphasized in the Lange-Lerner-Taylor claim of equivalence between price-guided socialist economies and market economies.

There are many aspects of the comparison of alternative systems with which we have not dealt adequately here, but which we believe can be incorporated into our framework, and which we hope to address in the future: for instance, the view that better incentive mechanisms can often be designed within decentralized economic systems (see Barry Nalebuff and Stiglitz, 1983), and the claim that centralized economic systems provide a better framework for dealing with externalities (which we have explicitly excluded from the analysis);[10] the view that a decentralized system's performance is less sensitive to the quality of the key decision makers; the view that natural selection mechanisms work more effectively in decentralized economics, and the view that decentralized structures provide greater stimulation for innovation.

Our analysis has, however, cast light on several other aspects of the debate concerning the relative merits of polyarchies vs. hierarchies: advocates of polyarchies point out that a good project has many opportunities of being accepted in their system, whereas critics contend that polyarchies fail to provide adequate checks against incompetent decision making. Critics of heirarchical structures claim that there are high costs to providing these checks; there are direct costs of additional evaluations, and there are indirect costs because good projects get rejected in the process of ensuring that bad projects do not get undertaken. Advocates of polyarchies point further to its virtues in

[10] In addition, we have assumed that only one firm can undertake a project. The inefficiencies which arise when many firms undertake similar projects within decentralized systems have been a source of criticism leveled at these systems.

economies of communication. There is a grain of truth in each of these views. In this paper, we have provided a framework within which one can assess the circumstances under which the grain of truth in one view is greater than that in the other.

REFERENCES

Lange, Oskar and Taylor, Fred M., *On the Economic Theory of Socialism*, New York: McGraw-Hill, 1964.

Nalebuff, Barry and Stiglitz, Joseph E., "Information, Competition and Markets," *American Economic Review Proceedings*, May 1983, *73*, 278–83.

Sah, Raaj Kumar and Stiglitz, Joseph E., (1985a) "The Architecture of Economic Systems: Hierarchies and Polyarchies," Economic Growth Center Discussion paper No. 476, Yale University, 1985.

______ **and** ______, (1985b) "Perpetuation, Self-Reproduction, and Evolution of Organizations: The Selection and Performance of Managers," paper presented at the 1985 World Congress of the Econometric Society, Cambridge, August 1985.

______ **and** ______, (1985c) "Economics of Committees," Economic Growth Center Discussion Paper No. 486, Yale University, 1985.