





## **MOTIVATION**



## **Key Characteristic of Networks**

Network effects captured one widely observed phenomenon of the behavior of agents in a network, especially in social network settings



## **Ideal Model for Implementation**

The majority of the analysis is based on the generic form of the utility function:  $u(s_{i,t}, x_{i,t}, S_t, S_{t-1}) = x_i[\lambda h(S_{t-1}) + (1-\lambda)h(S_t)]s_{i,t} - cs_{i,t}$  and the distribution G(x)



## **Intuitive Study Tool**

A graphical representation of the result we got in class will be a very intuitive study tool for students to better understand the theories on network effect





# **Infinite Agents**

Modelling a community with a large number of agents. For instance, a continuum I := [0,1] of agents



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## **Homogeneous Utility Function**

Each agent has the utility function:  $u(s_{i,t}, x_i, S_t, S_{t-1}) = [x_i[\lambda h(S_{t-1}) + (1-\lambda)h(S_t)] - c]s_{i,t}$ , where  $s_{i,t} \in \{0,1\}$  denotes the strategy of agent i at time t, and  $S_t$  denotes the fraction of population that choose s = 1



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## **Type Distribution**

Each agent has a type  $x_i$ , which describes his/her utility from taking action s = 1.  $x_i$  comes from the distribution G(x) (CDF)



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#### **Network Effect Function**

h(S) is an increasing function that captures positive network effect on agents



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#### **Other Constant Parameters**

c: cost of taking action s = 1

 $\lambda$ : weight of the network effect from the previous period

#### **NETWORK EFFECT RECAP – MAIN RESULT**

## **Fixed Point Equation**

Only agents that that will have positive utilities from taking action s = 1 will take this action. Recall that the utility function has the form:

$$u(s_{i,t}, x_i, S_t, S_{t-1}) = [x_i[\lambda h(S_{t-1}) + (1 - \lambda)h(S_t)] - c]s_{i,t}$$

Thus, only the agents with the following type will take action s = 1:

$$x_i > \frac{c}{\lambda h(S_{t-1}) + (1-\lambda)h(S_t)}$$

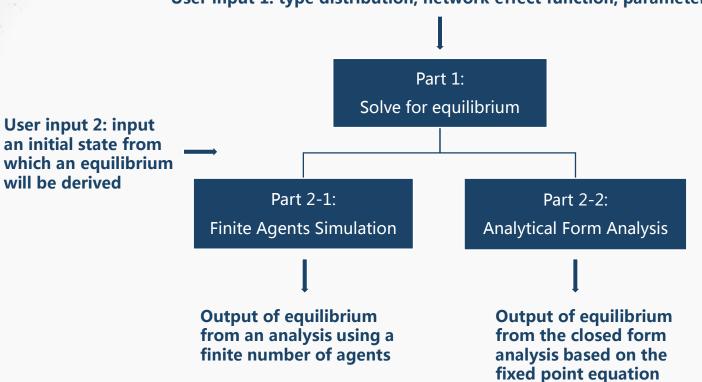
Thus, we have the fixed point equation for the fraction of population that take action s = 1:

$$S_t = 1 - G(\frac{c}{\lambda h(S_{t-1}) + (1 - \lambda)h(S_t)})$$



# **PROGRAM DEMO – STRUCTURE**

User input 1: type distribution, network effect function, parameters...



## **PROGRAM DEMO – ASSUMPTIONS**

## Type Distribution

$$G(x) = \frac{\gamma + \beta}{1 + \beta} \frac{(1 + \beta)e^{-\frac{\alpha'}{x}}}{\gamma + \beta e^{-\frac{\alpha'}{x}}}$$

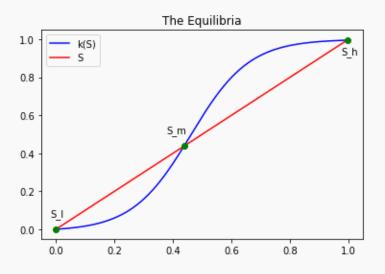


$$G^{-1}(U) = \frac{-\alpha'}{\log(\frac{\gamma U}{\gamma + \beta - \beta U})}$$

## **Network Effect Function**

$$h(S) = S$$

# **PROGRAM DEMO – ASSUMPTIONS**



# **Demo in Spyder**

